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THEORETICAL AND SIMULATION STUDY OF DISPERSIVE ELECTRON COOLING *

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Abstract

In electron cooling, the transverse cooling rate is usually smaller than the longitudinal rate, especially at high energies. By introducing dispersive cooling, it is possible to redistribute the cooling rate between longitudinal and transverse planes. Theoretically, achieving dispersive electron cooling requires an ion dispersion and a transverse gradient of longitudinal cooling force. The latter depends on many factors such as beam energy offset, transverse displacement, e-beam density distribution and space charge effect. Therefore, several methods can be employed to achieve dispersive electron cooling based on these factors. In this paper, these factors and their respective impacts on the cooling rate are discussed. Based on a linear friction force model, we propose a simple formula to numerically estimate the cooling rate redistribution effect of these methods. The analytical results are in good agreement with Monte-Carlo calculation and numerical simulation.

INTRODUCTION

In theory, dispersive cooling requires both ion beam dispersion and a transverse gradient of the longitudinal cooling force [1]. To simply explain that, we assume an off-momentum particle passing through the cooling section with a dispersion function D, and only consider the longitudinal cooling with a linear cooling force $\Delta\delta_p = -\lambda\delta_p$, the particle coordinate after cooling can be written by

$$x_{\beta 2} = x - D\delta_{p2} = x_{\beta 1} + D\lambda\delta_{p1}, \qquad (1)$$

where x_β denotes the betatron oscillation, and x is the real coordinate which is assumed to be unchanged during passing through the cooling section. If the cooling coefficient λ is a constant, the amplitude of the betatron oscillation keeps unchanged $x_{\beta 2} = x_{\beta 1}$, which means that there is no cooling contribution from the longitudinal direction to the transverse. Otherwise, if the cooling force has a transverse gradient, for example $\lambda(x) = (M - |x|)\lambda_0$ with $M > Max[x]$, the amplitude of the oscillation turns to $x_{\beta 2} \simeq (1 - \lambda_0 |D\delta_{p1}|)x_{\beta 1}$. It indicates amplitude damping of the betatron motion. A schematic plot of these two processes is shown in Fig. 1, where the x-axis represents betatron oscillation under longitudinal cooling of the y-axis. It clearly shows the amplitude damping process with an appropriate longitudinal cooling force setting, i.e. dispersive cooling.

We see that the transverse gradient of the longitudinal cooling force plays a key role in dispersive electron cooling. Some experimental and simulation studies have demonstrated several approaches to obtain this gradient. As indicated in Ref. [2, 3], one approach is to introduce a displacement between electron and ion beams, utilizing the parabolic velocity profile of the electrons caused by its space charge. Another method is by using an energy offset, a displacement and a transverse density gradient of the e-beam [4]. Recently, it has been demonstrated that an e-beam with Gaussian transverse distribution can naturally provide this transverse gradient, thus achieving dispersive cooling [5]. At the same time, it shows that electron dispersion is also beneficial to dispersive cooling. In this article, we conduct theoretical and simulation studies of these methods and show how they affect the cooling rate. Based on the linear cooling force model, we finally propose a simple formula to numerically estimate the cooling rate redistribution effect of these methods, and the analytical result agrees well with Monte-Carlo calculation and numerical simulation.

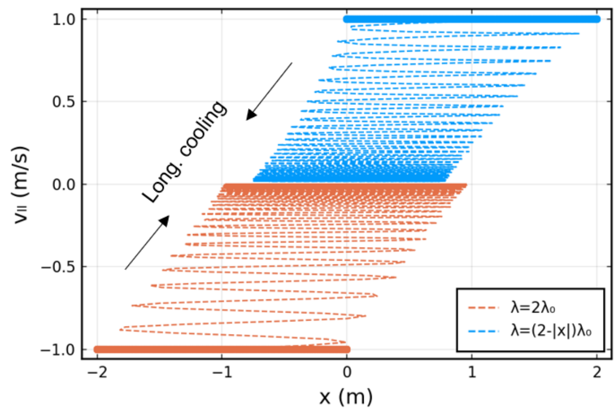

Figure 1: Comparison of the cooling process with two sets of longitudinal cooling force. It demonstrates that a dispersion and a transverse gradient of the longitudinal cooling force are necessary for dispersive cooling ($x_\beta = 1$ m, $D = 1$ m, $\delta_p = \pm 1$, $\lambda_0 = 0.01$).

