Magnetized Dynamic Friction Force in the Strong-Field, Short-Interaction-Time Limit

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Relativistic cooling: short interaction time, new physics

- Magnetized electron cooling is a relevant Plan B for the polarized electron-ion collider (EIC) at BNL
- EIC requires pre-cooling and cooling at high energy
  - *24 GeV/n → $\gamma \approx 26$ → 13 MeV bunched electrons*
  - *41 GeV/n → $\gamma \approx 44$ → 22 MeV bunched electrons*
  - *275 GeV/n → $\gamma \approx 300$ → 150 MeV bunched electrons*
- Electron cooling at $\gamma > 10$ or higher requires different thinking
  - *beam-frame interaction time T_{int} is Lorentz-contracted by a factor of γ*
  - *electron density dilation in the beam frame by a factor γ means longer plasma period*
    - challenging to achieve the required dynamical friction force (scales like $1/\gamma^2$)
    - not all of the processes that reduce the friction force have been quantified in this regime → significant technical risk
  - *normalized beam-frame interaction time is reduced to order unity or less*
    - $\tau = T_{int}\omega_{pe} >> 1$ for nonrelativistic coolers
    - $\tau = T_{int}\omega_{pe} < 1$ (in the beam frame), for $\gamma > 10$ or higher
      - *violates the assumptions of introductory beam & plasma textbooks*
      - *breaks the intuition developed for non-relativistic coolers*
      - *as a result, the problem requires careful analysis*

Asymptotics of the Derbenev-Skrinsky model for cold, strongly magnetized electrons

Asymptotics for $V_{ion} >> \Delta_{e,\|}, \quad B \to \infty$:

$$F_{\|} = -2\pi Z^2 n_e m_e \left(r_e c^2\right)^2 \left[3\left(\frac{V_\perp}{V_{ion}}\right)^2 \ln\left(\frac{\rho^A_{\max}}{\rho^A_{\min}}\right) + 1\right] \frac{V_\|}{V^3_{ion}}$$

$$F_\perp = -2\pi Z^2 n_e m_e (r_e c^2)^2 \left[\frac{V_\perp^2 - 2V_\|^2}{V_{ion}^2} \ln\left(\frac{\rho^A_{max}}{\rho^A_{min}}\right)\right] \frac{V_\perp}{V_{ion}^3}$$

Asymptotic result for *large* V_{ion} *parallel to* $\boldsymbol{B}$:

$$F_{\|}\left(V_\perp = 0\right) = -2\pi Z^2 n_e m_e \left(r_e c^2\right)^2 \frac{1}{V_\|^2} \qquad \text{(no dependence on } \tau\text{)}$$

$$\rho_{max} = min\left\{max\left(\rho_{sh}, \left(\frac{3Z}{n_e}\right)^{1/3}\right), V_{ion}\tau\right\}$$

$$\rho_{min} = Z r_e c^2 \frac{1}{|\vec{V}_{ion} - \vec{v}_e|^2}$$

$$\rho_{sh} = \frac{\sqrt{V_{ion}^2 + V_{e,rms}}}{\omega_e}$$

$$r_L = V_{rms,e,\perp} / \Omega_L\left(B_\|\right)$$

$$\rho^A_{\min} = \max\left(r_L, \rho_{\min}\right)$$

$$\rho^A_{\max} = \min\left(r_{beam}, \rho_{\max}\right)$$

$$V^2_{ion} = V^2_\| + V^2_\perp$$

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

Parametric model of Parkhomchuk for including finite B and thermal effects

$$\mathbf{F} = -4Z^2 n_e m_e \left(r_e c^2\right)^2 \ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \frac{\mathbf{V}_{ion}}{\left(V_{ion}^2 + V_{eff}^2\right)^{3/2}}$$

$$r_L = V_{rms,e,\perp} / \Omega_L\left(B_{\|}\right)$$

$$V_{eff}^2 = V_{e,rms,\|}^2 + \Delta V_{\perp e}^2$$

$$\rho_{min} = Z r_e c^2 / V_{ion}^2 \qquad \text{(as in the original paper)}$$

$$\rho_{max} = V_{ion} / (\omega_e + 1/\tau)$$

$$\rho_{min}^B = Z r_e c^2 / (V_{ion}^2 + V_{eff}^2) \qquad \text{(as implemented in BETACOOL)}$$

In the limit of $B \rightarrow \infty$:

$$\mathbf{F} = -4Z^2 n_e m_e (r_e c^2)^2 \ln\left(\frac{\rho_{max} + \rho_{min}}{\rho_{min}}\right) \frac{\mathbf{V}_{ion}}{(V_{ion}^2 + V_{eff}^2)^{3/2}}$$

In the limit of strong B, cold e-beam, and small V_{ion}:

$$F_{\|} = -4Z n_e r_e c^2 \frac{V_{ion,\|}}{\omega_e + 1/\tau} \qquad \text{with plasma frequency } \omega_e = \sqrt{4\pi n_e r_e c^2}$$

V.V. Parkhomchuk, “New insights in the theory of electron cooling,” *NIM* **A 441** (2000).

