COOLING AND DIFFUSION RATES IN COHERENT ELECTRON COOLING CONCEPTS

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1 November 2021

Introduction

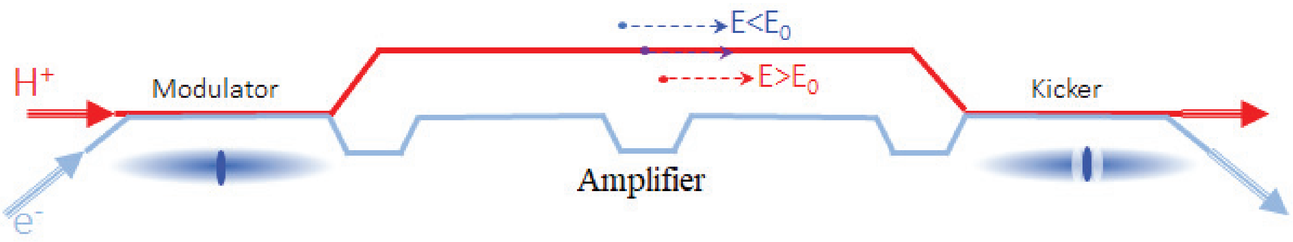
We present analytic cooling and diffusion rates for a simplified model of coherent electron cooling (CEC), based on a proton energy kick at each turn. This model also allows to estimate analytically the rms value of electron beam density fluctuations in the "kicker" section. Having such analytic expressions should allow for better understanding of the CEC mechanism, and for a quicker analysis and optimization of main system parameters. Our analysis is applicable to any CEC amplification mechanism, as long as the wake (kick) function is available.

Based on https://arxiv.org/abs/2102.10239

EIC Machine Parameters (CDR Table 3.3*)

Species	proton	electron	proton	electron	proton	electron	proton	electron
Energy [GeV]	275	18	275	10	100	10	100	5
CM energy [GeV]	140.7		104.9		63.2		44.7	
Bunch intensity [10^{10}]	19.1	6.2	6.9	17.2	6.9	17.2	4.8	17.2
No. of bunches	290		1160		1160		1160	
Beam current [A]	0.69	0.227	1	2.5	1	2.5	0.69	2.5
RMS norm. emit., h/v [µm]	5.2/0.47	845/71	3.3/0.3	391/26	3.2/0.29	391/26	2.7/0.25	196/18
RMS emittance, h/v [nm]	18/1.6	24/2.0	11.3/1.0	20/1.3	30/2.7	20/1.3	26/2.3	20/1.8
β^*, h/v [cm]]	80/7.1	59/5.7	80/7.2	45/5.6	63/5.7	96/12	61/5.5	78/7.1
IP RMS beam size, h/v [µm]	119/11		95/8.5		138/12		125/11	
K_x	11.1		11.1		11.1		11.1	
RMS $\Delta\theta$, h/v [µrad]	150/150	202/187	119/119	211/152	220/220	145/105	206/206	160/160
BB parameter, h/v [10^{-3}]	3/3	93/100	12/12	72/100	12/12	72/100	14/14	100/100
RMS long. emittance [10^{-3}, eV·s]	36		36		21		21	
RMS bunch length [cm]	6	0.9	6	2	7	2	7	2
RMS $\Delta p/p$ [10^{-4}]	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8
Max. space charge	0.007	neglig.	0.004	neglig.	0.026	neglig.	0.021	neglig.
Piwinski angle [rad]	6.3	2.1	7.9	2.4	6.3	1.8	7.0	2.0
Long. IBS time [h]	2.0		2.9		2.5		3.1	
Transv. IBS time [h]	2.0		2		2.0/4.0		2.0/4.0	
Hourglass factor H	0.91		0.94		0.90		0.88	
Luminosity [10^{33}cm^{-2}s^{-1}]	1.54		10.00		4.48		3.68	

CEC concepts



- Y. S. Derbenev, "On possibilities of fast cooling of heavy particle beams," *AIP Conference Proceedings*, vol. 253, no. 1, pp. 103–110, 1992, https://aip.scitation.org/doi/pdf/10.1063/1.42152
- V.N. Litvinenko and Y. S. Derbenev, "Coherent electron cooling," Phys. Rev. Lett., vol. 102, https://link.aps.org/doi/10.1103/PhysRevLett.102.114801
- D. Ratner, "Microbunched electron cooling for high-energy hadron beams," Phys. Rev. Lett., vol. 111, https://link.aps.org/doi/10.1103/PhysRevLett.111.084802