DISPERSIVE ELECTRON COOLING

To begin with, we assume a linear cooling force $\Delta u = -Cn_e u$ both in transverse and longitudinal directions, where n_e is the electron beam density, C is the cooling coefficient which depends on the velocity distribution of the electron beam. Consider a beam displacement x_o, an energy offset δ_o, and horizontal dispersion D of ions in the cooling section, the momentum change of a single particle after cooling can be described by $\Delta\delta \simeq -C_p n_e(\delta - \delta_e - \delta_o)$, where $\delta_e = K_{sc}(x^2 + y_\beta^2)$ is the electron momentum deviation due

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to space charge, and $x = x_\beta + x_o + D\delta$. Then we have

$$\Delta\delta^2 \simeq -2C_p n_e \delta^2 + 2C_p n_e \delta\delta_o + 2C_p n_e K_{sc}\delta(x^2 + y_\beta^2). \tag{2}$$

In the transverse direction, we only discuss the horizontal direction. Assume $\alpha = 0$ and ignore the betatron evolution in the cooling section, the single particle emittance is $\epsilon_x = (x - x_o - D\delta)^2/2\beta_x + \beta_x x'^2/2$, and the cooling effect can be written by

$$\Delta\epsilon_x \simeq -Dx_\beta \Delta\delta/\beta_x + \beta_x x' \Delta x', \tag{3}$$

where $\Delta x' = -C_x n_e (x' - x_e')$, $x_e' = L_{sc}\sqrt{x^2 + y_\beta^2}$ is the drift velocity caused by the space charge and magnetic fields.

Expanding Eqs. (2)–(3) and ignoring high-order and non-correlated terms, the longitudinal and horizontal cooling effects of the ion beam can be described as

$$\begin{aligned}\langle\Delta\delta^2\rangle &= -2C_p\langle n_e\delta^2\rangle + 2C_p\delta_o\langle n_e\delta\rangle \\ &\quad + 2C_pK_{sc}x_o\left(x_o\langle n_e\delta\rangle + 2\langle n_e x_\beta\delta\rangle + 2D\langle n_e\delta^2\rangle\right) \\ \langle\Delta\epsilon\rangle &= -C_x\epsilon_0\langle n_e\rangle + \frac{C_pD}{\beta_x}\langle n_e x_\beta\delta\rangle - \frac{C_pD\delta_o}{\beta_x}\langle n_e x_\beta\rangle \\ &\quad - \frac{C_pDK_{sc}x_o}{\beta_x}\left(x_o\langle n_e x_\beta\rangle + 2\langle n_e x_\beta^2\rangle + 2D\langle n_e x_\beta\delta\rangle\right),\end{aligned} \tag{4}$$

where $\langle\rangle$ denotes averaging over the ion beam phase space, $n_e(x, y, s)$ is the local density of the e-beam which depends on the position of ion particles. The equations show that energy offset, beam displacement, space charge effect as well as e-beam density are the main factors affecting the cooling rate, and that the coupling between position, momentum, and e-beam density caused by dispersion is the reason for this. The term related to drift velocity is eliminated as it has no correlation with longitudinal cooling. Based on the ion and electron beam distributions, Eq. (4) can be calculated analytically according to the law of the unconscious statistician (LOTUS) [6]. As shown in Ref. [5], the cooling rate redistribution for the e-beam with a Gaussian profile is studied. In this section, we will discuss the effect of rate redistribution for both Gaussian and uniform electron beams, while including all the mentioned factors.

