Observation of circular attractors in cooled ion bunches at LEReC

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Outline

- Introduction
- Low energy RHIC electron cooler (LEReC)
- Theory of coherent excitations
- Coherent excitations in operational settings vs dedicated studies
- Observation of circular attractor
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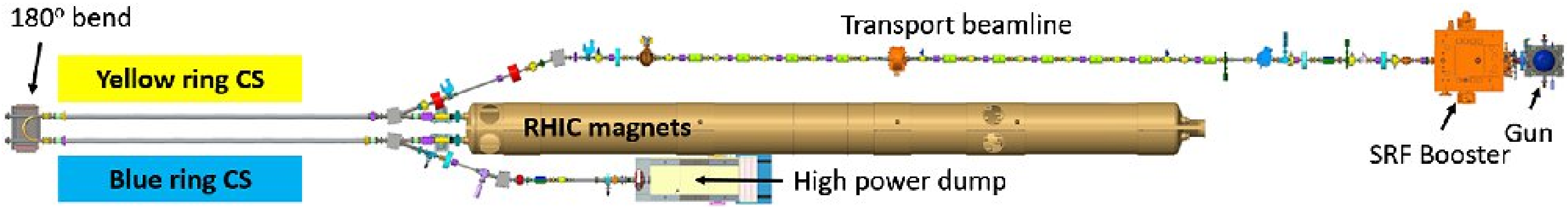
- Implications for high energy coolers at Electron Ion Collider
- Summary

Introduction

- We are reporting the experimental studies of the effect (known as monochromatic instability, coherent excitations etc.) first described in *YA. S. Derbenev, A. N. Skrinsky, Particle Acc. 1977, Vol. 8, pp. 1-20*
- This effect was observed at several non-relativistic coolers utilizing DC e-beam (a flat velocity distribution)
- LEReC is the first RF-based e-cooler (e-bunch velocity distribution is almost spherical).
- LEReC is the relativistic e-cooler ($\gamma = 4.1$ & $\gamma = 4.9$).
- The discussed effect sets strict requirements to the high-energy electron coolers (EIC precooler, EIC SHC option based on ring cooler)
- The very similar effect is expected in the CeC scheme and it sets strict requirements to the CeC design as well.

LEReC



- The e-bunches are produced at the photo-cathode illuminated by a green 704 MHz laser modulated with the 9 MHz frequency to match the frequency of RHIC ion bunches.
- The electrons are accelerated to 375 keV in the dc gun followed by a 704 MHz superconducting rf accelerating cavity bringing the beam energy to 1.6–2. MeV.
- The e-beam is transported in a 40 m long transport line and merged to the cooling section (CS) in the “Yellow” RHIC ring . After passing the Yellow CS, the beam is sent to the cooling section in the “Blue” RHIC ring by a 180° bend.
- Finally, the electron beam is extracted and sent to the beam dump.

- LEReC is the first:
  - RF-based electron cooler
  - cooler using neither e-beam magnetization nor continuous solenoidal field in the cooling section.
  - electron cooler which is applied directly to the ions in the collider at top energy (**LEReC was designed to counteract IBS heating and during the operational stores the IBS rate is comparable to the cooling rate**)
  - electron cooler that utilizes the same electron beam for cooling ions in two consecutive cooling sections in two rings of the collider
- LEReC approach can be easily scaled for high energy applications
- LEReC was successfully applied to colliding gold ions in 2020 run (@ $\gamma = 4.9$) and 2021 run (@ $\gamma = 4.1$), routinely providing a substantial luminosity increase.
- Recent peer-reviewed LEReC publications:

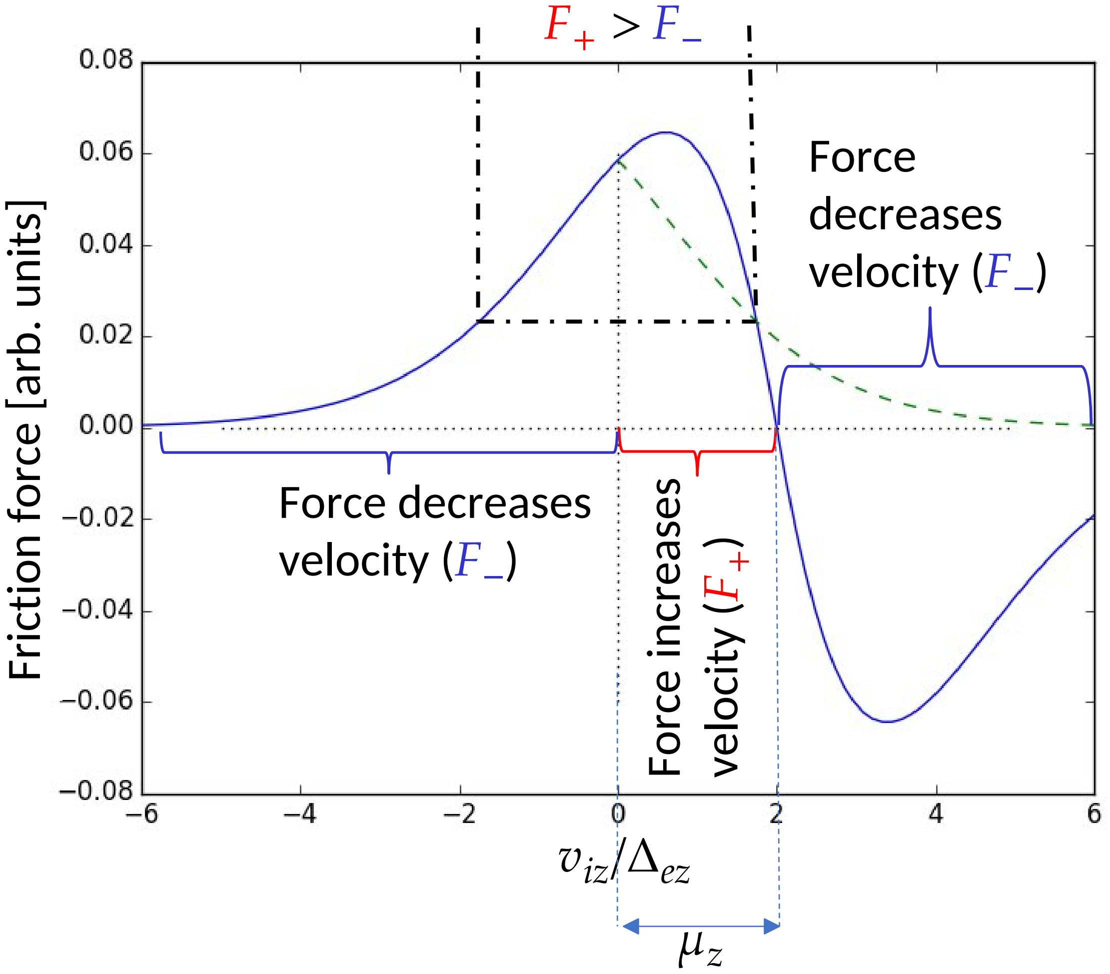
A. Fedotov et al., PRL 124, 084801 (2020).
D. Kayran et al., PRAB 23, 021003 (2020).
S. Seletskiy et al., PRAB 21, 111004 (2019).
X. Gu et al., PRAB 23, 013401 (2020).
H. Zhao et al., PRAB 23, 074201 (2020).
S. Seletskiy et al., PRAB 23, 110101 (2020).

Coherent excitations theory (I)

- The ion-electron interaction in E-cooling can be described by a friction force
- The friction force acting on an individual ion:

$$\vec{F} = -\frac{4\pi n_e e^4 Z^2}{m_e} \int L_C \frac{\vec{v} - \vec{v}_e}{|\vec{v} - \vec{v}_e|^3} f(v_e) d^3 v_e \qquad f(v_e) = \frac{1}{(2\pi)^{3/2}\Delta_x\Delta_y\Delta_z} \exp - \left(\frac{(v_{ex} - \mu_x)^2}{2\Delta^2} + \frac{(v_{ey} - \mu_y)^2}{2\Delta^2} + \frac{(v_{ez} - \mu_z)^2}{2\Delta^2} \right)$$