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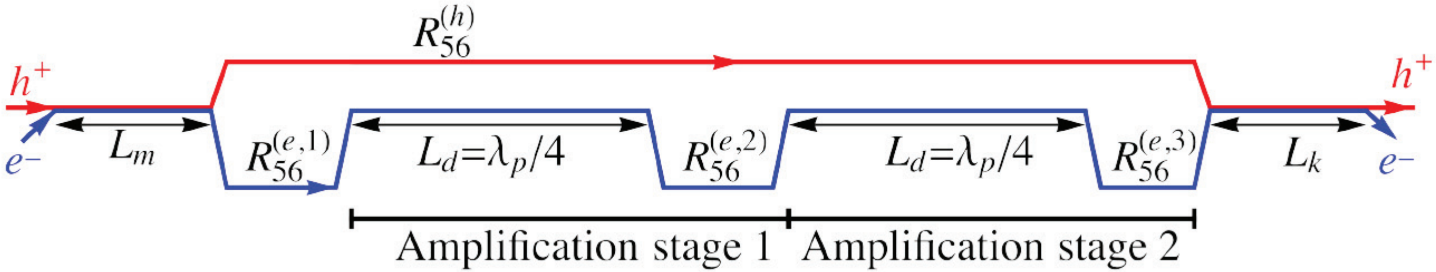
The MBEC concept

Micro-bunched electron cooling (MBEC) was proposed by D. Ratner (PRL, **111**, 084802 (2013)). It has an advantage of *broad-band amplification* (in contrast to the FEL).

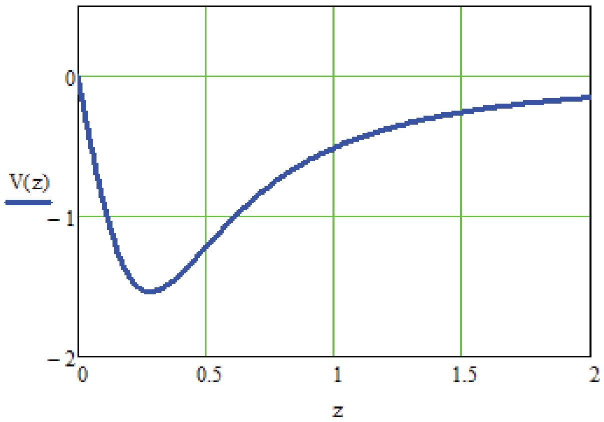


One stage of amplification is achieved through a combination of a drift of length=$\frac{1}{4}$ plasma oscillation length followed by a chicane. For the nominal EIC parameters, one stage amplification gain $G \approx \frac{1}{\Delta E/E}\sqrt{\frac{I_e}{I_A\gamma}} \approx 10\text{–}20.$

Courtesy of G. Stupakov

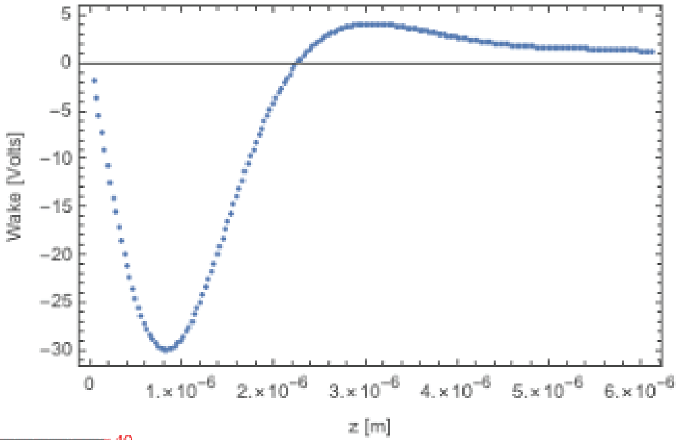
Wake field for a proton at the center of the electron beam

