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SIMULATION STUDY OF A MULTI-STAGE RECTILINEAR CHANNEL FOR MUON COOLING

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Abstract

The muon collider has the potential to be a powerful tool for the exploration of frontiers in particle physics. In order to reach the high luminosity, the 6D emittance of the muon beam needs to be reduced by several orders of magnitude. Ionization cooling, which has recently been demonstrated in 4D by the Muon Ionization Cooling Experiment (MICE), is a promising cooling method for the muon beam. In the future, muon production and 6D ionization cooling experiments are planned at the High Intensity Accelerator Facility (HIAF) at the Institute of Modern Physics, Chinese Academy of Sciences (IMP, CAS). In this paper, a multi-stage rectilinear 6D ionization cooling channel is developed and the cooling simulation results using G4Beamline are presented, indicating good performance for muon beams with large emittance. This work serves as a good starting point for future research at HIAF.

INTRODUCITON

Electron (e+e-) colliders have many advantages over hadron colliders mainly because they produce much cleaner and simpler collision events, allowing physicists to analyze the resulting particles more easily. However, the multi-TeV collision energy is hard to achieve for (e+e-) colliders due to the small mass of the electron which will lead to the significant radiative energy loss. Muons have much larger mass compared with electrons which makes them almost not affected by the synchrotron radiation. Meanwhile, muons also have electron like nature, thus it seems wiser and more cost effective to choose muon colliders for high-energy physics study [1].

One technical challenge for the muon collider is that the muon beam emittance from the pion decay is too large which significantly exceeds the acceptance of the downstream accelerator parts and a dedicated cooling channel is needed to shrink the beam volume space [2]. A conceptual rectilinear cooling channel containing 12 stages has been designed during the MAP (muon accelerator program) project. Its basic idea is using stronger focus for the later stage to achieve better transverse cooling and tilting the solenoid coils to generate dispersion for longitudinal cooling. However, tilting the solenoid might have technical issues. So, using extra dipole magnet for dispersion generation might be a better choice. Here we present a multi-stage rectilinear cooling channel design with additional dipole magnets which shows a better cooling performance compared with the design of previous studies and this conceptual design of rectilinear cooling channel would be a preparation work for the muon cooling experiments at HIAF [3] in the future, as shown in Fig. 1.

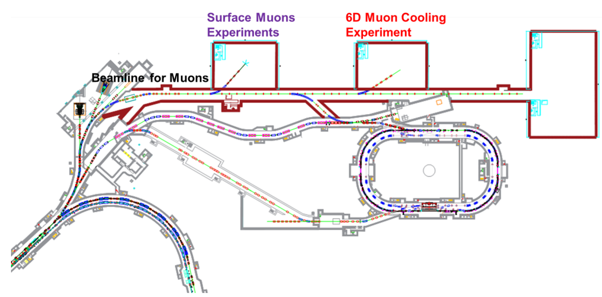

Figure 1: Muon Experiments Planned at HIAF.

DESCRIPTION OF RECTILINEAR COOLING LATTICE DESIGN

Formulas of Ionization Cooling

Ionization cooling involves muon beams losing both transverse and longitudinal momentum by the ionization of the atoms in the absorber material. The longitudinal momentum will get compensated in the RF cavities and the transverse momentum will not. Thus the momentum of the muons is more parallel and the emittance is reduced. The emittance evolution neglecting energy straggling is decribed as [4]:

$$\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_T(13.6\,\mathrm{MeV})^2}{2E_\mu m_\mu c^2 L_R} \quad (1)$$

where ϵ_n is the normalized transverse emittance, E_μ is the muon beam energy in GeV, m_μ is the muon mass, β is the muon particle velocity, c is the speed of light, β_T is the transverse beta value, dE_μ/ds is the energy loss per unit length and $L_R$is the radiation length of absorber material. The first part of this equation can be regarded as cooling term and the second heating term. The equilibrium transverse emittance is defined when dE_μ/ds in Eq.(1) is 0 [4]:

$$\epsilon_{n,eq} = \frac{\beta_T(13.6\,\mathrm{MeV})^2}{2\beta m_\mu c^2 L_R|\frac{dE_\mu}{ds}|}. \quad (2)$$

It can be easily seen from the Eq. (2) that, to reach a lower equilibrium transverse emittance, the focusing at the

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absorber should be tight (smaller transverse beta value) and the absorber material should have a large product of L_R and $|\frac{dE_\mu}{ds}|$ (e.g., liquid hydrogen and lithium hydride).