I. Meshkov, A. Sidorin, A. Smirnov, G. Trubnikov, R. Pivin, “BETACOOL Physics Guide,” http://lepta.jinr.ru/betacool (2008).

Our approach is motivated by the work of *Ya. Derbenev*

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

THEORY OF ELECTRON COOLING

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- The E-fields associated with friction must be carefully identified
  - *these are the fields generated by the presence of the ion*

$$\vec{E}(\vec{r},\vec{v},t) = \underbrace{\langle\vec{E}^{0}\rangle(\vec{r},t)}_{\text{bulk fields}} + \underbrace{\langle\Delta\vec{E}\rangle(\vec{r},\vec{v},t)}_{\text{friction}} + \underbrace{\vec{E}^{fl}(\vec{r},\vec{v},t)}_{\text{statistical fluctuations}} \qquad (1.1)$$

- Friction force must be calculated along the ion trajectory:

$$\vec{F} = -ze\langle\Delta\vec{E}\rangle(\vec{r},\vec{v},t)\Big|_{\vec{r}=\vec{r}(t),\dot{\vec{r}}(t)=\vec{v}} \qquad (1.2)$$

  - *we do this numerically for each individual ion-electron interaction*
    - **total force obtained by summing over e⁻ distribution (i.e. no shielding)**
  - *bulk forces are removed by subtracting force from unperturbed e⁻'s*

Our model: strongly magnetized, relativistic cooling regime → short interaction time, strong magnetic field

- Prototyping is done in the parameter regime of Fedotov *et al*. (2006)

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Numerical study of the magnetized friction force

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Analysis of the magnetized friction force *

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- For our test case, we considered the following beam frame parameters:
  - *e^- density, $n_e = 2\text{x}10^{15}\ m^{-3}$*
  - *$V_{e,rms,||} = 0$ or 1.0×10^5 m/s and $V_{e,rms,\perp} = 4.2 \times 10^5$ m/s*
  - *ideal solenoid, B = 1T and 5T (theoretical models) and infinitely strong field (theoretical models and our simulations)*
  - *interaction time, $T_{int} = 4\text{x}10^{-10}$ s ~ 56 T_L ~ 0.16 T_{pl} (T_L for B = 5T)*
    - **16% of a plasma period → no shielding of the interaction**
  - *expectation value of distance to nearest e^-, $r_1 \sim 4.9\text{x}10^{-6}$ m ~ 10 r_L*
    - **small Larmor radius → strong B-field assumption is reasonable**

Gyrokinetic averaging yields *1D e⁻* oscillations

- Hamiltonian perturbation theory for single ion and e^-
  - *unperturbed motion: drifting ion and magnetized e^-*
  - *strong B assumption: D (impact parameter) >> r_L (Larmor radius)*
  - *longitudinal dynamics: $V_{ion,\perp} = 0$ (to be relaxed in future work)*
- choose ion to be stationary at the origin (convenient reference frame)
- to the leading order in perturbation theory, e^- gyrocenters stay on cylinder of constant radius D (different for different e^-'s)
  - *gyrocenters move in an effective nonlinear 1D potential:*

$$\ddot{z}(t) = -Zr_e c^2 \frac{z}{(D^2+z^2)^{3/2}}$$

- a "soft" nonlinear potential:
  - *larger amplitudes <=> longer oscillation periods;* $T_{min} = 2\pi\sqrt{D^3/Zr_e c^2}$
  - *both unbound and oscillatory e^- orbits, incl. trajectories with T > or >> T_{int}*
  - *impact parameter above which there are no oscillatory orbits is determined by V_{ion} for large V_{ion}, by the interaction time T_{int} for small V_{ion}*
  - *net friction force is determined by contributions from different orbit types*
  - *1D numerical simulations are required to capture these effects*