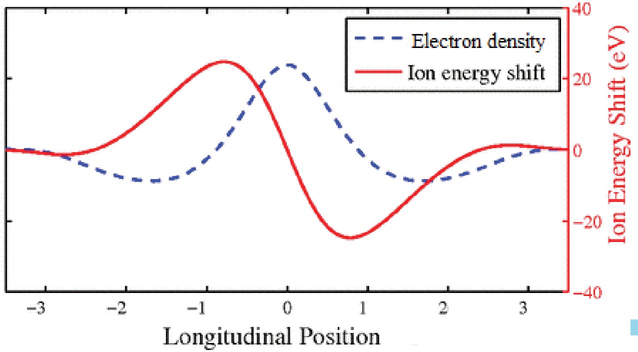
Initial modulation after "Modulator"



Amplification with some limited BW and some electron R56

Kick in the "Kicker" section





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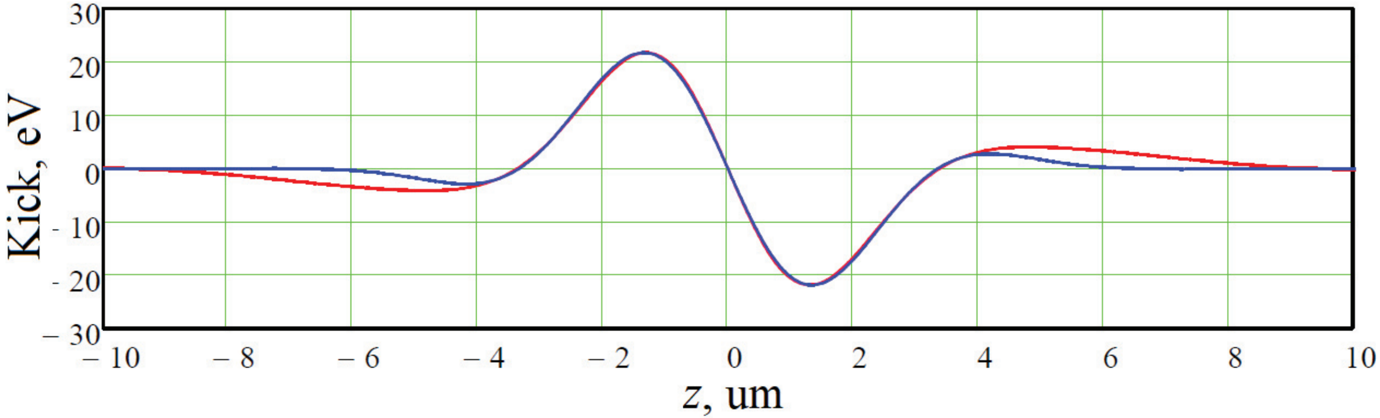
CEC system parameters (example)

Parameter	Symbol	Value	Unit
Proton energy	E_0	275	GeV
Lorentz factor	γ	290	
Ring circumference	C	3834	m
Revolution frequency	f_0	78.3	kHz
Protons per bunch	N_p	6.9	10^{10}
Prot. rms moment. spread	δ_p	6.8	10^{-4}
Prot. rms bunch length	σ_{pz}	6.0	cm
Electrons per bunch	N_e	6.3	10^9
El. rms bunch length	σ_{ez}	4.0	mm
El. rms beam size (vert)	σ_{ey}	0.6	mm
El. rms beam size (hor)	σ_{ex}	0.6	mm
Kicker section length	L_k	40	m

Wake approximation (Blue curve) and calculated (red)



We will approximate the wake by:

$$V(z) = V_0 \sin\left(2\pi \frac{z}{z_0}\right)\exp\left(-\frac{z^2}{\sigma_0^2}\right)$$

with $z_0 = 6.7$ um in this example

$$V_0 = 28\ \mathrm{V};\ \sigma_0 = 3.0\ \mu\mathrm{m}$$

$$V(z) = V_0 \sin\left(2\pi \frac{z}{z_0}\right) \exp\left(-\frac{z^2}{\sigma_0^2}\right)$$