For the longitudinal ionization cooling, dispersion is needed at the absorber and a wedge-shaped absorber can be used. The dispersion will spread the beam transversely so that particles with higher momentum go through a thicker part of the absorber and lose more energy leading to the reduction of the longitudinal emittance.

Optics Design of Rectilinear Cooling Lattice

Transverse beta value, dynamic aperture and dispersion are three important factors in rectilinear cooling channel design. Transverse beta value represents the focusing strength of the solenoid and evolves as [5]:

$$2\beta_T\beta_T'' - \beta_T''^2 + 4\beta_T^2 k^2 - 4(1 + L^2) = 0 \tag{3}$$

where L is the normalized canonical angular momentum and k is the solenoid focusing strength,

$$k = \frac{qcB_z(z, r = 0)}{2p_z} \tag{4}$$

Assuming $L = 0$, one can solve Eq.(3) periodically to obtain the beta values in the rectilinear cooling channel. Phase advance is defined as:

$$\phi = \int \frac{1}{\beta_T} dz \tag{5}$$

Phase advance is closely related to the dynamical aperture and it must avoid the resonance which can cause serious particle loss.

The dispersion is important for the longitudinal cooling and can be obtained from the difference of closed orbit in different energies as:

$$D_x = \frac{\delta x}{\delta p} \tag{6}$$

where δx is the difference of the closed orbit relative to the reference particle and δp is the difference of the z-momentum relative to the reference particle.

Once the transverse beta value, dynamic aperture and dispersion are decided, then one can start the cooling simulation by choosing the suitable RF and wedge parameters.

Layout of Rectilinear Cooling Lattice

The basic lattice layout of one cell used in this paper is shown in Fig. 2 which closely follows Stratakis design [6] but using the additional dipole magnets for the longitudinal cooling as tilting the solenoids might have some technical issues.

The whole cooling channel consists of 5 stages and each stage includes repeated cells. The cell length will gradually reduce for stronger focusing at the wedge absorber in order to achieve smaller emittance. Main parameters of the cooling cell in each stage are listed in Table 1. Fringe field of dipole magnets is considered as well and for now a simple quadratic function is used to describe the fringe. The absorber material in all stages is liquid hydrogen (LH_2) and 100 µm Be safety window is used.

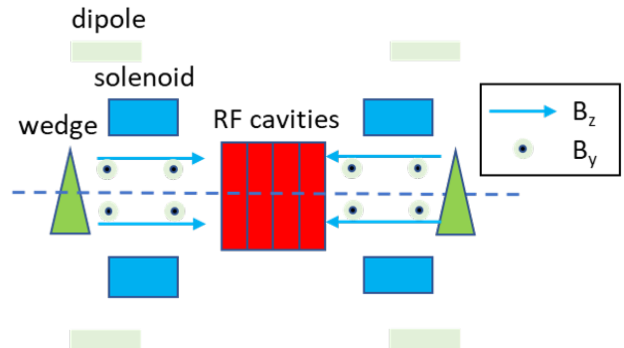

Figure 2: Layout of One Cooling Cell.

ANALYSIS OF SIMULATION RESULTS

As can be seen from Table 1, the transverse beta value drops gradually from 35 cm to 10 cm. The reason for choosing a large beta value at the early stage is to keep the dynamic aperture high. The on-axis length of the wedge absorber is always close to the beta value at the position of the wedge (3~5 cm longer). During the simulation, we also find that some high mean z-momentum (~220 MeV) of the beam will cause very bad cooling performance, so it's crucial to keep the mean z-momentum at a reasonable range (200~210 MeV) and this can be done by changing the length of the wedge absorber. The dispersion and wedge apex angle influnce the longitudinal cooling greatly and their values are decided empirically trying to reach a best balance between longitudinal cooling and transverse heating. Meanwhile, the RF settings also matter especially the RF phase which can not be too large otherwise instability in beam longitudinal motion will occur.