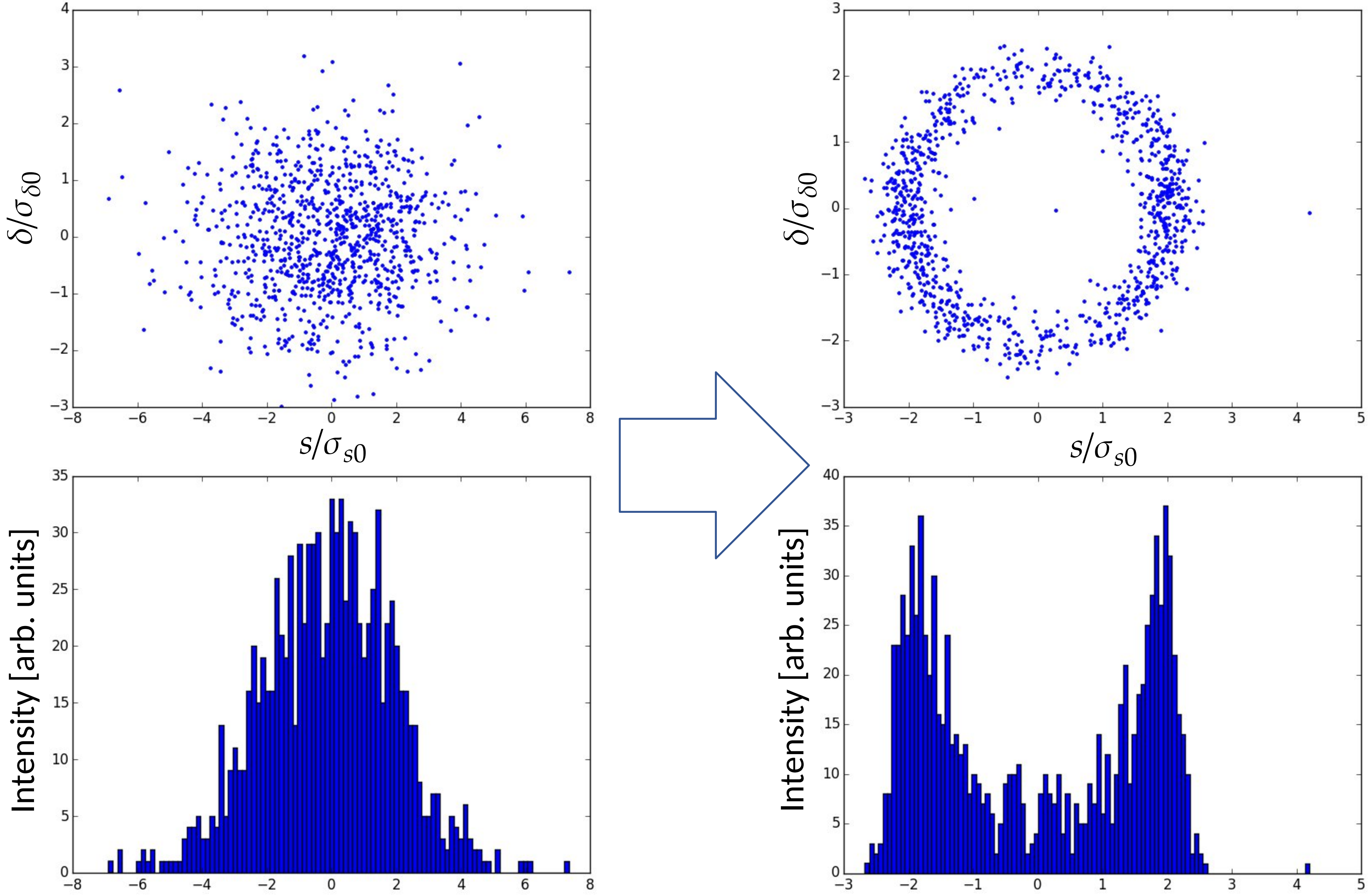
- If μ is larger than the velocity at which the force's first derivative is changing sign, then:
  - ✓ An ion with a positive velocity $v < \mu$ experiences the force (F_+) which increases v
  - ✓ When the same ion interacts with electrons while having a negative velocity it experiences a force (F_-) reducing v
  - ✓ There is a range of velocities where $F_+ > F_-$ and the average effect of the friction force is the excitation of the synchrotron (or betatron) oscillations.
  - ✓ For velocities outside of this range the friction force is always damping the oscillations' amplitude.
  - ✓ Hence, the friction force "tries" to set the oscillation amplitudes of all ions to the same non-zero value.



Coherent excitations theory (II)

- The described coherent offset results in creation of a circular attractor in the phase space.

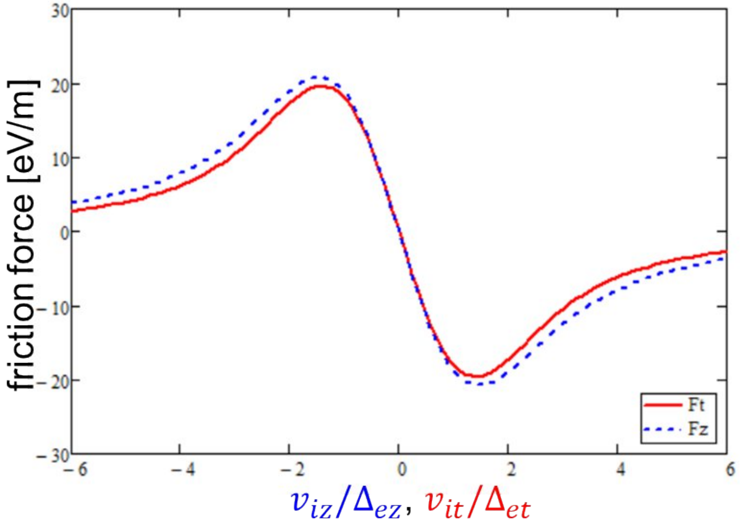
- In the physical space it manifests as a “two-horn” density distribution

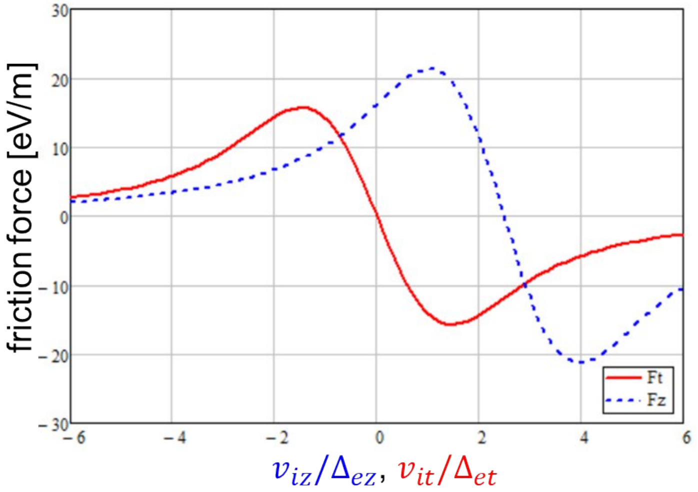


LEReC in operational conditions

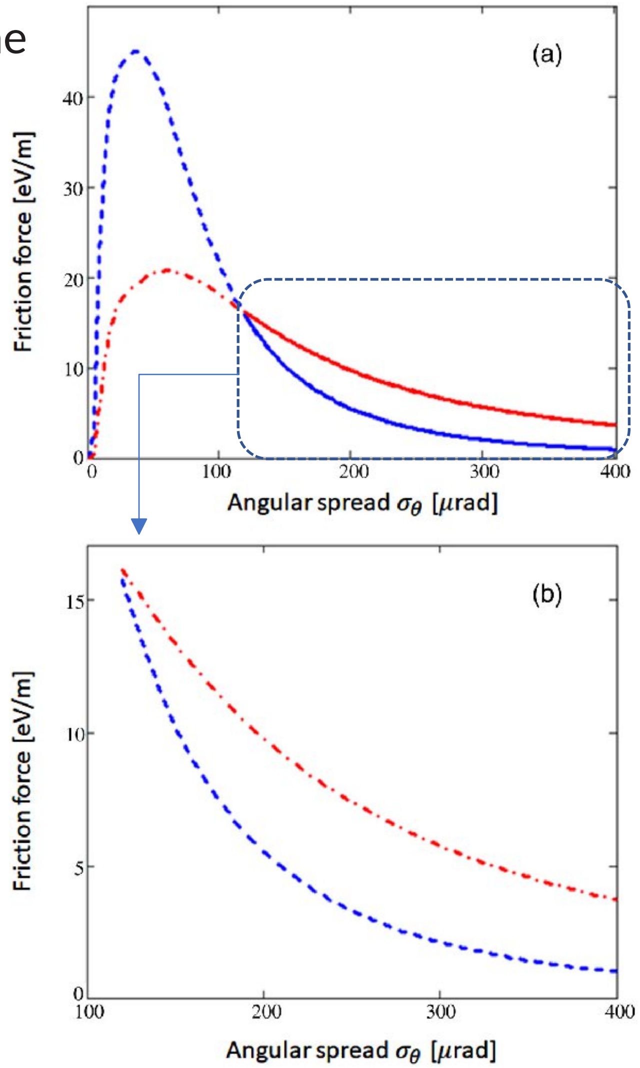
- Friction force for LEReC design parameters:

$$\sigma_\delta = \frac{\Delta_z}{\beta c} = 5 \times 10^{-4}; \qquad \sigma_\theta = \frac{\Delta_t}{\gamma \beta c} = 150\ \mu\text{rad}$$





- Friction force depending on the average angular spread of the electron bunch:



- By design the cooling time in LEReC is several minutes
- The IBS in ion bunches is relatively strong: under operational conditions the IBS rate is on average about 70% of the cooling rate

- Under such circumstances we don't expect to observe a well defined "two-hump" distribution in the presence of an attractor.
- We expect to see a special case of beam "heating"

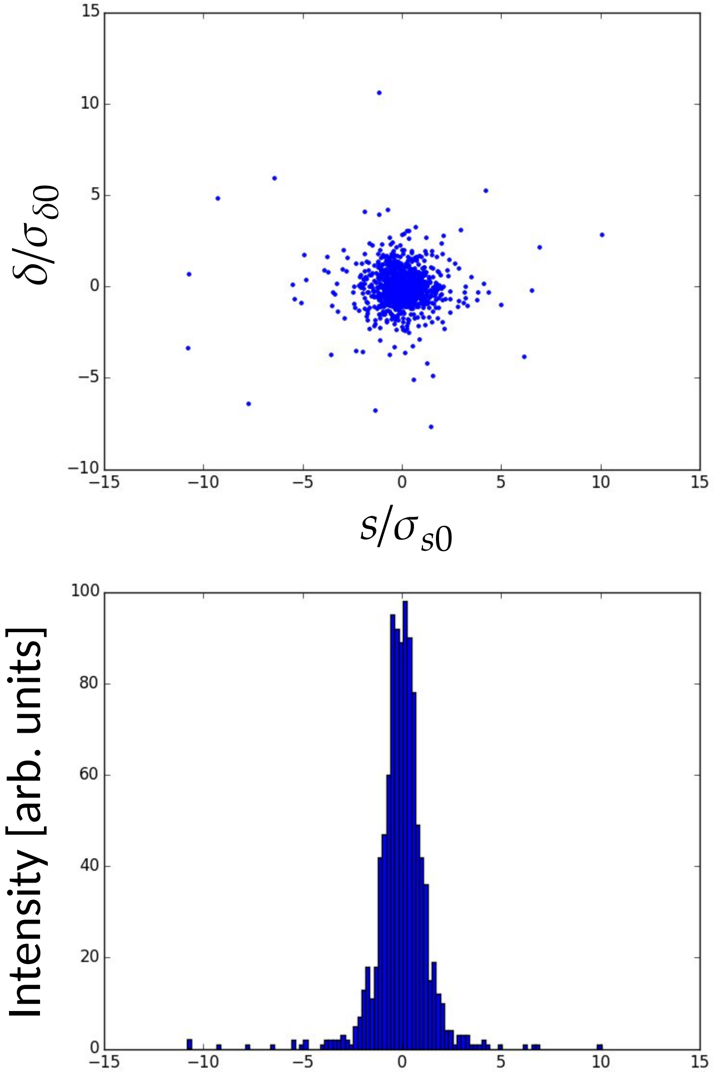
- Indeed, the longitudinal "heating" was noticed in operations when the γ-matching was lost.