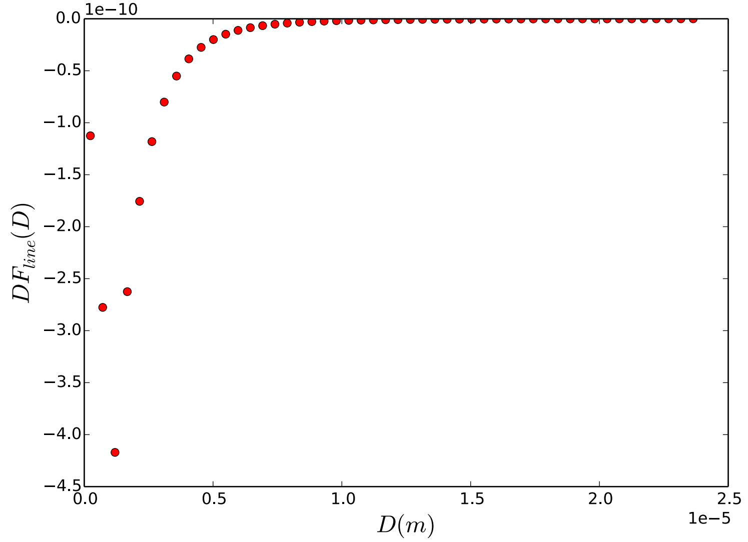
Key aspects of the numerical simulations

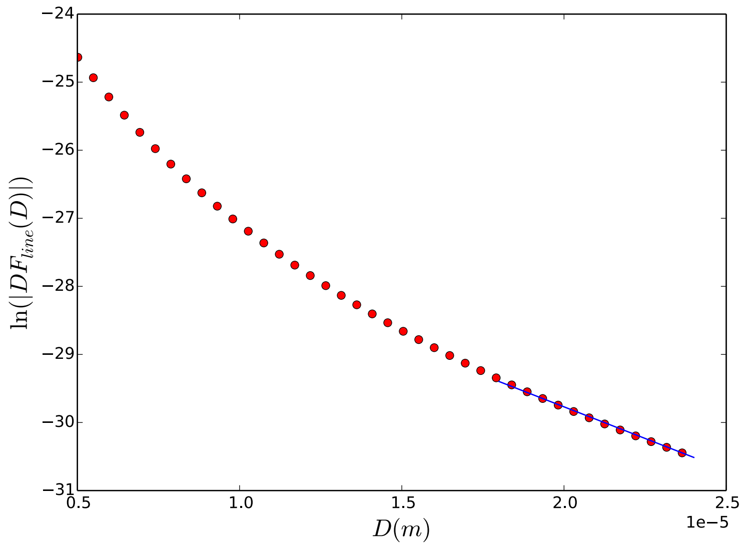
- Work in the system of reference where the ion is at rest
  - *assume ion velocity along the field lines of B (→ axial symmetry)*
  - *cold electrons → all have the same initial velocity w.r.t. the ion*
  - *momentum kicks add up, averaged over T_{int}*
- Dynamical friction comes from the ion-induced *density perturbation*
  - *add up the difference between force from e^-'s on perturbed & unperturbed paths*
    - **hence, we track pairs of electrons with identical initial conditions**
  - *this approach eliminates all bulk forces, both physical and numerical*
- Compute ensemble-average expectation value of friction
  - *we assume a locally-uniform electron density n_e*
  - *transversely, e^--s are uniformly distributed on lines of constant D*
    - **there is no logarithmic singularity for D→0, nor for D→∞**
  - *longitudinal distribution is uniform in initial z position, z_{ini}*
    - **finite range of z_{ini} values contributes non-negligibly to the friction force**
    - **range depends on: D (impact parameter), V_{ion}, Z (ion charge state)**
- Friction force for warm e^-'s is obtained from the friction force for cold electrons via convolution with electron distribution in velocity space

Finite friction for all impact parameters *(no logarithmic singularities)*

- First add up contributions to the friction force from initial conditions on lines of constant D, then integrate over the impact parameter:

$$F_{\|}(V_{\perp}=0)=2\pi n_e\int_0^{\infty} DF_{line}(D)dD, \quad where \quad F_{line}(D)=\int_{-\infty}^{\infty} F_{i-e}(z_{ini},D)dz_{ini}$$



