Wiggler Enhanced Plasma-Cascade Amplifier for Coherent Electron Cooling

G. Stupakov and A. Zholents

COOL21

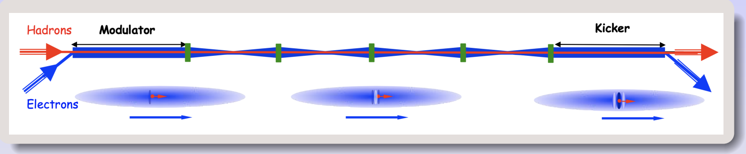
November 1-5, 2021

Introduction

Outline

- Introduction
- Plasma oscillations in a beam and parametric amplification
- Wiggler Enhanced Parametric Amplifier (WEPA)
- Frequency range and amplification
- Wigglers+chicanes
- Experiment at AWA facility at ANL
- Summary

PCA



In the plasma cascade amplifier[1] (PCA) the amplification occurs in an electron beam with periodically varying transverse size. This is a parametric instability. In principle, there is no need to separate the hadron and electron beam orbits.

This method is currently being investigated experimentally at RHIC at the electron beam energy 14.6 MeV.

[1] V. Litvinenko et al. arXiv:1802.08677, (2018); V. Litvinenko et al. PRAB 24, 014402 (2021).

Plasma oscillations in relativistic beams

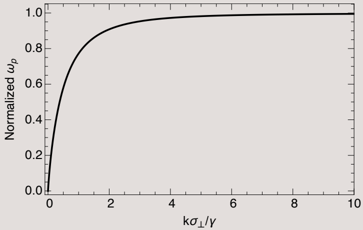
When a relativistic beam travels along a straight line, small-scale density perturbations, $\delta n(z)$, interact through the space charge forces and exhibit *plasma oscillations*. For the Fourier transform, $\delta\hat{n}_k = \int \delta n(z) e^{-ikz} dz$, the equation of plasma oscillations is

$$\frac{d^2 \delta\hat{n}_k}{ds^2} + \frac{\omega_p^2(s,k)}{c^2}\delta\hat{n}_k = 0$$

In the model of disks, assuming a round beam with the transverse size $\sigma_\perp$, $\omega_p = \Omega_p f(k\sigma_\perp/\gamma)$

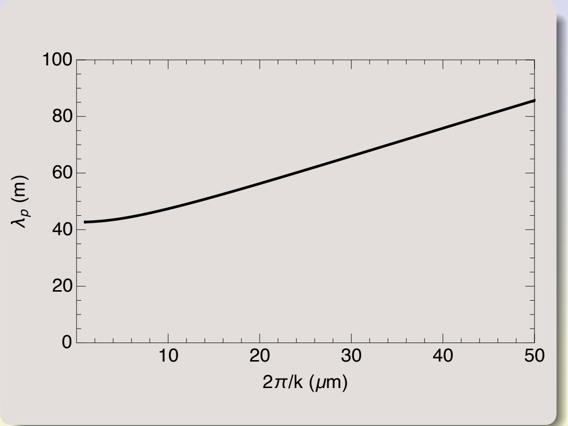
$$\Omega_p = \frac{c}{\sigma_\perp}\sqrt{\frac{I_e}{I_A \gamma^3}}$$

Note that the plasma frequency, $\omega_p \propto 1/\sigma_\perp$. In PCA, $\sigma_\perp$ is varied periodically with $s \to$ parametric resonance $\to$ instability $\to$ amplification of initial signal.



Numerical example

For $I_e = 100$ A, $\sigma_\perp = 100$ μm, $\gamma = 300$. Plotted is $\lambda_p = 2\pi c/\omega_p$ versus $2\pi/k$ (note that $2\pi/k = 10$ $\mu\mathrm{m}$ corresponds to the frequency 30 THz).



We would like to make λ_p shorter and also vary it along the beam line. This can be done with wigglers.

Plasma oscillations in wiggler. WEPA

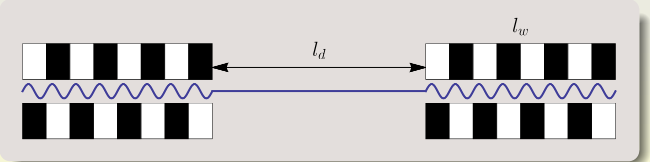
Inside a wiggler electrons travel with a smaller z-velocity v_z

$$\gamma_z = \frac{1}{\sqrt{1 - v_z^2/c^2}} = \frac{\gamma}{\sqrt{1 + K^2/2}}$$

$K = eB\lambda_w/2\pi mc^2$ is the wiggler parameter. The plasma frequency in the wiggler is larger than in the drift (plasma period is smaller)

$$\omega_p^{(w)} = \Omega_p^{(w)} f\left(\frac{k\sigma_\perp}{\gamma_z}\right), \qquad \Omega_p^{(w)} = \sqrt{1 + K^2/2}\,\frac{c}{\sigma_\perp \gamma}\sqrt{\frac{I_e}{I_A \gamma}}$$

Here we use a cold beam approximation.