Expectations for operational settings

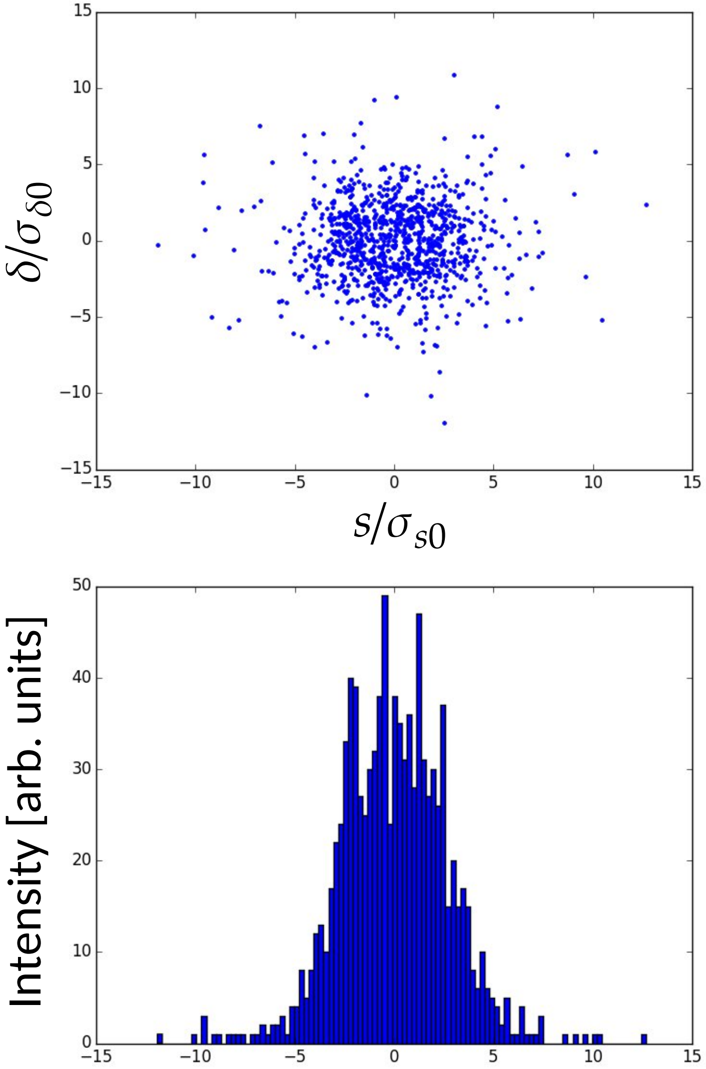
- When the IBS-driven diffusion is about 2 times weaker than the cooling:

- This effective heating can be present both in longitudinal dynamics (when there is a mismatch in γ-factors) and in the transverse direction (in a presence of the angular misalignment of the trajectories).

Cooling when $\mu = 0$



Cooling turns into “heating” for $\mu = 2\Delta_z$

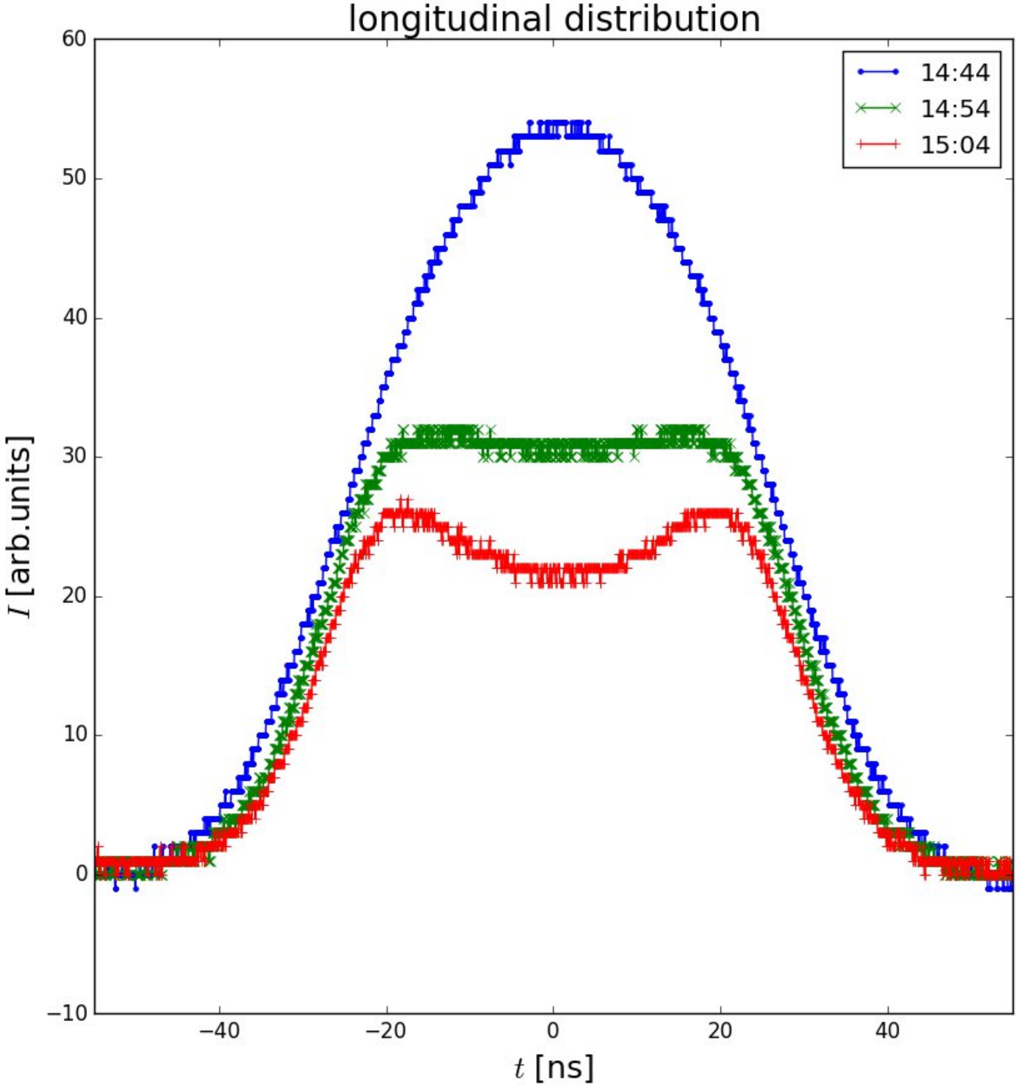


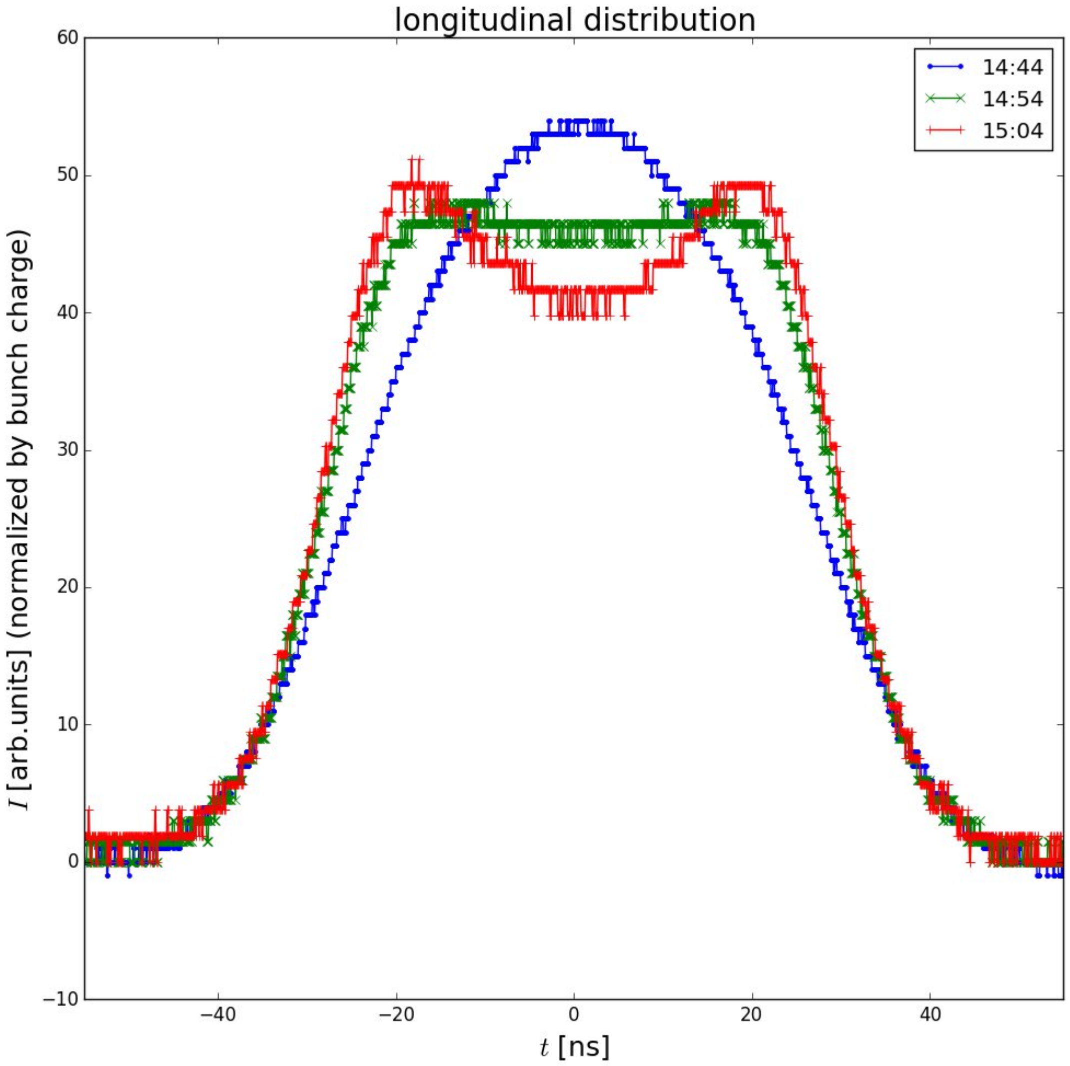
In dedicated studies

- We reduced the IBS-driven heating rate by reducing the ion bunches intensity by a factor of ~5 as compared to the operational intensity
- The low intensity ion bunches were still resolved by a wall current monitor utilized for obtaining the longitudinal profile of the bunches
- In our measurements we were varying the energy offset of the e-beam
- The obtained 2-hump profiles allowed us to measure the attractors' radii corresponding to each energy offset