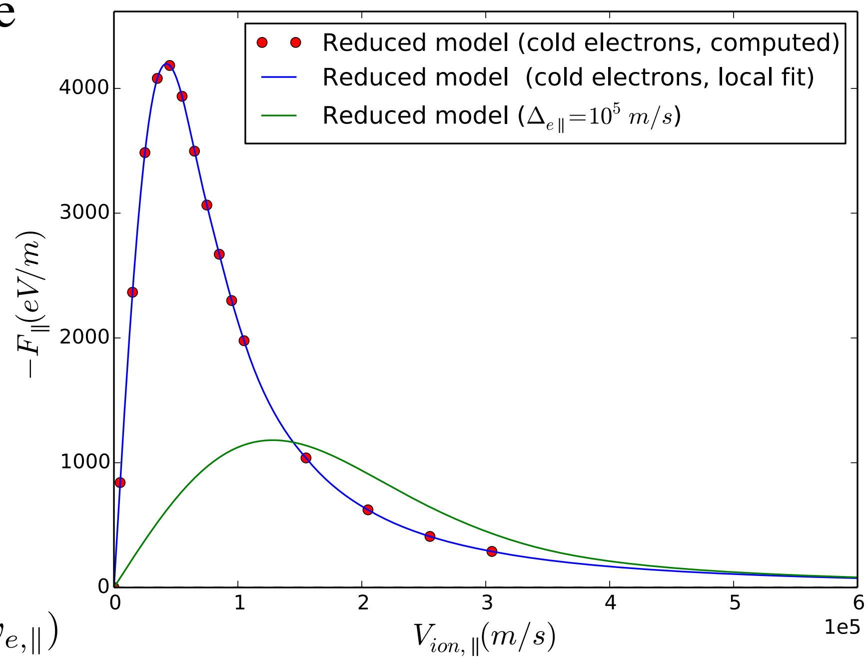


- Integrand is finite for small D & tails off exponentially => finite $F_{\|}$
- Exponential fall-off for large D makes it possible to correct (analytically) for finite values of D_{max} used in simulations
- Repeat for different values of $V_{ion,\|}$ to compute $F_{\|}\,(V_{ion,\|})$

Physically reasonable behavior of $F_{\|}(V_{ion,\|})$ seen for both small and large $V_{ion,\|}$, cold and warm electrons

- For *cold* electrons and Au^{+79} ion, reasonable qualitative behavior of $F_{\|}(V_{ion,\|})$ seen for both small and large $V_{ion,\|}$:
  - *linear in V for small V*
  - *$1/V^2$ for large V*
- For an arbitrary distribution $f(v_{e,\|})$ of *warm* electrons, $F_{\|}(V_{ion,\|})$ is computed by convolution of $f(v_{e,\|})$ with $F_{\|}(V_{ion,\|})$ for cold electrons:

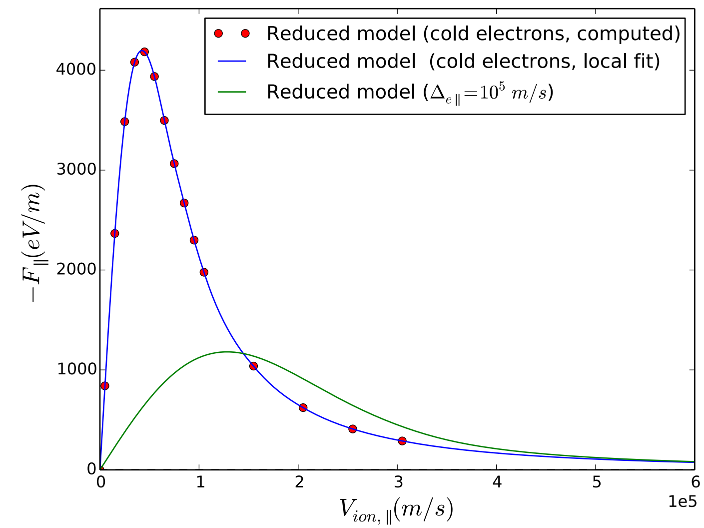
$$F_{\|}^{(warm\ \ els.)}(V_{i,\|}) = \int_{-\infty}^{+\infty} dv_e F_{\|}^{(cold\ \ els.)}(V_{i,\|} - v_{e,\|}) f(v_{e,\|})$$



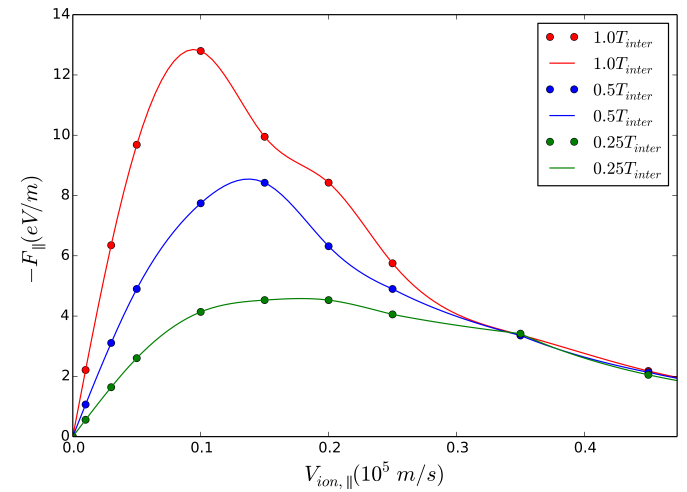
- Convolution with $f(v_{e,\|})$ acts as a smoothing filter => peak of $F_{\|}(V_{ion,\|})$ for warm electrons is lower and shifted towards larger $V_{ion,\|}$
- Just as for cold e^- gas, for warm electrons $F_{\|}(V_{ion,\|})$ is linear in $V_{ion,\|}$ for small $V_{ion,\|}$ and scales as $1/V^2$ in the large $V_{ion,\|}$ region
- As expected, $F_{\|}(V_{ion,\|})$ for different electron temperatures converge as $V_{ion,\|}$ gets larger