We define the gain factor as the ratio of the two cooling rates with and without dispersion and other factors $k = \lambda/\lambda_0$, where $\lambda_p = \langle\Delta\delta^2\rangle/\delta_p^2$, $\lambda_x = \langle\Delta\epsilon\rangle/\epsilon_0$ are the longitudinal and horizontal cooling rates, and δ_p, ϵ_0 are the rms momentum spread and emittance, respectively. Also, we always assume a DC e-beam, so that only transverse distribution is discussed. Using the same method in Ref. [5], several cases that can realize dispersive electron cooling are studied based on the e-beams with transverse Gaussian and uniform distributions. Moreover, a Monte-Carlo calculation based on Eq. (4) is performed and compared with the analytical formula.

Case 1: Gaussian E-Beam with Energy Offset Δ_o and Beam Displacement X_o

As Y. S. Derbenev introduced in Ref. [1, 4], dispersive electron cooling can be achieved by a longitudinal velocity offset, a beam displacement as well as a transverse gradient of electron density. For this case, we use a Gaussian e-beam to produce the transverse density gradient

$$n_e = n_{e0}exp[-\frac{(x_\beta + x_o + D\delta)^2}{2\sigma_{ex}^2} - \frac{y_\beta^2}{2\sigma_{ey}^2} - \frac{s^2}{2\sigma_{es}^2}]. \tag{5}$$

Furthermore, we assume that the ion beam also has a Gaussian distribution in the transverse direction. Then, Eq. (4) can be calculated and the final result of the gain factor is

$$\begin{aligned} a &= \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2} \\ b &= \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2 + D^2\delta_p^2} \\ k_p &= e^{-\frac{x_o^2}{2b^2}}[\frac{a^3}{b^3} + \frac{a}{b^5}D^2\delta_p^2x_o^2 + \frac{a}{b^3}D\delta_o x_o] \\ k_x &= e^{-\frac{x_o^2}{2b^2}}[\frac{a}{b} + \frac{C_pa}{C_xb^5}D^2\delta_p^2(b^2 - x_o^2) - \frac{C_pa}{C_xb^3}D\delta_o x_o]. \end{aligned} \tag{6}$$

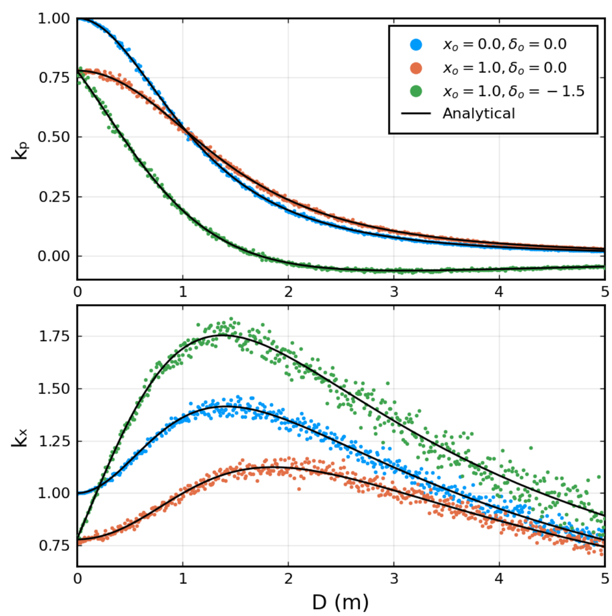

Figure 2: Monte-Carlo and analytical results of the gain factor dependence on dispersion for Case 1 with $\sigma_{ex} = 1$ m, $\sigma_{ix} = 1$ m, $\delta_p = 1$, $C_p/C_x = 2$.