Measurements (I)

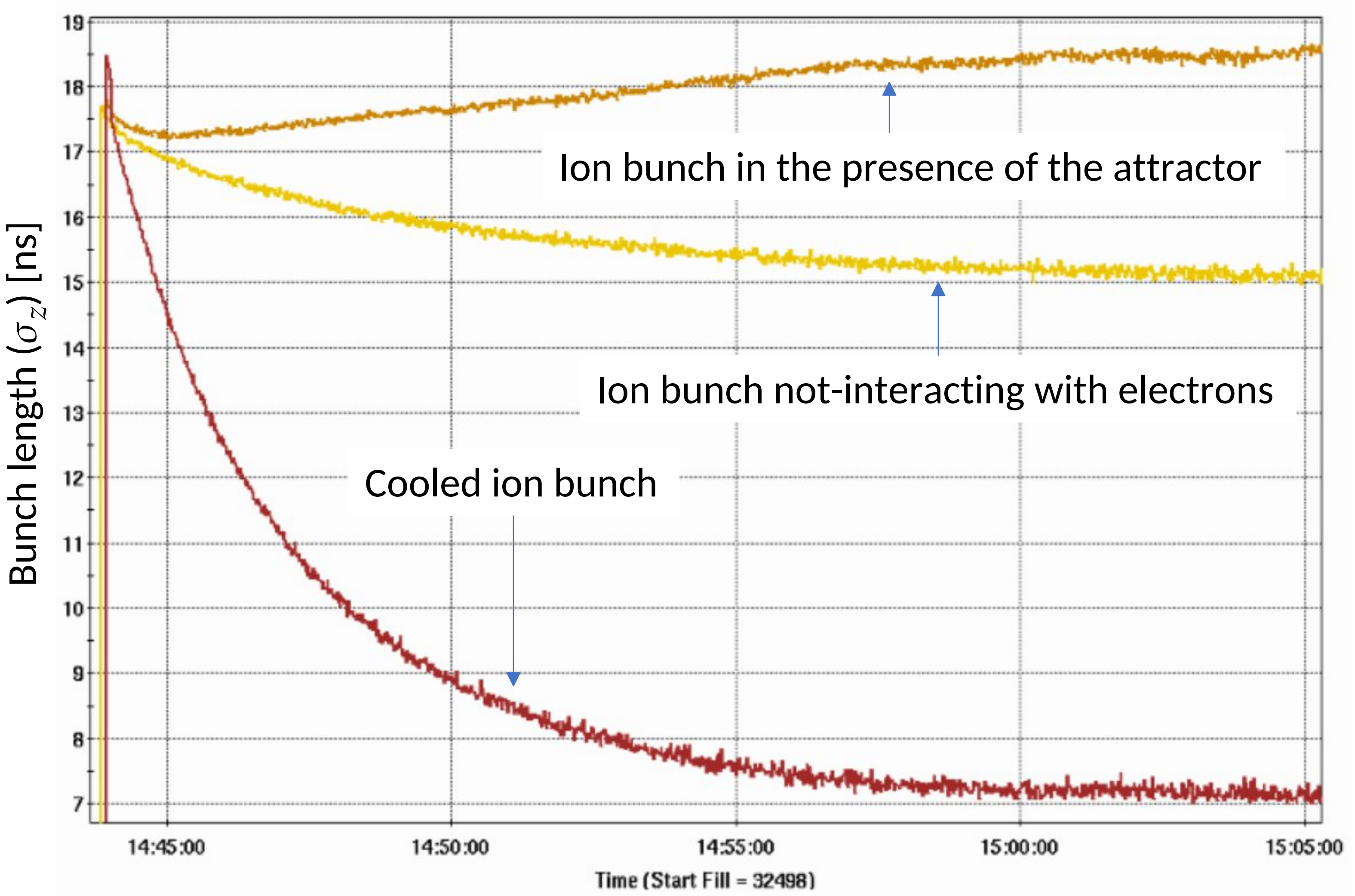
- Evolution of longitudinal profile of the ion bunch affected by the presence of a circular attractor in the longitudinal phase space:





Measurements (II)

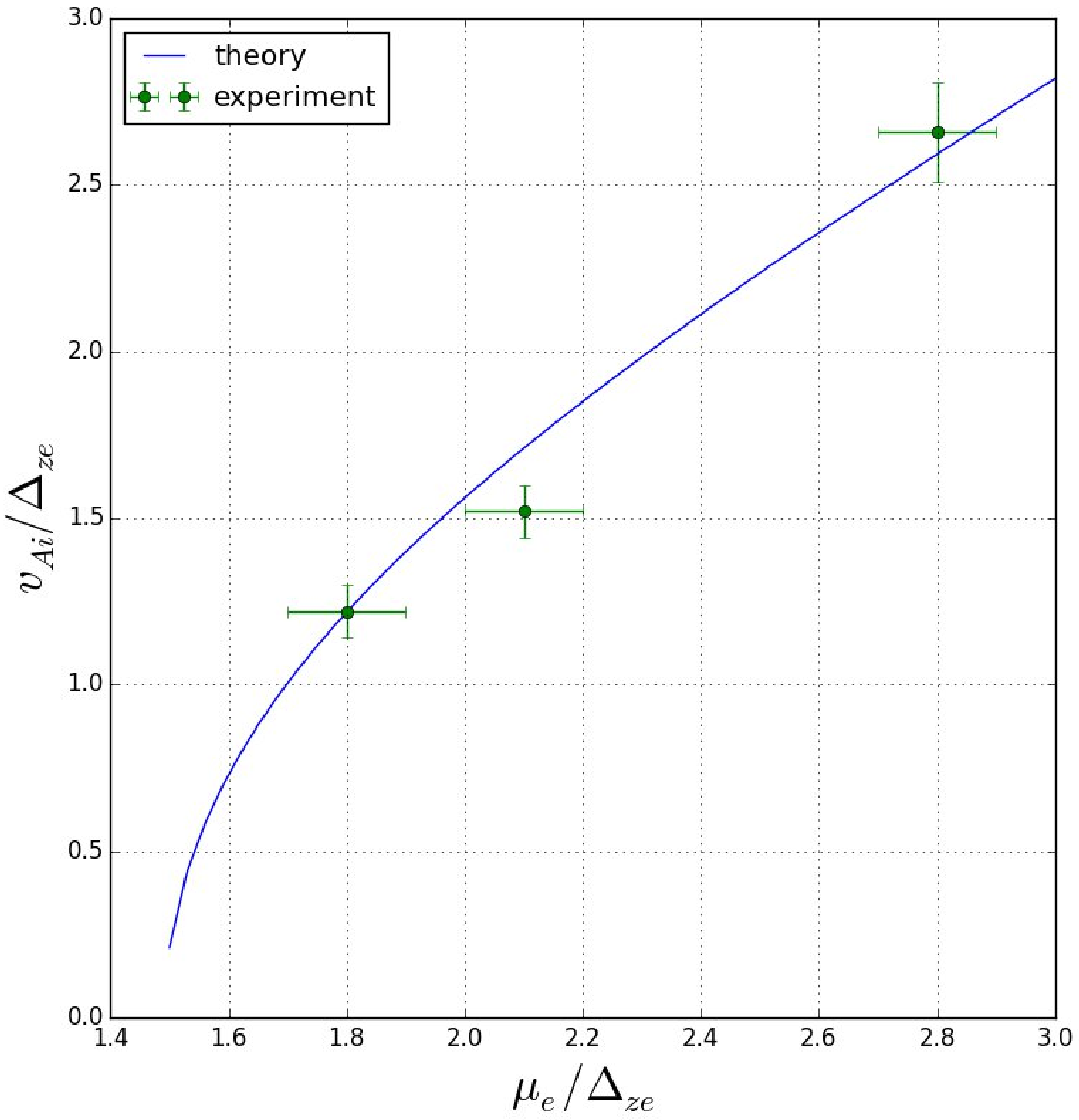
- Rms length of a cooled, an excited and an unaffected ion bunches during the measurement



- When IBS is strong enough to smear out the peculiarities of longitudinal distribution defined by the presence of an attractor, the effect of coherent excitations becomes indistinguishable from incoherent longitudinal heating.

Comparing experiment to theory

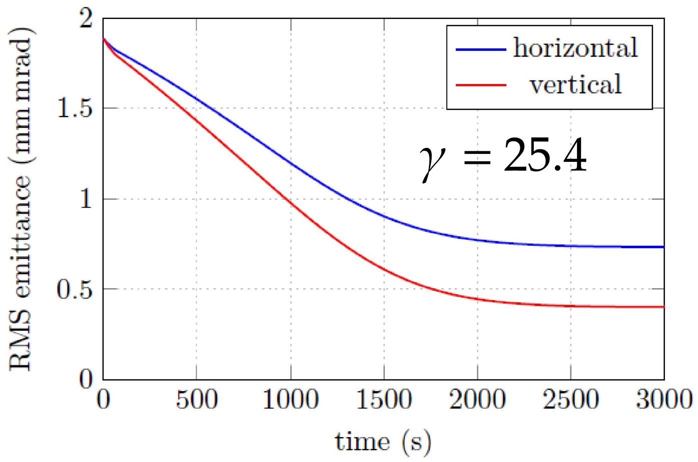
- We performed three measurements
- From each measurement we can calculate the “radius” of the attractor
- The obtained results are in good agreement with theoretical predictions