$F_{||}(V_{ion,||})$ for *cold* electrons: scaling in Z and T_{int}

- For cold electrons, looked at protons and Au^{+79} ions and different interaction times in the cooler (interaction-time-averaged force):
  - *for small* $V_{ion,||}$: $F_{||}(V) \sim V$; *slope* $dF_{||}(V)/dV \approx -2Z\, n_e\, m_e\, r_e\, c^2\, T_{int}$
  - *large-V tail is well approximated by* $F_{||} = -2\pi Z^2 n_e m_e (r_e c^2)^2 / V^2$, with no dependence on T_{int}
  - *for a given* T_{int} *, peak friction force scales approximately as* $Z^{4/3}$
- For $T_{int} < T_{pl}$ and small-to-moderate V_{ion}, $F_{||}(V)$ goes up with interaction time; large-V tail is T_{int} –independent
- $F_{||}(V)$ is linear in n_e by construction



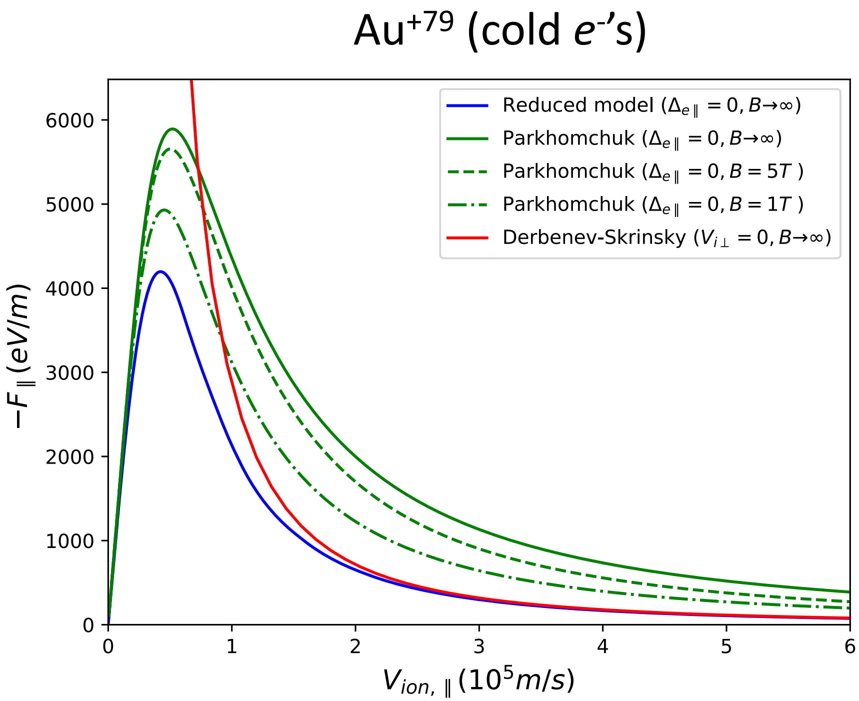
Above: Gold ion, cold and warm e⁻'s

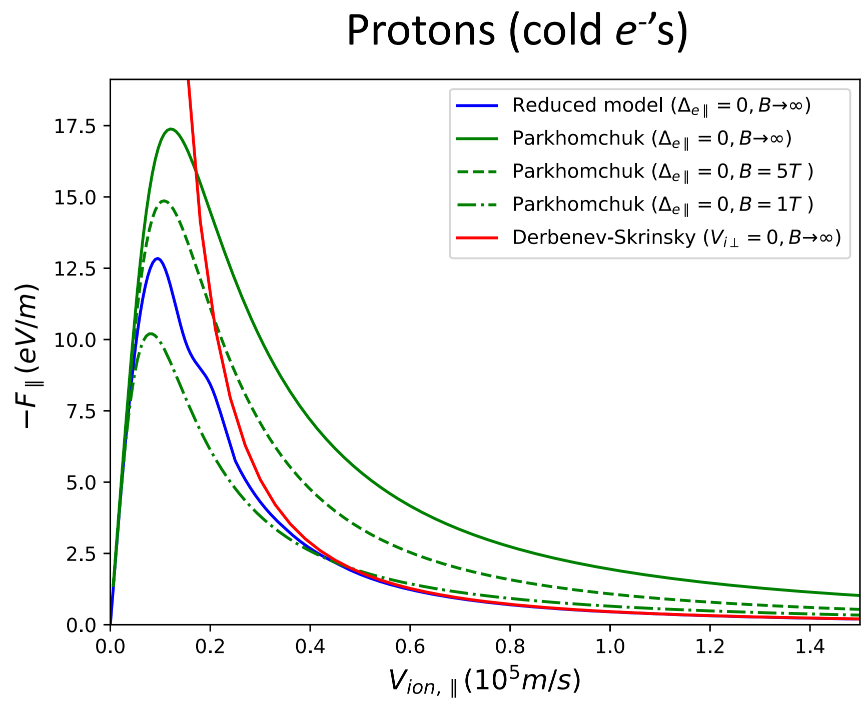


Above: Protons, cold e⁻'s, varying T_{int}

Compare with Derbenev-Skrinsky and Parkhomchuk (1)

