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INFLUENCES OF THE LONGITUDINAL SHIFT OF THE ELECTRON BUNCH TO THE LONGITUDINAL COOLING RATE*

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Abstract

As two major techniques of cooling a bunched hadron beam in a storage ring, both coherent electron cooling and rf-based traditional electron cooling involve overlapping the cooling electron bunches with the circulating ion bunch. A longitudinal offset of a cooling electron bunch with respect to the ion bunch centre is often introduced, either to cool a single ion bunch with multiple electron bunches or to cool the ions with large synchrotron amplitude more efficiently, i.e. painting. In this work, we derive how the cooling rate is affected by such a longitudinal offset. We use the EIC pre-cooler as an example to study how different overlapping pattern of the cooling electron bunches, e.g. the number of the cooling electron bunches and their longitudinal positions, affect the evolution of the circulating hadron bunches.

INTRODUCTION

Cooling hadrons with a bunched electron beam often involves overlapping one hadron bunch with multiple electron bunches so that the cooling rate can be increased. Examples for such cooling systems include the Low Energy RHIC electron Cooling (LEReC) system [1] and the precooler designed for the Electron Ion Collider (EIC) [2]. Due to the variation of the ions' longitudinal density and their synchrotron oscillations, electrons sitting at different location along the ion bunch have different contributions to the cooling process. It is important to evaluate how the cooling performance changes with the locations of cooling electron bunches with respect to the hadron bunch so that they can be optimized to achieve more efficient cooling. Another example for cooling the hadron bunch with the longitudinally shifted electron bunches is cooling with painting. Since the cooling rate is usually more efficient for hadrons with small synchrotron oscillation amplitudes, the longitudinal profile of the hadron bunch can deviate from Gaussian and a dense core may form after being cooled for some time, which may lead to single bunch instability and degradation of beam lifetime due to large space charge tune shift. One way to counteract the non-uniformity of the cooling rate is to modulate slowly the longitudinal locations of the electron bunches, i.e. painting. Evaluating the cooling rate in the presence of painting also requires calculation of how the cooling rate depends on the longitudinal offset of the electron bunch.

In this work, we derived an analytical formula to calculate the cooling rate as a function of the ion's synchrotron oscillation amplitude with the cooling electron bunch longitudinally shifted away from the ion bunch centre.

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AVERAGING THE COOLING RATE OVER SYNCHROTRON OSCILLATION

We consider a hadron circulating in a storage ring. Due to the energy kick in the RF cavities, the ion carries out synchrotron oscillation as shown in Fig. 1.