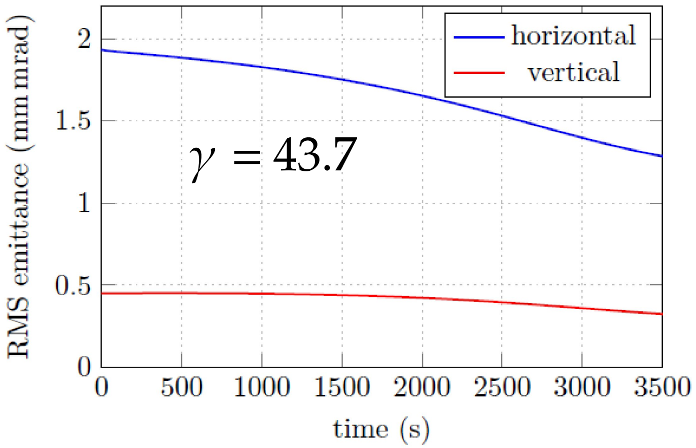


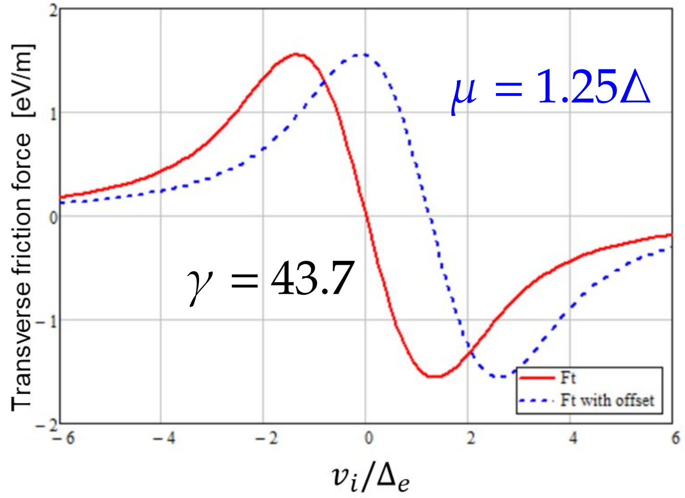
Precooler for Electron Ion Collider

- EIC needs a pre-cooling of the proton bunches at an injection energy of 23.8 GeV to a normalized vertical emittance of 0.45 mm·mrad
- For the lowest proton collision energy of 41 GeV in the EIC, there is a need for an electron cooler which counteracts the beam emittance growth due to IBS.
- LEReC-type cooler with a single pass ERL:

γ	25.4	43.7
E_p [GeV]	23.8	41
E_e [MeV]	13	22.3
CS length [m]	130	
Q_e/p-bunch [nC]	2×2.5	1
Average I_e [mA]	123	99
$\sigma_{\theta e}$ [urad]	30	20
σ_δ	5e-4	5e-4







- It is important to keep the angular alignment of the e-p trajectories within 25 urad (@ $\gamma = 43.7$) to avoid an effective heating of the vertical emittance by a circular attractor

For more details see: ***A. Fedotov, S. Benson et al., BNL-220686-2020-***

CeC-based Strong Hadron Cooler for EIC

- In the baseline EIC design the cooling at top energy is CeC-based
- In CeC the energy kick obtained by an individual ion in the kicker is a non-monotonic function of the ion's relative momentum
- An effect similar to coherent excitations in the regular EC is present in CeC scheme

$$\Delta E(z) = -V_0 \sin\left(2\pi \frac{z - z_1}{z_0}\right) \exp\left(-\frac{(z - z_1)^2}{\sigma_0^2}\right)$$

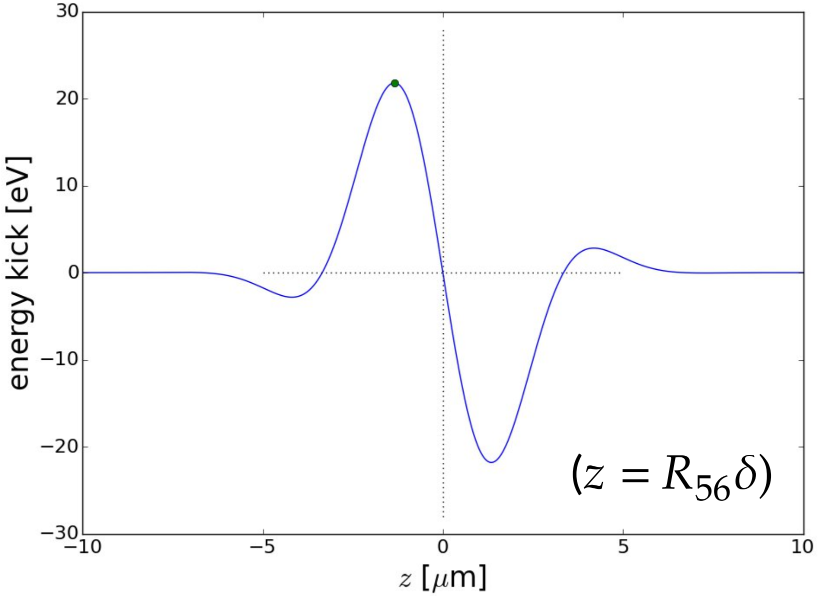


TABLE I. EIC cooler parameters

Kick amplitude (V_0) [eV]	28
Kick characteristic wavelength (z_0) [μm]	6.7
Kick characteristic width (σ_0) [μm]	3
Proton beamline R_{56} [mm]	2.2
Electron beamline R_{56}^e [mm]	1.6
Protons per bunch (N_p)	$6.9 \cdot 10^{10}$
Protons relative momentum spread ($\sigma_{\delta 0}$)	$6.8 \cdot 10^{-4}$
Proton bunch length (σ_{ps0}) [cm]	6
Electron bunch length (σ_{es}) [mm]	7

- For current EIC parameters, the systematic longitudinal e-p mismatch from the modulator to kicker must be kept below 1.3 um
- The mismatch (z_1) can result from γ–mismatch between two beams or from errors in magnets' settings.

Papers on CeC theory:

Y. S. Derbenev, AIP Conference Proceedings 253, 103 (1992)
V. N. Litvinenko and Y. S. Derbenev, Phys. Rev. Lett. 102, 114801 (2009)
G. Stupakov and P. Baxevanis, Phys. Rev. Accel.Beams 22, 034401 (2019)
S. Nagaitsev, V. Lebedev, G. Stupakov, E. Wang, W.Bergan, arXiv:2102.10239v1 [physics.acc-ph] (2021)

For more details on attaractor in CeC: ***S. Seletskiy, A. Fedotov, D. Kayran, arXiv:2106.12617v2[physics.acc-ph], 2021.***

Conclusion

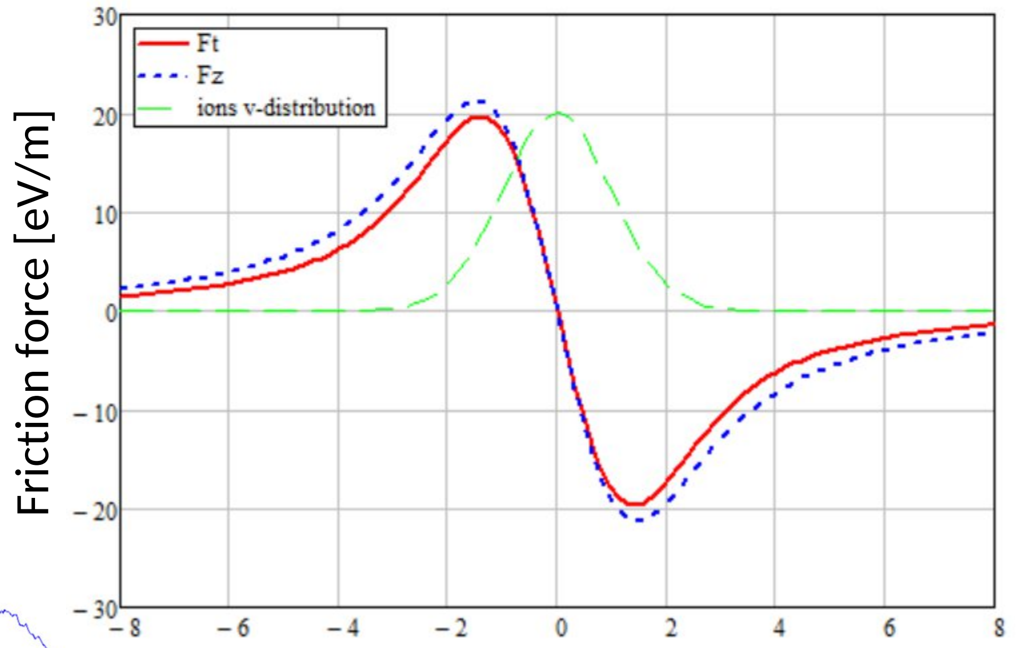
- We studied the effect of coherent excitations at LEReC and formation of the circular attractor in the longitudinal phase space of the ion bunch
- The experimental results are in good agreement with theoretical predictions
- The studied effect is important both for the high-energy electron coolers and for coherent electron coolers and must be taken into account in realistic design of the EIC coolers of any type