- Comparison of new model for Au^{+79} ions and protons (*cold e⁻'s*), with:
  - *Derbenev and Skrinsky (D&S) for $V_{ion,\perp} = 0$, cold e⁻'s, strong B and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 effective longitudinal e⁻ temperature for $V_{ion,\perp} = 0$*



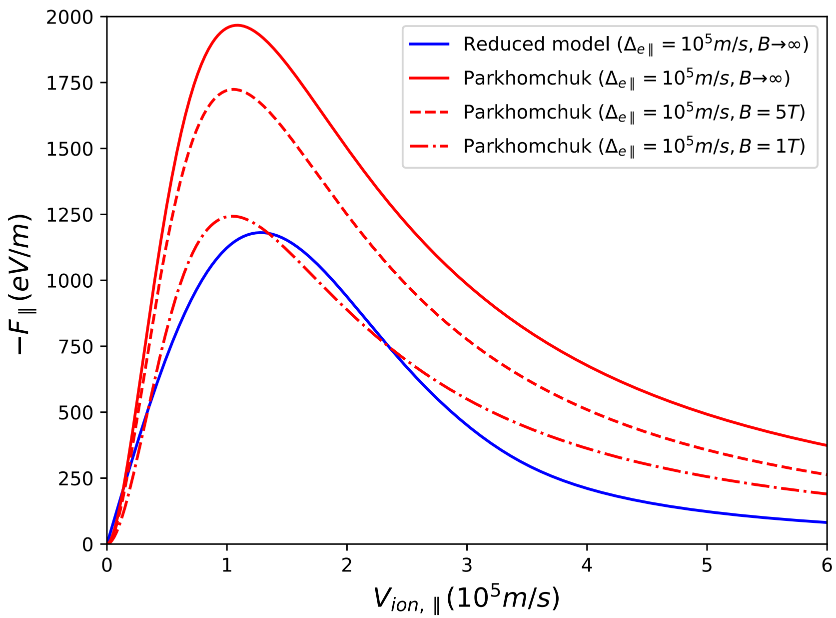


- All models predict $F(V) \sim 1/V^2$ for large V
  - *our simulations and semi-analytic model agree exactly with D&S*
  - *consistently lower strong-B force than Parkhomchuk for ALL velocities*

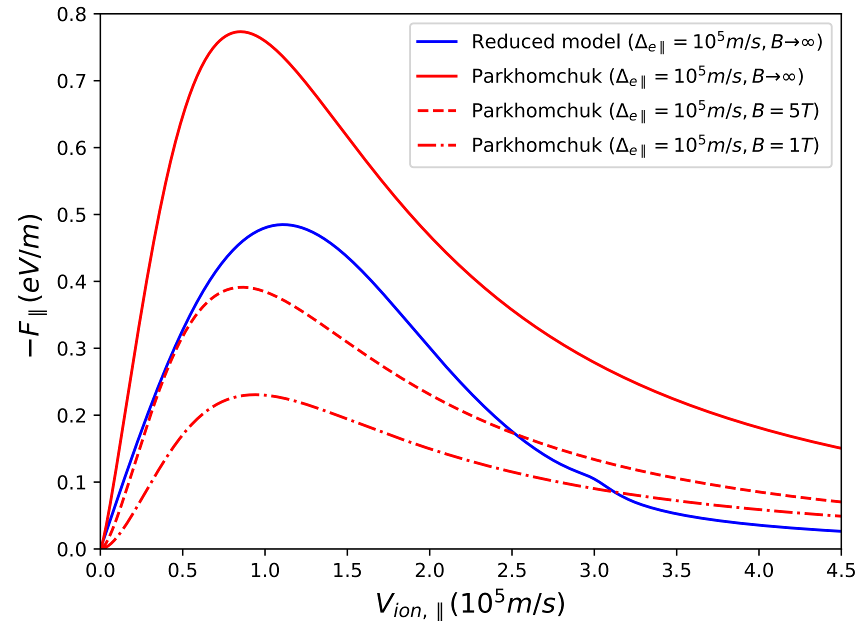
Compare with Derbenev-Skrinsky and Parkhomchuk (2)