We can have parametric instability in a wiggler-drift periodic system.

The hope is that the amplifier can be shorter. Wigglers also focus electrons in the transverse direction—a smaller transverse beam size.

WEPA matrix analysis

We analyzed such amplifier assuming $\omega_p = 0$ in the drift and $\omega_p^{(w)} > 0$ in the wiggler. We write the transformation matrices for $(\delta\hat{n}_k, (\gamma_z^2/\gamma^2)d\delta\hat{n}_k/ds)^T$ which is continuous through the transition from the wiggler to drift[2]. In the wiggler

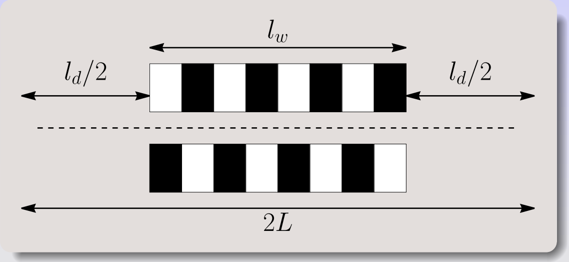
$$M_w(s, \omega_p^{(w)}) = \begin{pmatrix} \cos(\omega_p^{(w)} s/c) & \dfrac{\gamma^2 c}{\gamma_z^2 \omega_p^{(w)}} \sin(\omega_p^{(w)} s/c) \\ -\dfrac{\gamma_z^2 \omega_p^{(w)}}{\gamma^2 c} \sin(\omega_p^{(w)} s/c) & \cos(\omega_p^{(w)} s/c) \end{pmatrix}$$

In the drift

$$M_d(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

[2] This is because $(\gamma_z^2/\gamma^2)d\delta\hat{n}_k/ds \propto \Delta\hat{E}_k$—the energy perturbation.

WEPA cell

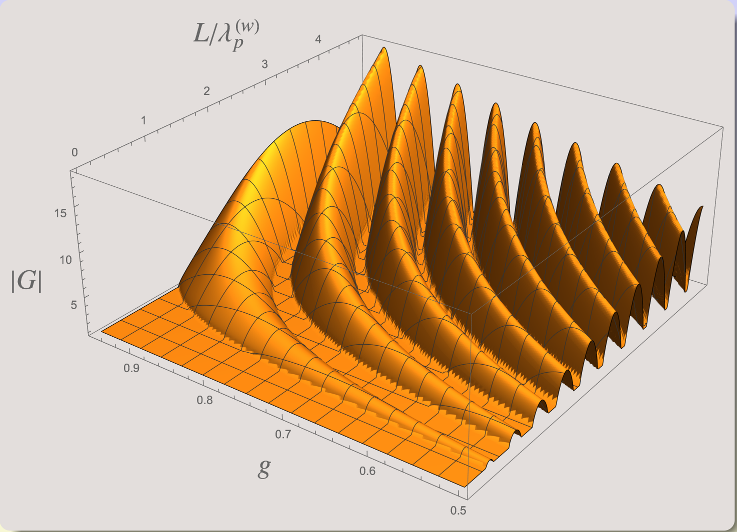


Define the length of the cell $2L = \ell_d + \ell_w$ and $g = \ell_d/2L$ then the total matrix is

$$M_{\text{cell}} = M_d(gL) \cdot M_w(2(1-g)L, k) \cdot M_d(gL)$$

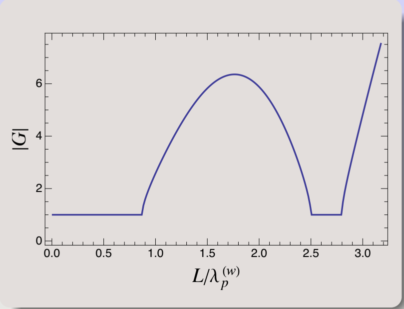
We calculate the eigenvalues G of this matrix. If $|G| > 1$ then this is an amplification per cell.

One cell gain for WEPA, $K = 2$

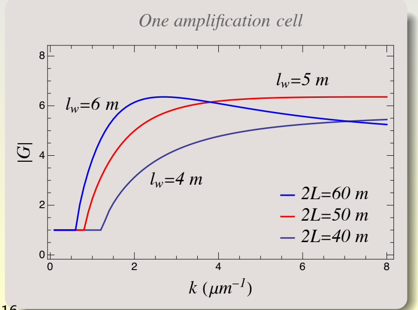


To increase gain we choose 90% drift, 10% wiggler length, $g = 0.9$.

WEPA amplifier



Amplification versus $L/\lambda_p^{(w)}$ for $g = 0.9$. We choose $\sigma_\perp = 70$ µm and $I_e = 150$ A.



With four cells, the amplification is $\sim 6^4 \approx 1300$ and it is broadband, but the cells are long (45 m drift + 5 m wiggler). They however can be made considerably shorter.

What is the role of the drifts in WEPA?