Backup slides

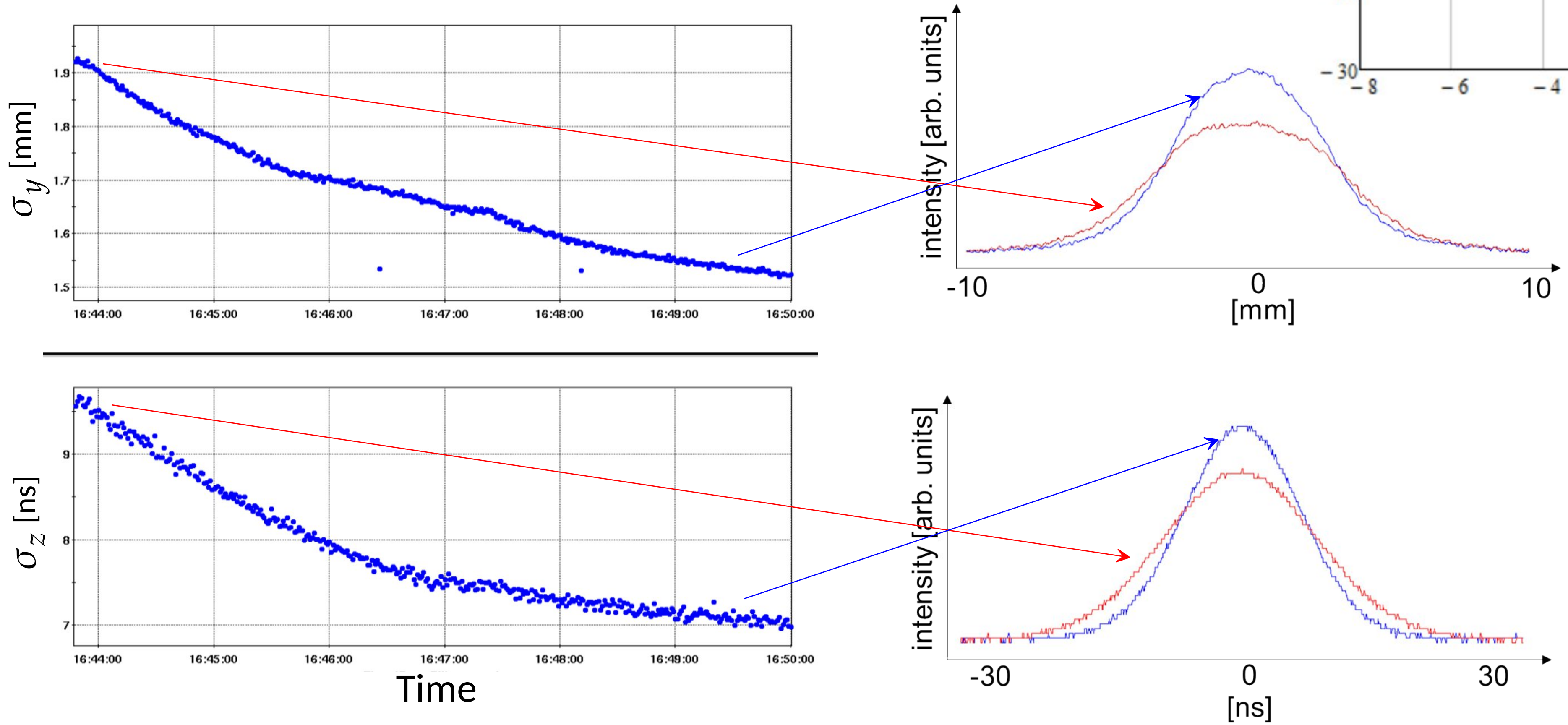
Cooling in LEReC

Typical 3D cooling for optimized e-beam parameters

- e&i-beams γ-factors are matched
- e-bunch energy spread is ~5e-4
- average e-bunch angular spread ~150 urad



$$\frac{v_{it}}{\sigma_{v_{et}}};\ \frac{v_{iz}}{\sigma_{v_{ez}}}$$



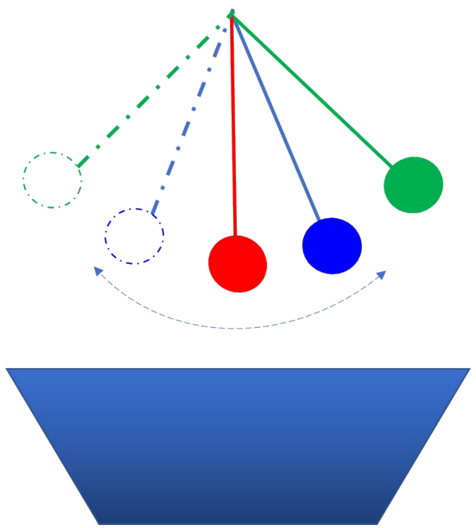
Friction force

- The friction force acting on an individual ion:

$$\vec{F} = -\frac{4\pi n_e e^4 Z^2}{m_e} \int L_C \frac{\vec{v} - \vec{v}_e}{|\vec{v} - \vec{v}_e|^3} f(v_e) d^3 v_e$$

$$f(v_e) = \frac{1}{(2\pi)^{3/2} \Delta_x \Delta_y \Delta_z} \exp\left(-\frac{v_{ex}^2}{2\Delta_x^2} - \frac{v_{ey}^2}{2\Delta_y^2} - \frac{v_{ez}^2}{2\Delta_z^2} \right)$$

- An analogy for the cooling process is a pendulum, which is briefly immersed, with some periodicity, into a bucket of water.



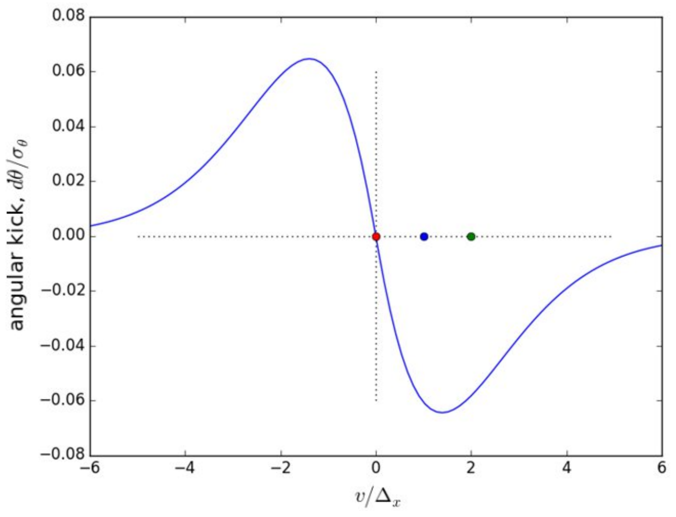
- If the pendulum’s oscillation amplitude is non-zero, then it will be experiencing a friction reducing its velocity.
- Consider just one component of $\vec{F}$, for example a horizontal friction force F_x:

$$\xi'' + \xi = \frac{L_{CS}\sqrt{\beta_{CS}}}{m_i c^2 \beta^2} F_x(v_x(\zeta))\, \mathbb{C}(\psi)$$

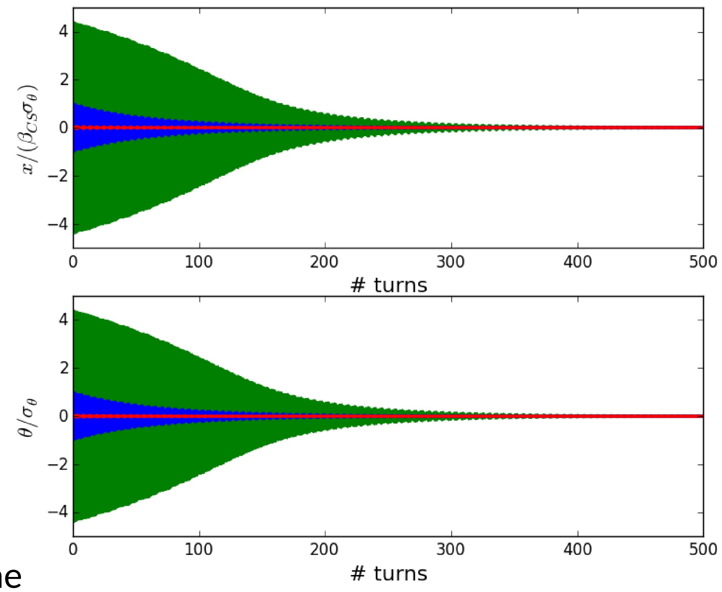
$$\xi = \frac{x}{\sqrt{\beta_T}}, \quad \zeta = \frac{\alpha_T}{\sqrt{\beta_T}} x + \sqrt{\beta_T}\theta_x$$

$$v_x(\zeta) = \zeta \gamma \beta c / \sqrt{\beta_{CS}}$$

$$\mathbb{C} = \sum_{n=0}^{\infty} \delta_D(\psi - 2\pi n Q_x)$$



an angular kick accumulated over the length of the cooling section (L_{CS}): $d\theta = \frac{F_x L_{CS}}{m_i c^2 \beta^2}$

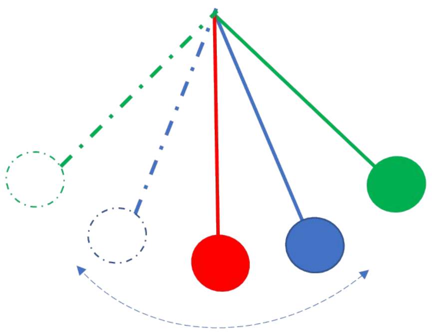


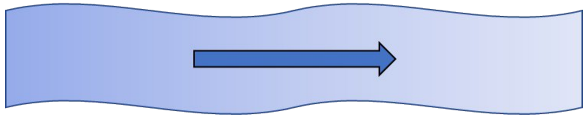
Friction force with coherent offset

- Assume that the e-bunch velocity distribution has an offset (for example μ_x)

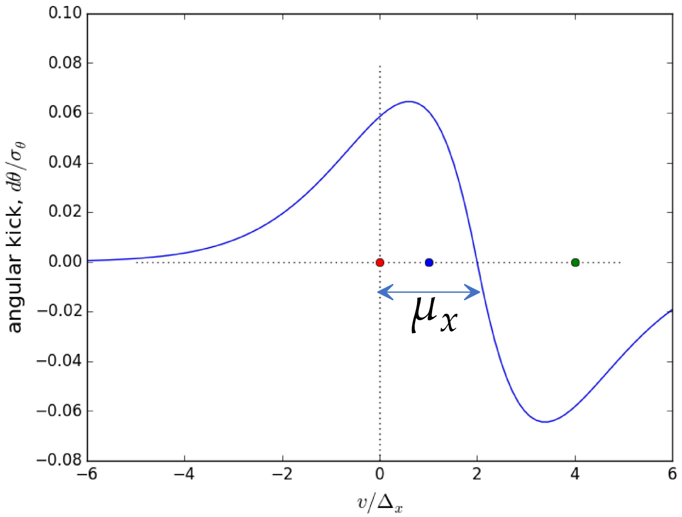
$$\vec{F} = -\frac{4\pi n_e e^4 Z^2}{m_e} \int L_C \frac{\vec{v} - \vec{v}_e}{|\vec{v} - \vec{v}_e|^3} f(v_e) d^3 v_e$$

$$f(v_e) = \frac{1}{(2\pi)^{3/2}\Delta_x \Delta_y \Delta_z} \exp - \left(\frac{(v_{ex} - \mu_x)^2}{2\Delta_x^2} + \frac{(v_{ey} - \mu_y)^2}{2\Delta_y^2} + \frac{(v_{ez} - \mu_z)^2}{2\Delta_z^2} \right)$$





- The friction force will be shifted by μ_x w.r.t. the zero ion velocity:



- In the mechanical analogy the pendulum is periodically immersed into a stream rather than a bucket of water