Using arbitrary parameters, the dependence of the gain factors on the dispersion function under different conditions is calculated and shown in Fig. 2. The Monte-Carlo results show a good agreement with the analytical formula. We see that using dispersion alone can realize dispersive cooling and a factor of 1.4 is achieved for the horizontal cooling rate. This effect is mainly due to the Gaussian e-beam distribution, which naturally provides the transverse gradient of the longitudinal force. The details of the explanation can be found in Ref. [5]. When beam displacement is applied, it shows that the horizontal gain factor drops off and the maximum value of k_x decreases to 1.1. This is due to the fact that the displacement reduces the average cooling force on the

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ion beam, which is density-dependent, thereby weakening the dispersive cooling effect as shown by the second term in the bracket of Eq. (6). Meanwhile, the third term in the bracket depends on both energy offset and beam displacement. The increase and decrease of the horizontal cooling rate can be adjusted by the product of the two values. This conclusion agrees with Ref. [1,4]. As shown in Fig 2, the maximum value of k_x can reach 1.75 by using $x_o = 1.0$ m and $\delta_o = -1.5$, even though the beam displacement introduces a certain degradation of the horizontal cooling rate.

However, an energy mismatch between electron and ion beams may result in a circular attractor in the longitudinal phase space of the ion bunch. If the relative shift exceeds a critical value, beam heating instead of cooling may occur [7]. Therefore, the method using energy offset to realize dispersive cooling needs to be carefully calculated and evaluated in practical applications.

Case 2: Gaussian E-Beam with Space Charge K_{sc} and Beam Displacement X_o

Another method to realize dispersive cooling relies on the space charge effect of the e-beam, which in combination with a beam displacement can generate the transverse gradient of the longitudinal force [2,3]. The radial-dependant velocity deviation in the longitudinal direction will be produced due to the space charge effect. Here we assume a parabolic velocity profile, the final result of the two gain factors is

$$\begin{aligned} a &= \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2} \\ b &= \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2 + D^2\delta_p^2} \\ c &= \sqrt{\sigma_{ex}^2 - \sigma_{ix}^2 - D^2\delta_p^2} \\ k_p &= e^{-\frac{x_o^2}{2b^2}}\left[\frac{a^3}{b^3} + \frac{a}{b^5}D^2\delta_p^2x_o^2\right. \\ &\qquad \left. - \frac{a}{b^5}DK_{sc}x_o(2\sigma_{ex}^2b^2 - x_o^2c^2)\right] \\ k_x &= e^{-\frac{x_o^2}{2b^2}}\left[\frac{a}{b} + \frac{C_pa}{C_xb^5}D^2\delta_p^2(b^2 - x_o^2)\right. \\ &\qquad \left. + \frac{C_pa}{C_xb^5}DK_{sc}x_o(2\sigma_{ex}^2b^2 - x_o^2c^2)\right]. \end{aligned} \tag{7}$$

It shows that the first and second terms in the bracket come from the Gaussian e-beam distribution, which has already been discussed above. The third term is of interest to us, and it is directly determined by the e-beam space charge and beam displacement. We see that the sign of this term also depends on the electron and ion beam parameters. Considering arbitrary parameters, a comparison between the Monte-Carlo calculation and analytical formula is shown in Fig. 3. It is clear that beam displacement and the space charge effect contribute to dispersive electron cooling. As discussed in Ref. [2,3], an outward displacement of the e-beam is required for the increase of the horizontal cooling rate, which is consistent with our result. The equation also shows that a larger K_{sc} can improve the rate redistribution

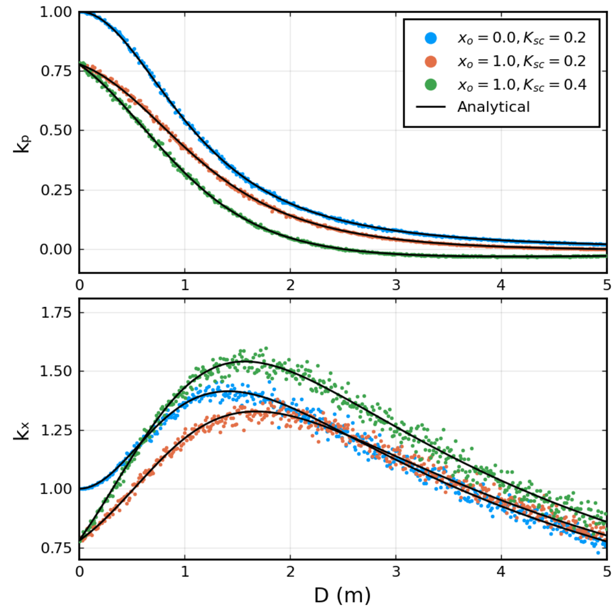

Figure 3: Monte-Carlo and analytical results of the gain factor dependence on dispersion for Case 2 with $\sigma_{ex} = 1$ m, $\sigma_{ix} = 1$ m, $\delta_p = 1$, $C_p/C_x = 2$.