- Comparison of the new model for Au^{+79} and protons *(warm e⁻'s)* with Parkhomchuk:
  - *Parkhomchuk model with finite effective* longitudinal *e⁻ temperature*
  - *ϱ_{min} as implemented in BETACOOL* $\rho^B_{min} = Zr_e c^2/(V^2_{ion} + V^2_{e,rms,\parallel})$

Au+79 ions (warm e⁻'s)



Protons (warm e⁻'s)



- For warm electrons, new model agrees approximately with Parkhomchuk (but details depend on Z, V_{ion}); consistently weaker friction than the *strong-field limit* of the BETACOOL variant of the Parkhomchuk model

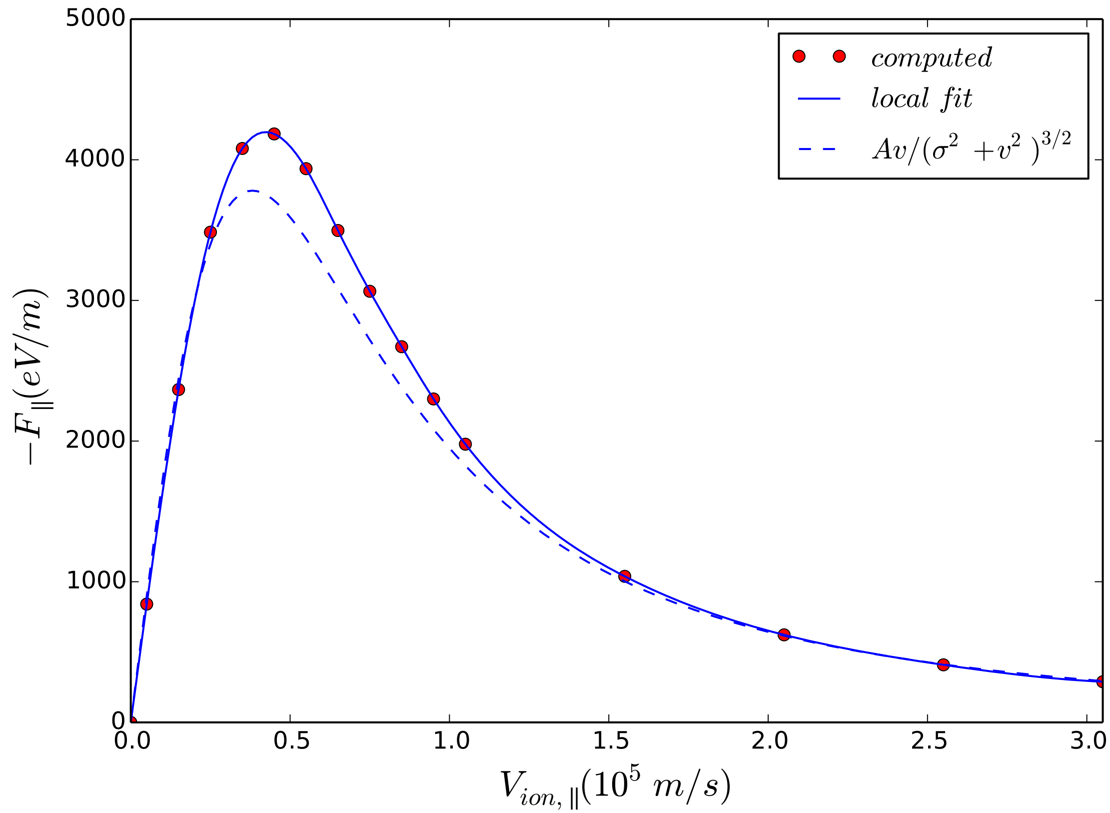
Simple, approximate 2-parameter model

- The physical system is described by 3 parameters: n_e, Z, and T_{int}

$$F_{\|}(v) = -\frac{Av}{(\sigma^2 + v^2)^{3/2}} \qquad A = 2\pi Z^2 n_e m_e (r_e c^2)^2$$

$$\sigma \approx (\pi Z r_e c^2 / T_{int})^{1/3}$$

- Large v:
  - $F_{\|} \sim A/v^2 \sim Z^2/v^2$
  - *A found via fit and dimensional and scaling analysis*
- Small v:
  - $F_{\|} \sim Av/\sigma^3 \sim Z\, T_{int}\, v$
  - *σ found via fit, dimensional and scaling analysis*
  - *scaling confirmed by analytic calculation*
- Peak force is underestimated by ~10-15%
  - $F_{\|,max} \sim Z^{4/3} T_{int}^{2/3}$



Simulation results and the parametric model (Au+79 ion)