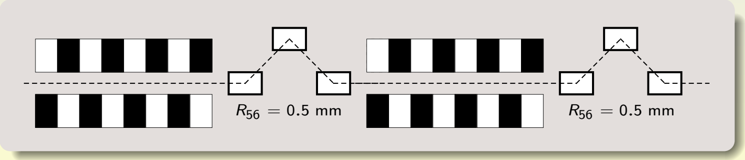
In the drift

$$M_d(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

Analysis shows that the drift provides the transport matrix element R_{56} that converts an energy perturbation in the plasma oscillation into the density perturbation:

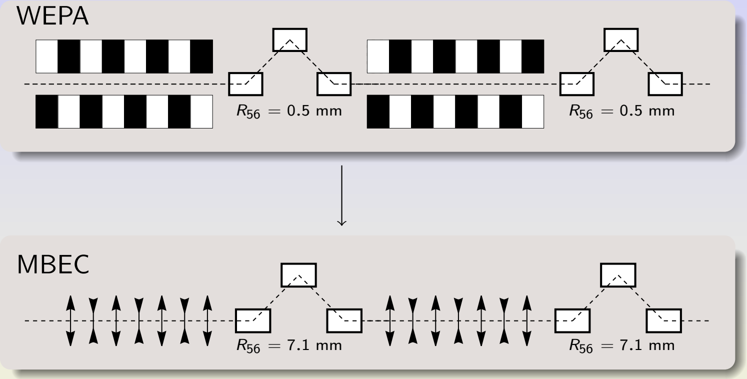
$$R_{56} = \frac{1}{\gamma^2} \times 45 \text{ m} = 0.52 \text{ mm}$$

We can replace 45 m drift by a short chicane with $R_{56} = 0.52$ mm with the length less then 1 m. Then the cell length is ~ 6 m.



Comment: connection between WEPA and MBEC

There is a similarity between WEPA to MBEC.



MBEC has relatively large R_{56} in chicanes. This, however, suppresses the gain at large values of k.

Landau damping and CSR wakefields

In our analysis we assumed a cold beam and hence ignored the Landau damping. We can estimate this effect as smearing of the density perturbations with wave number k due the slippage caused by the energy spread in the beam σ_η

$$\propto \exp\left[-\frac{1}{2}k^2 R_{56}^2 \sigma_\eta^2\right]$$

Assume $\sigma_\eta = 2 \times 10^{-4}$. This will suppress the amplification at $k \sim \sqrt{2}/R_{56}\sigma_\eta \approx 13.5\ \mu\mathrm{m}^{-1}$ (this corresponds to the frequency 640 THz).

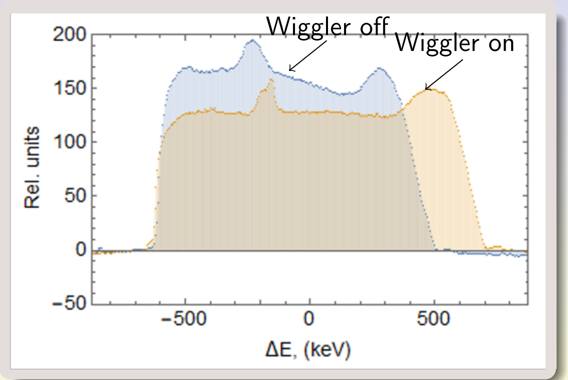
There are also CSR wakefields in the wiggler that add to the space charge forces. They should be taken into account in the dynamics of plasma oscillations (we plan to include the CSR wake in future analysis).

Experiment at Argonne Wakefield Accelerator (AWA) facility (S. Lee, E. McCarthy, M. Qian, E. Trakhtenberg, I. Vasserman, J. Xu, A. Zholents (ANL/APS); S. Doran, G. Ha, J. Power, J. Seok (ANL/AWA); A. Adelmann, A. Alba, R. Bellotti (PSI).)

A section of AWA beamline used in the experiment. A wiggler was assembled using spare magnetic structures and a new strong back frame and a new vacuum chamber. The wiggler length is 1 m, the period = 8.5 cm, K=10.8. Beam energy 45.4 MeV, $Q = 0.3$ nC, $\sigma_y = 0.4$ mm, $\sigma_x = 0.6$ mm, $\sigma_z = 0.25 - 0.5$ mm.

Wiggler affects beam dynamics—preliminary results

The idea is to measure the change in the bunch distribution propagating with wiggler on and wiggler off and compare them with computer simulations carried out with OPAL. The simulations includes both the space charge and CSR wakefields.



Data analysis is in progress.

Summary/Discussion

- WEPA with small chicanes can provide and alternative to PCA with a considerably shorter cell length. However, adding small chicanes would require an adjusting the path length of the hadron beam (similar to MBEC).
- WEPA is a broadband amplifier; conceptually it allows to extend the bandwidth of the coherent cooler into even higher frequency range.
- An experiment in ANL has been carried out to demonstrate the combined effect on the beam dynamics of the CS and CSR wakefield in a strong wiggler.