effect. However, a strong space charge field is not desirable for cooling, since the transverse drift velocity and the longitudinal velocity deviation have a significant influence on the cooling process. Therefore, when studying dispersive electron cooling, the value of K_{sc} or the e-beam density should be carefully determined according to the beam energy and cooling requirements.

Case 3: Uniform E-Beam with Infinite Radius R_e, Space Charge K_{sc}, and Beam Displacement X_o

In the above, we investigated two methods that can be used to realize dispersive cooling for a Gaussian e-beam. In addition to the energy offset and space charge effect, we see that the e-beam itself also contributes to the rate redistribution effect, since the Gaussian density distribution naturally provides a transverse gradient of the longitudinal force. In fact, a uniform or hollow e-beam is much preferable for most electron coolers to avoid beam losses due to recombination, overcooling and instabilities [8,9]. Therefore, it is necessary to study the rate redistribution effect for these two e-beams. Here we only discuss the uniform e-beam.

We assume a uniform e-beam with an infinite radius that is $n_e \equiv$ constant. Since there is no density gradient, the energy offset does not affect the dispersion cooling as discussed in Case 1, and the density distribution will not provide the transverse gradient of the cooling force. So, using the space charge effect of the e-beam is the only useful approach. In this case, the gain factors can be easily calculated

$$\begin{aligned} k_p &= 1 - 2DK_{sc}x_o \\ k_x &= 1 + 2DK_{sc}x_oC_p/C_x. \end{aligned} \tag{8}$$

It shows the same conclusion that dispersive electron cooling

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can be achieved by the velocity deviation caused by the e-beam space charge, combined with a beam displacement.

Case 4: Uniform E-Beam with Finite Radius R_e, Space Charge K_{sc}, and Beam Displacement X_o

For a uniform e-beam with a finite radius R_e, the density distribution is

$$n_e(r) = \begin{cases} \text{constant}, & r \leqslant R_e \\ 0, & r > R_e. \end{cases} \tag{9}$$

For this distribution, we can say that there is a density gradient which is created by the density difference between the inside and outside of the e-beam. Due to the betatron motion, particles with large amplitude will cross the boundary of the e-beam back and forth, thereby creating a transverse gradient of the longitudinal cooling force. As a result, a uniform e-beam itself can also be applied to achieve dispersive cooling as well as a Gaussian e-beam.

For simplicity, we only consider the space charge effect $\delta_e(r) = K_{sc}r^2$ and beam displacement. Based on the uniform density distribution (Eq. (9)), the analytical result of the gain factors is calculated as below

$$\begin{aligned}
m &= Erf\left[\frac{R_e}{\sqrt{2\sigma_{ix}^2}}\right] \\
n &= \sqrt{\sigma_{ix}^2 + D^2\delta_p^2} \\
a &= Erf\left[\frac{R_e + x_o}{\sqrt{2}n}\right] + Erf\left[\frac{R_e - x_o}{\sqrt{2}n}\right] \\
b &= \frac{e^{-\frac{(R_e - x_o)^2}{2n^2}}(R_e - x_o) + e^{-\frac{(R_e + x_o)^2}{2n^2}}(R_e + x_o)}{n^3} \\
c &= \frac{e^{-\frac{(R_e + x_o)^2}{2n^2}} - e^{-\frac{(R_e - x_o)^2}{2n^2}}}{n} \\
k_p &= \frac{a}{2m} - \frac{D^2\delta_p^2 b}{\sqrt{2\pi}m} + \frac{DK_{sc}x_o}{\sqrt{2\pi}m}(2n^2 b - \sqrt{2\pi}a - x_o c) \\
k_x &= \frac{a}{2m} + \frac{C_p}{C_x}\left[\frac{D^2\delta_p^2 b}{\sqrt{2\pi}m} - \frac{DK_{sc}x_o}{\sqrt{2\pi}m}(2n^2 b - \sqrt{2\pi}a - x_o c)\right].
\end{aligned} \tag{10}$$