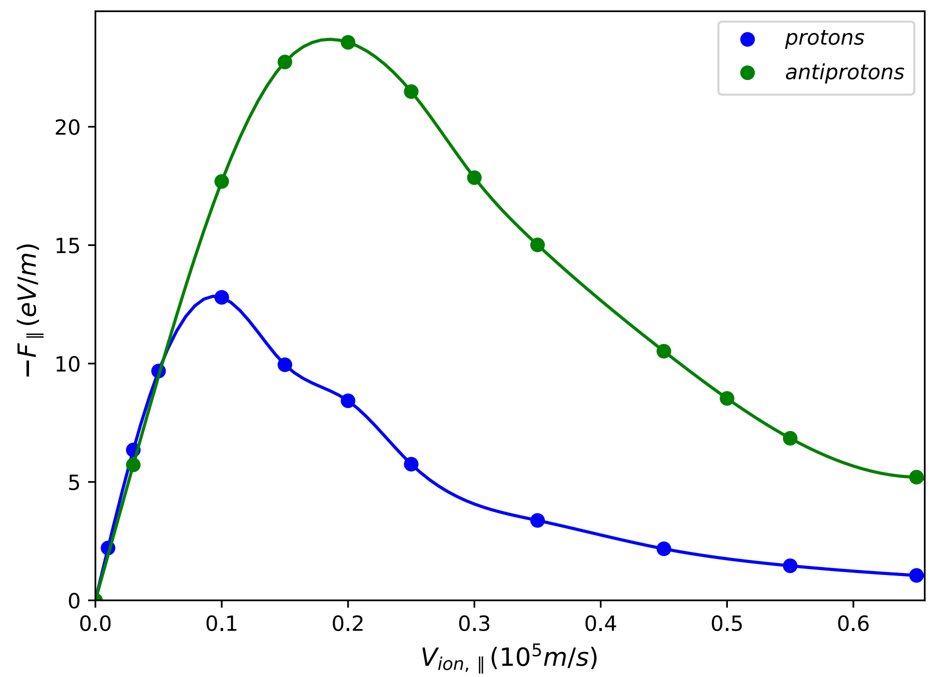
An estimate of the number of physical electrons contributing to the friction force is possible

- Assuming a locally-constant n_e, for a given T_{int}, Z, *and ion velocity* we can estimate the number of physical electrons contributing to the longitudinal friction force on a single pass through the cooler:
  - *for a given impact parameter, the longitudinal size of the (asymmetric) region from which non-negligible contributions come can be determined*
  - *from simulations, the contributions to the net force decrease approximately exponentially with impact parameter*
  - *one can therefore estimate the volume from which (say) 95% of the total force on the ion comes, thus the number N_{95} of physical electrons in this volume*
  - *clearly, these electrons do not contribute equally, so estimating the relative pass-to-pass variation of the force as ~1 / $N_{95}^{1/2}$ may be overly simplistic*
- A few estimates for the (beam-frame) $n_e = 2x10^{15}\ m^{-3}$, $T_{int} = 4x10^{-10}s$:
  - $Z = 79$, $V_{ion} = 2x10^5$ *m/s (in the 1/V^2 tail of F(V)):* $N_{95} \sim 2400$
  - $Z = 79$, $V_{ion} = 4.5x10^4$ *m/s (near the peak of F(V)):* $N_{95} \sim 220$
  - $Z = 1$, $V_{ion} = 1.0x10^4$ *m/s (near the peak of F(V)):* $N_{95} \sim 20$

Stronger friction for a repulsive ion-e^- force

- Other parameters being equal, first-principles simulations with our reduced *1D* potential show a stronger friction for negative ions:
  - *in practice, this is probably only relevant for Z = -1 (antiprotons)*
- Different topology of electron orbits compared to $Z > 0$:
  - *No oscillatory orbits possible, only unbound orbits and orbits having at most one turn-around point (depending on the T_{int} and initial conditions)*

- Most of the contribution to the ensemble-averaged net force for Z< 0 comes from strong, small-impact-parameter interactions
- Statistics of small-impact parameter interactions need be considered



Simulations with the reduced 1D model (cold electrons)

Future plans

- Improved parametrized models
- Better understanding of the role of trapped (oscillatory) *vs* unbound electron orbits, unidirectional *vs* bounce-off orbits for electrons in a repulsive force potential
- The case of finite B
- Modeling transverse dynamic friction (initial results show $F \sim 1/V^2$ friction for large V but *antifriction* for small V; this is puzzling and needs to be verified and understood)
- Statistical properties of $F(V)$: so far, only the expectation value was considered (in essence, the continuum limit)
- Adding new models to JSPEC as they become available, simulations in the EIC parameter regime (https://sirepo.com)

Thank You!

Спасибо!

Comments or Questions?