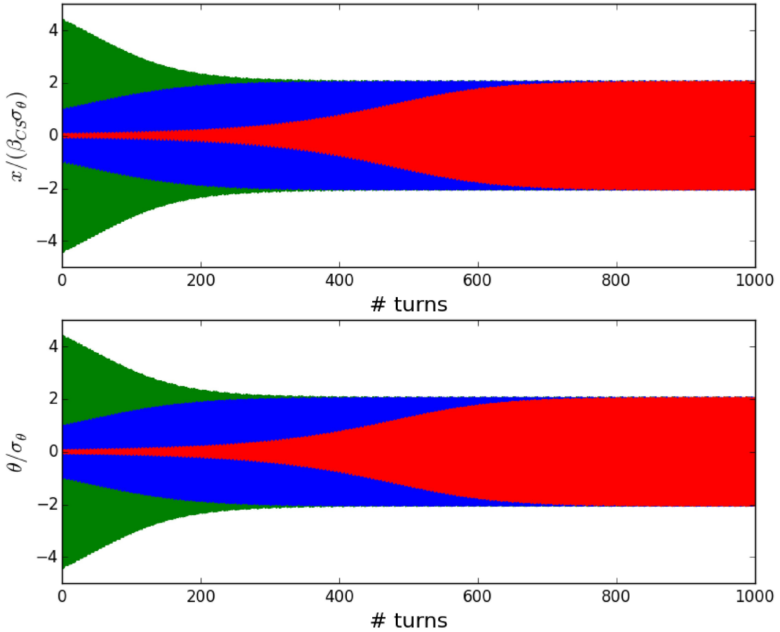
$$\xi'' + \xi = \frac{L_{CS}\sqrt{\beta_{CS}}}{m\gamma c^2 \beta^2} F_x(v_x(\zeta)) \mathbb{C}(\psi)$$

$$\xi = \frac{x}{\sqrt{\beta_T}}, \zeta = \frac{\alpha_T}{\sqrt{\beta_T}} x + \sqrt{\beta_T}\theta_x$$

$$v_x(\zeta) = \zeta\gamma\beta c / \sqrt{\beta_{CS}}$$

$$\mathbb{C} = \sum_{n=0}^{\infty} \delta_D(\psi - 2\pi n Q_x)$$

- If the shift is larger than the velocity at which the friction force's first derivative is changing sign, then the net force acting on an ion with small amplitude excites the oscillations. On the other hand, the ions with large amplitudes still experience a net damping force.



Binney formulas for friction force with an offset

$$\vec{F} = -C_0 \int \frac{\vec{\mathrm{v}} - \vec{v}_e}{\left|\vec{\mathrm{v}} - \vec{v}_e\right|^3} f(v_e)\, d^3 v_e \qquad C_0 = \frac{4\pi n_e e^4 Z^2 L_C}{m_e}$$

$$f(v_e) = \frac{1}{(2\pi)^{3/2}\Delta_x \Delta_y \Delta_z} \exp\left(-\frac{(v_{ex}-\mu_x)^2}{2\Delta_x{}^2} - \frac{(v_{ey}-\mu_y)^2}{2\Delta_y{}^2} - \frac{(v_{ez}-\mu_z)^2}{2\Delta_z{}^2} \right)$$

As long as $f(v_e)$ stays close to Gaussian

$$\begin{cases} F_{x,y} = -C\left(v_{x,y} - \mu_{x,y}\right) \displaystyle\int_0^\infty g_t(q)\, dq \\ F_z = -C\left(v_z - \mu_z\right) \displaystyle\int_0^\infty g_z(q)\, dq \end{cases}$$

$$g_t(q) = \frac{\mathrm{E}}{\Delta_t{}^2 (1+q)^2 \sqrt{\Delta_t{}^2 q + \Delta_z{}^2}} \qquad g_z(q) = \frac{\mathrm{E}}{(1+q)\left(\Delta_t{}^2 q + \Delta_z{}^2\right)^{\frac{3}{2}}}$$

$$\mathrm{E} = \exp\left[-\frac{(v_x-\mu_x)^2 + \left(v_y-\mu_y\right)^2}{2\Delta_t{}^2 (1+q)} - \frac{(v_z-\mu_z)^2}{2\left(\Delta_t{}^2 q + \Delta_z{}^2\right)} \right]$$

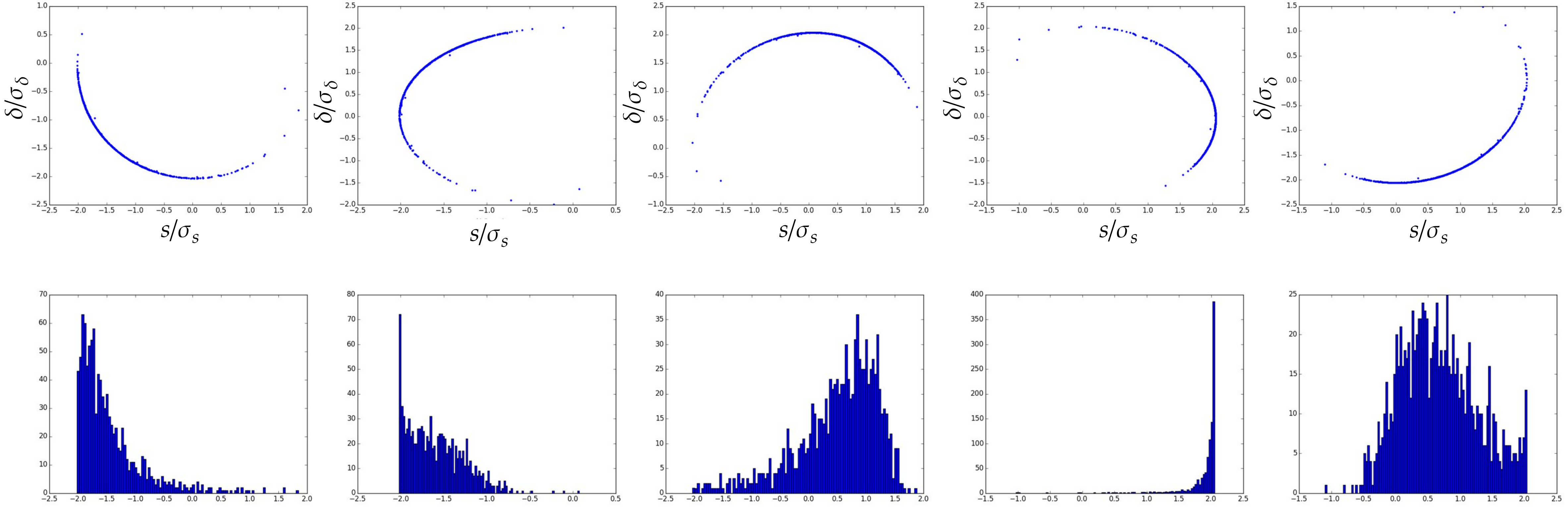
$$C = 2\sqrt{2\pi}\, n_e r_e{}^2 m_e c^4 Z^2 L_C$$

These are not the exact formulas when distribution becomes non-Gaussian, but they still give a fairly good approximation of the friction force for our examples

For more details see: *S. Seletskiy, A. Fedotov, D. Kayran, TUXA04, IPAC21 Proceedings, 2021*

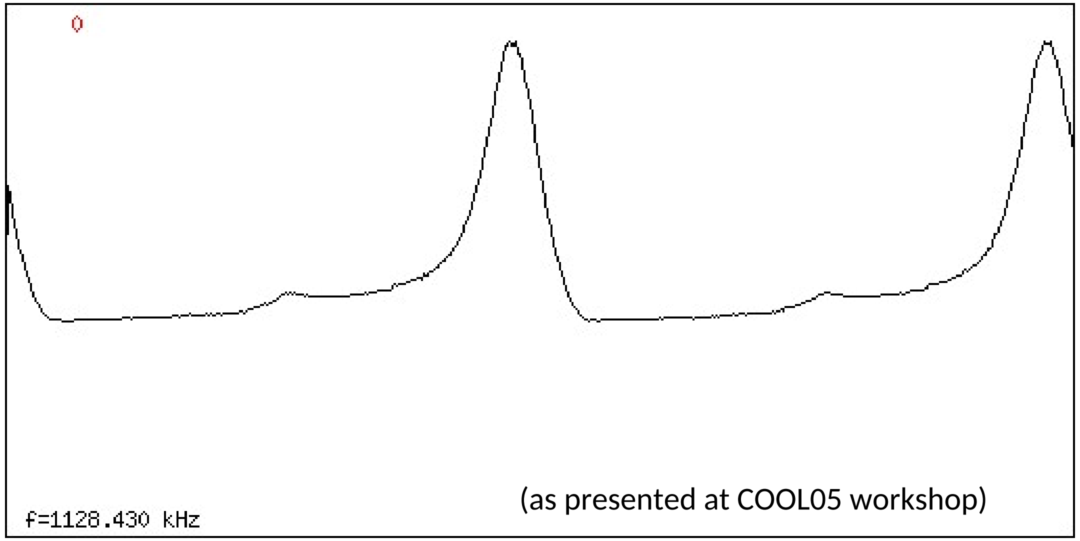
Bifurcations caused by circular attractor

- Introducing of a coherent offset to the pre-cooled bunches causes density bifurcations



Example of observations at non-relativistic cooler (CELSIUS, 2005)

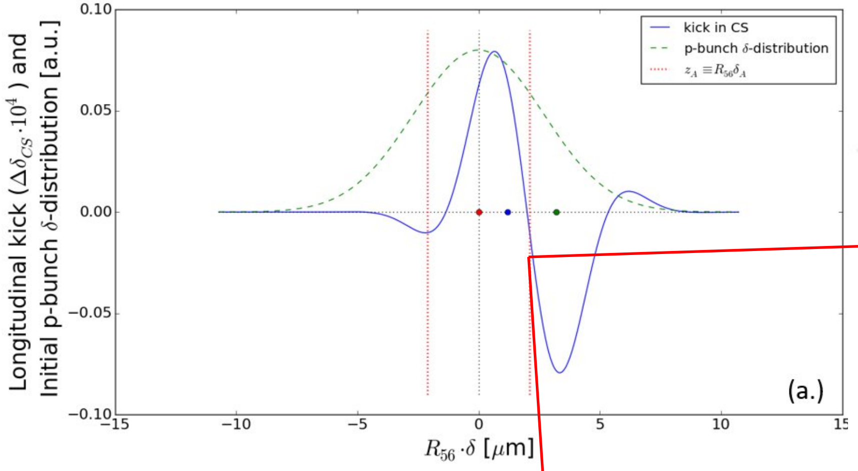
- The bifurcations of longitudinal density were obtained by first cooling the ion bunch to a small longitudinal emittance and then shifting the ring energy by $\approx 2\frac{\Delta_z}{\beta c}$