It shows that the first and second terms of the gain factor are due to the e-beam distribution, and the third term comes from the space charge effect. A comparison of different settings is shown in Fig. 4, and the Monte-Carlo results agree well with the analytical formula. We see that the rate redistribution effect strongly depends on the e-beam radius, since it directly determines how many particles can see the density gradient. If the e-beam radius is smaller than the ion beam, it is easy to realize dispersive cooling with small dispersion. Otherwise, dispersive cooling is less likely to occur unless the dispersion is large enough.

SUMMARY AND DISCUSSION

In electron cooling, transverse cooling is usually weaker than the longitudinal direction. For this reason, dispersive

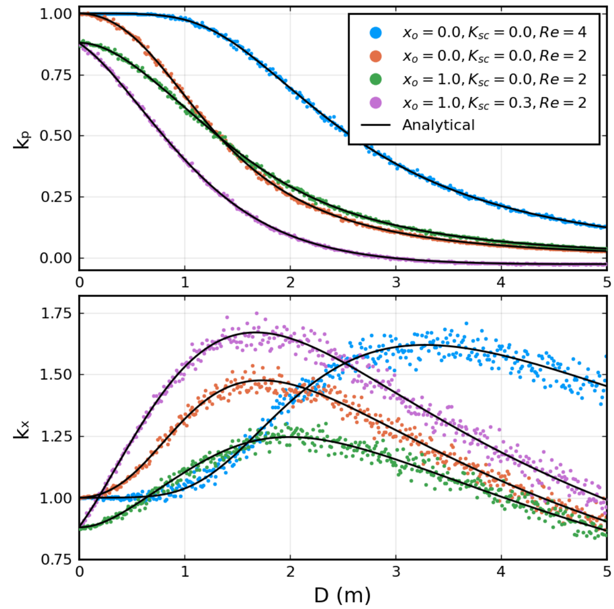

Figure 4: Monte-Carlo and analytical results of the gain factor dependence on dispersion for Case 4 with $\sigma_{ix} = 1$ m, $\delta_p = 1$, $C_p/C_x = 2$.

electron cooling is an effective scheme to redistribute the cooling rate, especially for future high-energy coolers. In this paper, we investigated several approaches that can be applied to achieve dispersive electron cooling. It is demonstrated that beam energy offset, transverse displacement, density distribution and space charge effect of e-beam all contribute to the rate redistribution in dispersive cooling. For the first time, we demonstrate that a transverse uniform e-beam with a finite radius can be applied in dispersive cooling. Based on a linear cooling force model, we present an analytical formula for numerically estimating the cooling rate redistribution effect. Moreover, a Monte-Carlo calculation and numerical simulation are also carried out, and all results show good agreement with the analytical model.

As previously discussed, the beam energy offset and displacement may affect the cooling performance and may induce some undesired effects such as circular attractor or even beam heating. So, these two approaches should be carefully calculated and evaluated in practice. Moreover, since the strong dependence of the space charge effect on beam energy, the method using the velocity deviation is only suitable for low-energy beam cooling, such as conventional electron cooling with electron energies below a few MeV. For high-energy beam cooling, such as EIC, where e-beam energy would reach tens or hundreds of MeV, the method employing beam density is much preferable. However, these factors in dispersive electron cooling have not been well explored through experiments. The influences of these factors on the cooling rate need to be further investigated. Additionally, the effects induced by the dispersion function, such as ion beam dynamics and IBS, require comprehensive exploration. It's also important to note that the dispersion function in accel-

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erators is quite limited. Therefore, considering all the above points, dispersive cooling needs further extensive research in the future.

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