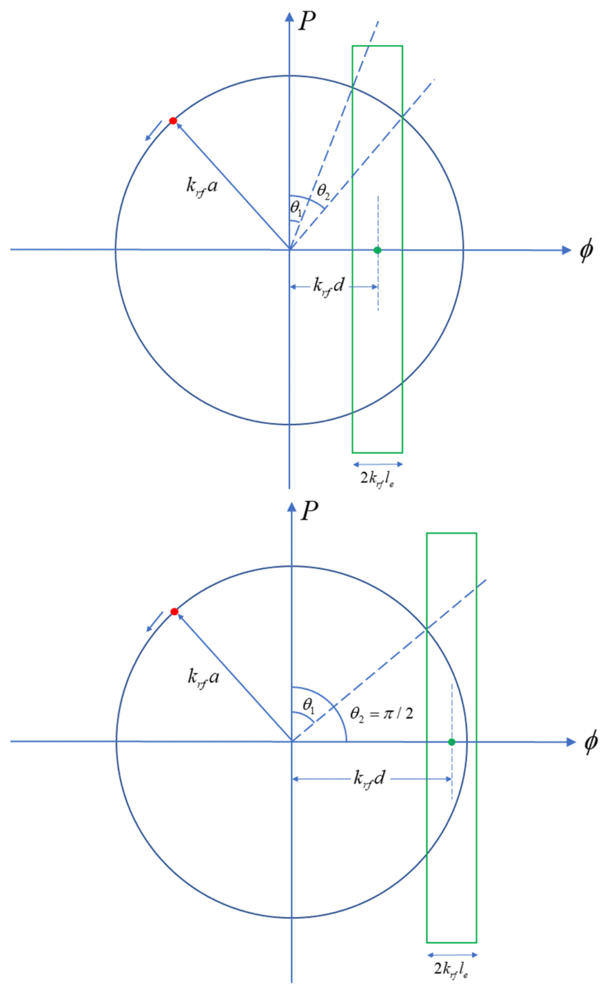

Figure 1: Illustration of the longitudinal cooling of an ion with synchrotron oscillation amplitude ϕrf_{max} and a cooling electron bunch with bunch length of $2l_e$ and offset of d. The red dot represents the ion and the green box represents the region in the ion's longitudinal phase space covered by the cooling electron bunch, i.e. the ion is overlapping with the electrons when it gets into the box. The green dot represents the centre of the electron bunch. The abscissa is the RF phase of the ion and the ordinate is the normalized energy deviation.

Figure 1 shows two cases of the ion conducting synchrotron oscillation in its phase space with the abscissa and

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ordinate axis defined as

$$P \equiv -h\frac{|\eta|}{\nu_s}\frac{\Delta p}{p}, \tag{1}$$

and

$$\phi \equiv \omega_{rf}\tau, \tag{2}$$

where h is the harmonic number of the RF cavity, η is the phase slipping factor, ν_s is the synchrotron tune, $\Delta p/p$ is the relative momentum deviation of the ion, ω_{rf} is the angular frequency of the RF cavity and τ is the arriving time of the ion. If we use the action-angle variables, (I,w), defined as

$$P = \sqrt{2I}\cos w \tag{3}$$

and

$$\phi = \sqrt{2I}\sin w \tag{4}$$

For every turn that an ion passing through the electrons, the reduction of its action is

$$\Delta I_c = \frac{1}{2}\Delta\left(P^2+\phi^2\right) = P\Delta P_c, \tag{5}$$

where

$$\Delta P_c = \frac{h|\eta|}{\nu_s\gamma}\Delta\delta\gamma_c, \tag{6}$$

and $\Delta\delta\gamma_c$ is the one turn energy kick due to cooling which is proportional to the energy deviation of the ion, $\delta\gamma = \gamma\Delta p/p$, through the following equation

$$\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \tag{7}$$

with ζ_0 being the local cooling rate. Eq. (7) can be rewritten as

$$\Delta P_c = -\zeta_0 T_{rev} P \tag{8}$$

and inserting Eq. (8) into Eq. (5) yields

$$\Delta I_c = -\zeta_0 T_{rev} P^2 = -2I\zeta_0 T_{rev}\cos^2 w. \tag{9}$$

For an ion with oscillation amplitude of

$$\phi_{\max} = \sqrt{2I} = \omega_{rf} a,$$

its cooling rate is obtained by averaging Eq. (9) over one synchrotron oscillation period, i.e.

$$\zeta(I) = -\frac{1}{I}\left\langle\frac{\Delta I_c}{T_{rev}}\right\rangle_{T_s} = \zeta_0\bar{\zeta}(I), \tag{10}$$

with

$$\bar{\zeta}(I) = \frac{4}{2\pi}\int_{\theta_1}^{\theta_2}\cos^2 w\,dw = \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(\cos(2w)+1\right)dw \tag{11}$$

$$= \frac{\theta_2-\theta_1}{\pi} + \frac{1}{2\pi}\cos(\theta_1+\theta_2)\sin(\theta_2-\theta_1)$$

$$\theta_1 = \arcsin\left(\frac{d-l_e}{a}\right), \tag{12}$$

and

$$\theta_2 = \arcsin\left(\frac{d+l_e}{a}\right). \tag{13}$$

As shown in Fig. 1 (bottom), if $d + l_e > a$, Eq. (10) and (11) are still valid if one take $\theta_2 = \pi/2$. Similarly, if $d - l_e > a$, one need to take $\theta_1 = \theta_2 = \pi/2$ and consequently, the cooling rate is zero since the ion with amplitude $\omega_{rf}a$ will never see the electrons. From the above analysis, we obtained the expression for the cooling rate to an ion with synchrotron oscillation amplitude of ϕrf_{max} and electron bunch with offset of $d > 0$ (it is obvious that the cooling rate for $d < 0$ is identical with that for the case of electrons with offset of $-d > 0$) and half bunch length of l_e, as the following:

$$\zeta(I) = \zeta_0\left\{\frac{\theta_2-\theta_1}{\pi} + \frac{1}{2\pi}\cos(\theta_1+\theta_2)\sin(\theta_2-\theta_1)\right\}, \tag{14}$$

with

$$\theta_1 = \begin{cases} \arcsin\left(\dfrac{d-l_e}{a}\right), & \text{for } a \ge |d-l_e| \\ \dfrac{\pi}{2}\operatorname{sgn}(d-l_e), & \text{for } a < |d-l_e| \end{cases}, \tag{15}$$

and

$$\theta_2 = \begin{cases} \arcsin\left(\dfrac{d+l_e}{a}\right), & \text{for } a \ge d+l_e \\ \dfrac{\pi}{2}, & \text{for } a < d+l_e \end{cases}. \tag{16}$$

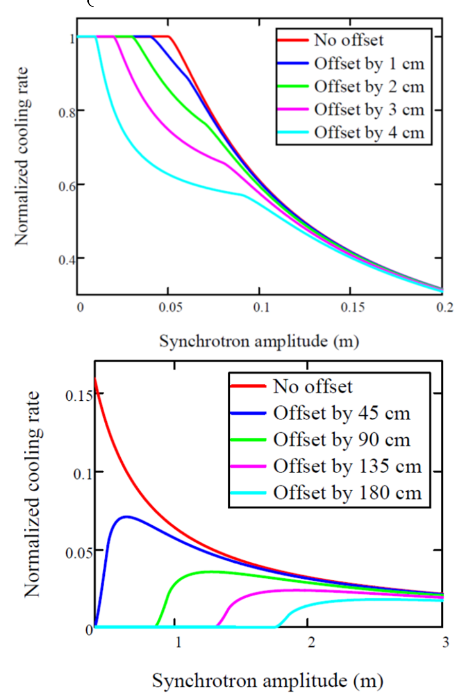

Figure 2: Normalized cooling rate for an electron bunch with various offsets. The abscissa is the synchrotron oscillation amplitude of the ion being cooled and the ordinate is the cooling rate as calculated from Eq. (14) in units of ζ_0, i.e. the local cooling rate as defined in Eq. (7). The half electron bunch length is $l_e = 5$ cm for these plots.

In the absence of the offset of the electron bunch, $d = 0$ and $\theta_1 = -\theta_2 = -arcsin(l_e/a)$, Eq. (14) reduces to the results derived in Ref. [3]. Figure 2 shows how the cooling rate depends on the synchrotron oscillation amplitude for various offsets of the electron bunch, with the half electron bunch length, $l_e = 5$ cm. It is worth noting that introducing

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a longitudinal offset of the electron bunch will not increase the cooling rate for the ions with large synchrotron oscillation amplitude and consequently, cooling with painting will decrease cooling rate for all the ions, as shown in Fig. 2.

AN EXAMPLE: PRE-COOLER FOR EIC

To achieve the initial proton beam parameters in the hadron storage ring (HSR), a precooler will be built to cool the proton beam at energy of $\gamma = 25.4$. The designed RMS bunch length for the cooling electrons is 5 cm and that for the proton beam is 70 cm. In the cooling section, a proton bunch overlaps with multiple electron bunches so that the desired cooling performance can be obtained. Since the ratio between the repetition frequency of the electron beam and that of the proton beam is not an integer, different proton bunch overlaps with the electron bunches differently in the cooling section. Two typical cases of the overlapping patterns are shown in Fig. 3, which corresponds to the proton bunch in the 0th bucket and that in the 80th bucket. The dependence of the cooling rate on the synchrotron oscillation amplitude is shown in Fig. 4, for a proton in the 0th bucket (red) and that in the 80th bucket (blue).