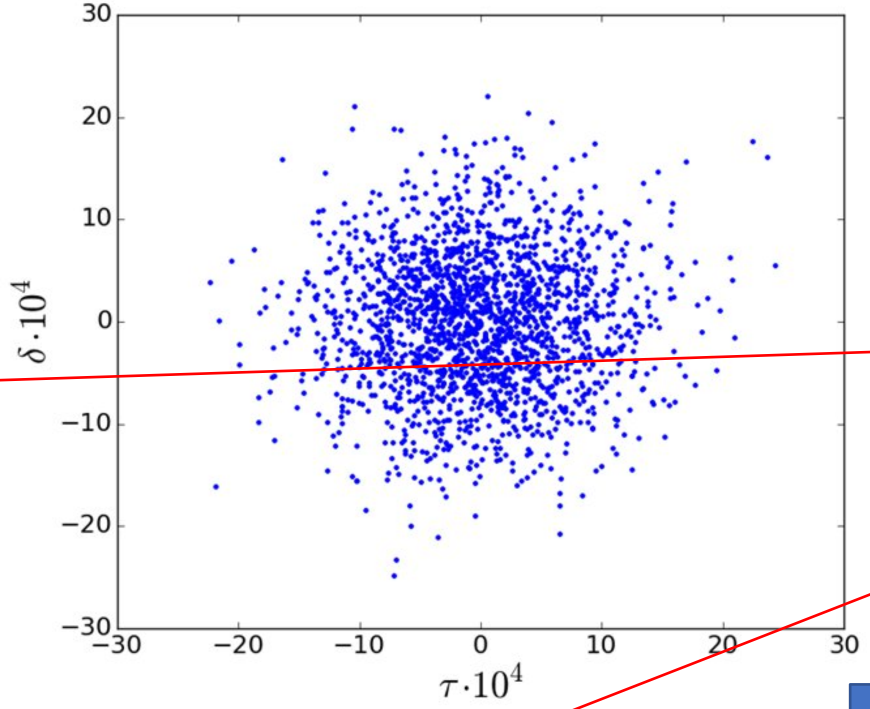


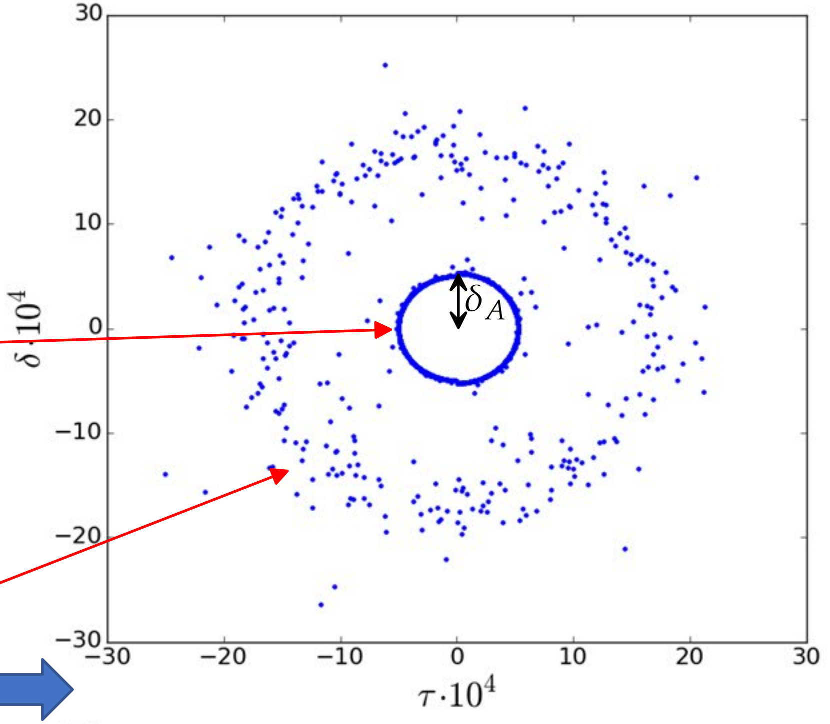
(as presented at COOL05 workshop)

A. Fedotov et al.: "Experimental benchmarking of the magnetized frictions force", *Proc. of COOL05 workshop*

CeC attractor

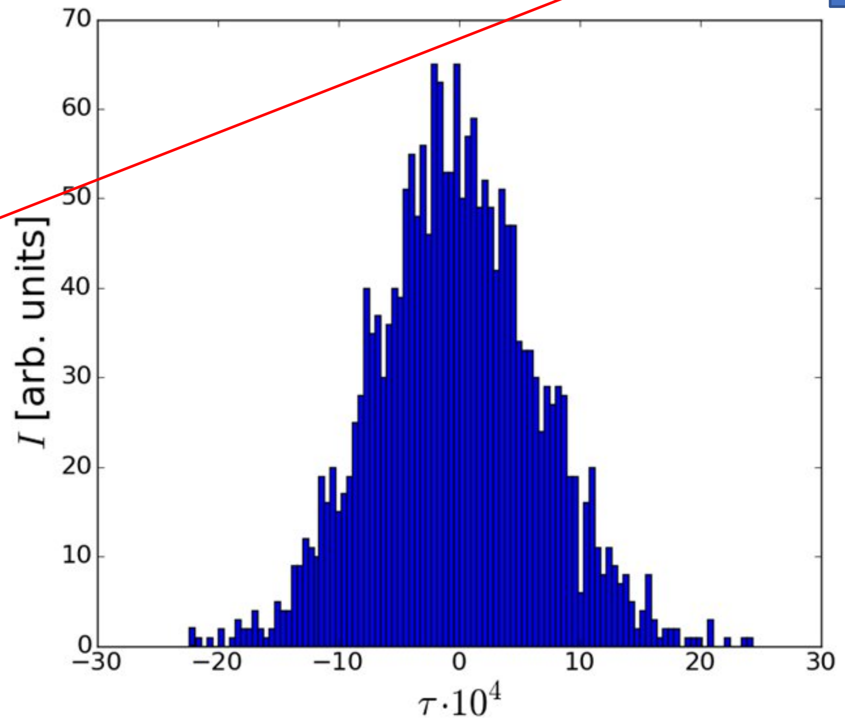


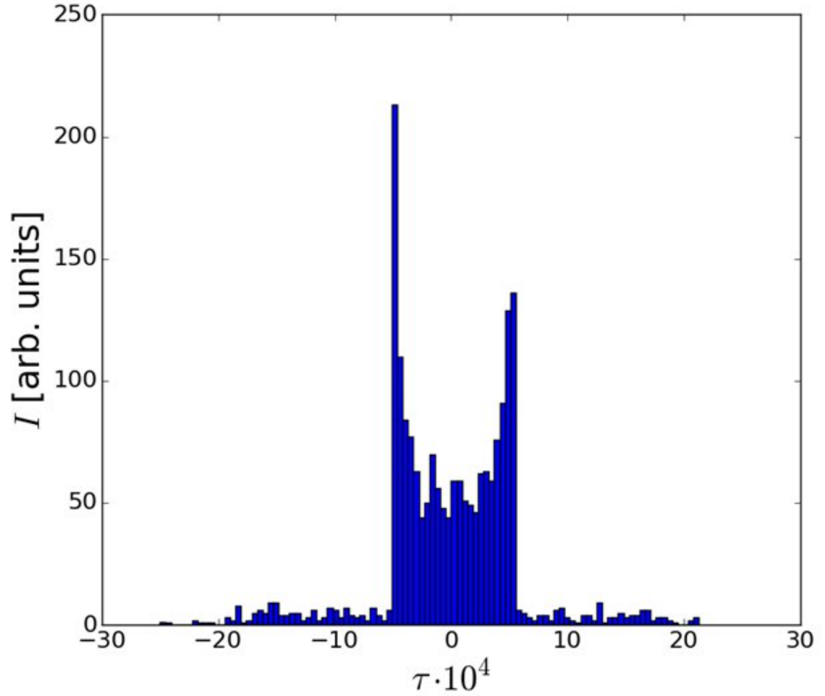




The attractor stops the cooling for all the protons with $\delta < \delta_A$. As a matter of fact, it "heats" all the small amplitude protons up.

A second "weak" attractor in the phase space exists because the cooling force has an actual "heating" portion and because the electron bunch longitudinally placed at center of the proton bunch is much shorter than the p-bunch.





Useful formulas for CeC attractor

$$\Delta E(z) = -V_0 \sin\left(2\pi \frac{z - z_1}{z_0}\right) \exp\left(-\frac{(z - z_1)^2}{\sigma_0^2}\right)$$

- Since we have a simple analytical expression approximating the wake, we can write a couple of useful formulas.

$$\boxed{\frac{z_1}{\sigma_0^2} = \frac{\pi}{z_0} \cos\left(\frac{2\pi}{z_0} z_1\right)}$$

$$\boxed{\begin{aligned} &\sin\left(2\pi \frac{R_{56}\delta_A - z_{off}}{z_0}\right) \exp\left(-\frac{(R_{56}\delta_A - z_{off})^2}{\sigma_0^2}\right) + \\ &\sin\left(2\pi \frac{R_{56}\delta_A + z_{off}}{z_0}\right) \exp\left(-\frac{(R_{56}\delta_A + z_{off})^2}{\sigma_0^2}\right) = 0 \end{aligned}}$$

$z_1 \approx 1.33\ \mu$m

Assuming that the systematic misalignment is caused by the error in p-beam energy ($\epsilon \equiv \frac{E - E_0}{E_0}$), we get for the critical relative energy error:

@ R56=4 mm, $\epsilon_1 \approx 3 \cdot 10^{-4}$

@ R56=1.3 mm, $\epsilon_1 \approx 1 \cdot 10^{-3}$