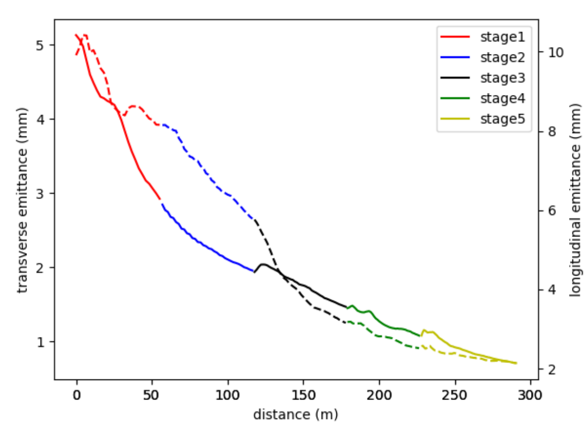

Figure 3: Emittance Evolution in the Cooling Channel (solid line: transverse emittance, dashed line: longitudinal emittance).

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Table 1: Parameters of Cooling Cells in Each Stage

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
Cell length (m)	2.3	1.8	1.4	1.1	0.8
$B_{z,max}$ (T)	3.1	4.1	4.8	6.2	8.8
β_T (cm)	35	30	20	15	10
B_y (T)	0.3	0.375	0.425	0.45	0.35
Dispersion (cm)	5	5	4.5	2.5	1.8
On-axis wedge length (cm)	37	32	24	20	12
Wedge apex angle	110°	120°	115°	110°	120°
RF frequency (MHz)	325	325	325	325	650
RF #	6	6	6	4	4
RF length (cm)	22	17.7	12	14.6	11.6
RF gradient (MV/m)	22	21.4	24.3	22.9	21.1
RF phase	27.7°	29.8°	27.9°	32°	28.3°

Similar to the previous study, we use the beam after bunch merging as the input instead of an idea Gaussian distribution. G4Beamline-3.08 [7] is adopted as the simulation tool and the reference particle is allowed to lose energy in the absorber and affected by the electromagnetic field so it travels with the whole beam. G4Beamline is based on Geant4 so it has all physics libraries of Geant4 and we choose the QGSP_BERT_EMX which includes all relevant process between the muons and the wedge material.

The emittance evolution and the initial and final beam distribution are plotted in Fig. 3 and Fig. 4. The emittance and transmission at the end of each stage are listed in Table 2. After passing through the rectilinear cooling channel which is about 290 m long, both the transverse and longitudinal emittance of initial muon beam get substantial reduction (transverse: 5.13 to 0.71 mm, longitudinal: 9.91 to 2.14 mm) with an overall transmission of 57.5 % including decay. There is a sudden rise in transverse and longitudinal emittance at the junction between two different stages which can be explained by the mismatching of the beta value and the rise can be limited by a soft reduction of beta value for each stage. Compared with the cooling performance at the end of stage 5 (transverse emittance: 0.68 mm, longitudinal emittance: 2.97 mm) from Ref. [6], we find the design of this paper has a smaller longitudinal emittance and a slightly larger transverse emittance and the length of the channel is reduced by 46 m.

Table 2: Emittance and Transmission at the End of Each Stage

	ϵ_T(mm)	ϵ_L(mm)	ϵ_{6D}(mm^3)	T(%)
Initial	5.13	9.91	260	
Stage 1	2.92	8.16	71.6	87.1
Stage 2	1.96	5.78	22.6	91.2
Stage 3	1.47	3.16	7.12	88
Stage 4	1.08	2.52	3.11	92.2
Stage 5	0.71	2.14	1.14	89.2

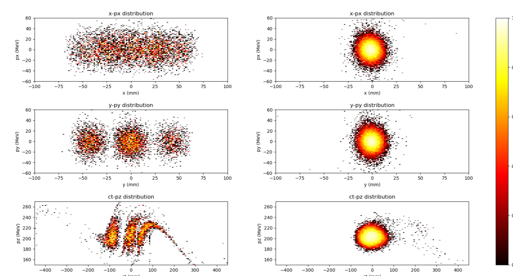

Figure 4: Beam Distribution at the Start and End of Cooling Channel (left column: start, right column: end).

CONCLUSION AND OUTLOOK

The 6D muon cooling scheme is essential for a muon collider. We have presented a 5-stage rectilinear cooling channel design with additional dipole magnets which shows a better performance than previous study. In the future, we will extend the cooling channel to more stages and check the limit of the cooling emittance. Meanwhile, we will explore the collective effects (e.g., space charge, short-range wakefield) in muon cooling by adding new modules in G4Beamline.

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