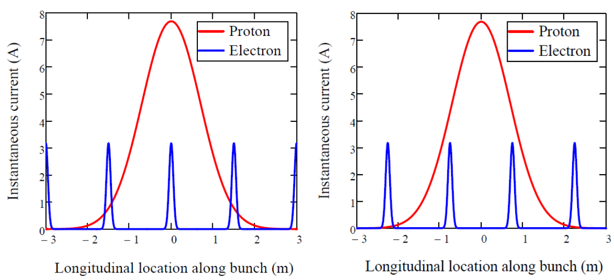

Figure 3: Overlapping of the electron bunches with the ion bunch in the 0th bucket (Left) and in the 80th bucket (Right). The electron bunches locate at $s = 0$ m, ± 1.5 m for the ion bunch in the 0th bucket (Left) and $s = \pm 0.75$ m, ± 2.25 m for the ion bunch in the 80th bucket (Right).

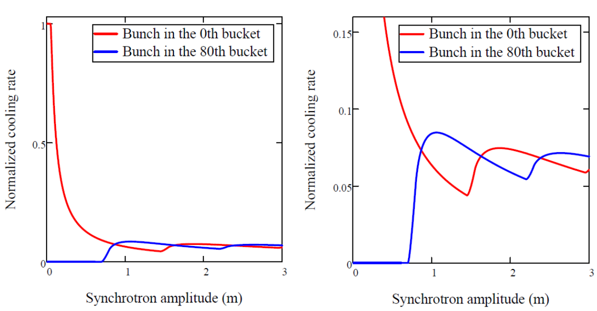

Figure 4: Normalized longitudinal cooling rate as a function of the synchrotron oscillation amplitude of an ion in the 0th bucket (red) and in the 80th bucket (blue) as shown in Fig. 3.

Figure 5 shows the profiles of the ion bunch in the 0th bucket (red), the 40th bucket (green) and the 80th bucket (blue) after 40 minutes of cooling, which suggests that the

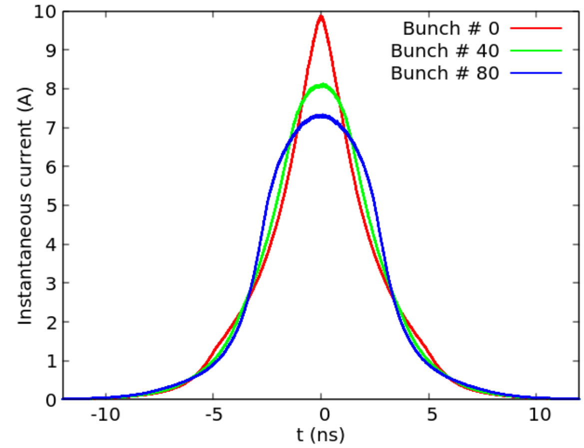

Figure 5: The proton bunch profiles after 40 minutes of cooling.

peak current of the ion bunches varies along the bunch train as a result of different overlapping patterns with the cooling electron bunches. One way to avoid the bunch-to-bunch variations is to paint the electron bunches slowly with respect to the ion bunches. If the painting is done uniformly with time, all proton bunches will be cooled identically and hence will have the same longitudinal profile.

SUMMARY

Assuming the cooling rate is independent of the location within the electron bunch, we derived an expression to calculate how the average cooling rate of an ion depends on its synchrotron oscillation amplitude when the cooling electron bunch is longitudinally shifted with respect to the ion bunch. We found that shifting the electron bunch away from the centre of the ion bunch will not increase the cooling rate for the ions with large synchrotron oscillation amplitude and hence painting will reduce the cooling rate for all ions in the bunch.

We have applied the results to study how the proton bunches in different rf bucket of the EIC will be cooled in the pre-cooler and found that their longitudinal profiles will be significantly different since every proton bunch in the train overlaps with the electron bunches differently. The variation of the proton bunch profile along the bunch train can be eliminated if a uniform painting of the cooling electron bunches is introduced.

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