- We are interested in 4 parameters:
  1. Cooling rate
  2. Diffusion rate
  3. Cooling range
  4. Electron beam rms density modulation
- The proposed approximation has 3 free parameters (amplitude V_0, wavelength z_0, and bandwidth σ_0)
  - Cooling rate depends on the slope ~V_0 / z_0
  - Diffusion rate depends on $V_0^2 \sigma_0$
  - Cooling range depends on R_{56} and z_0

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Fokker-Planck equation

$$\frac{\partial \psi}{\partial t} = -\sqrt{2\beta}\frac{\partial}{\partial J}\left(\sqrt{J}\tilde{F}(J)\psi\right) + \beta\frac{\partial}{\partial J}\left(J\tilde{D}(J)\frac{\partial \psi}{\partial J}\right)$$

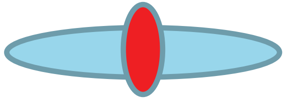
J is synchrotron action of a proton

β is the so-called longitudinal beta function, $\beta = \sigma_{pz}/\delta_p \approx 88$ m

$$\tilde{F}(J) = -\frac{f_0 V_0}{E_0}\frac{\sigma_{ez}}{\sqrt{2\pi\beta J}}\sin\left(2\pi\frac{R_{56}}{z_0}\sqrt{\frac{2J}{\beta}}\right)\exp\left(-\frac{R_{56}^2}{{\sigma_0}^2}\frac{2J}{\beta}\right)$$

$$\tilde{D} = \frac{D_0}{2}\frac{\sigma_{ez}}{2\sigma_{pz}}. \qquad D_0 = \frac{\langle (w(z)/E_0)^2\rangle}{T} = \frac{N_p f_0 {V_0}^2}{4{E_0}^2}\frac{\sigma_0}{\sigma_{pz}}\left(1 - \exp\left(-2\pi^2\frac{{\sigma_0}^2}{{z_0}^2}\right)\right)$$

- The electron bunch is much shorter than the proton bunch, we use the δ – function approximation for the electron density



$$\epsilon_L = \int_0^\infty \psi \, J \, dJ$$

-- Long emittance

$$\frac{1}{\epsilon_L}\frac{d\epsilon_L}{dt} = -\frac{1}{\tau_c} + \frac{\tilde{D}\beta}{\epsilon_L}$$

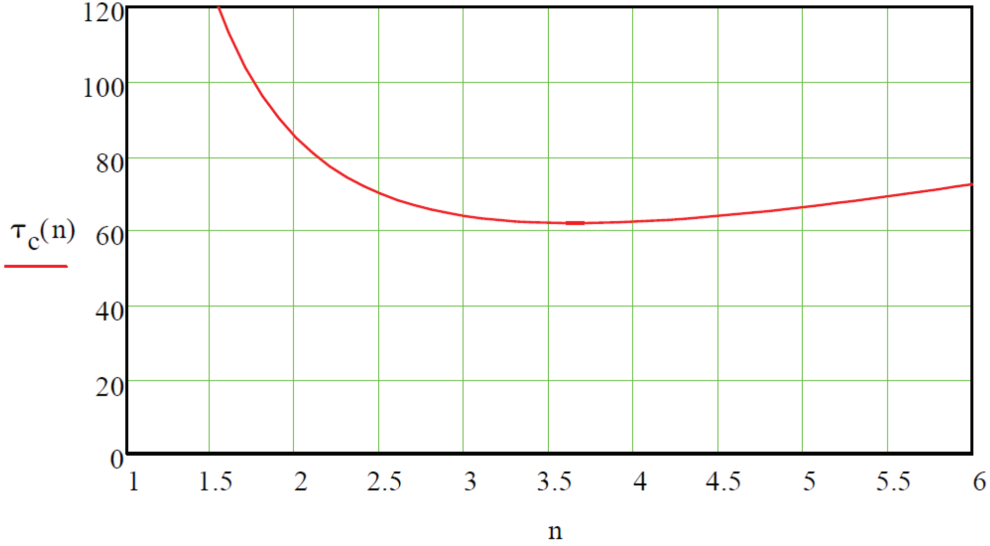
$$\frac{1}{\tau_c} = \frac{\sqrt{2\beta}}{\epsilon_L}\int_0^{+\infty} J \frac{\partial}{\partial J}\left(\sqrt{J}\tilde{F}(J)\psi\right) dJ,$$

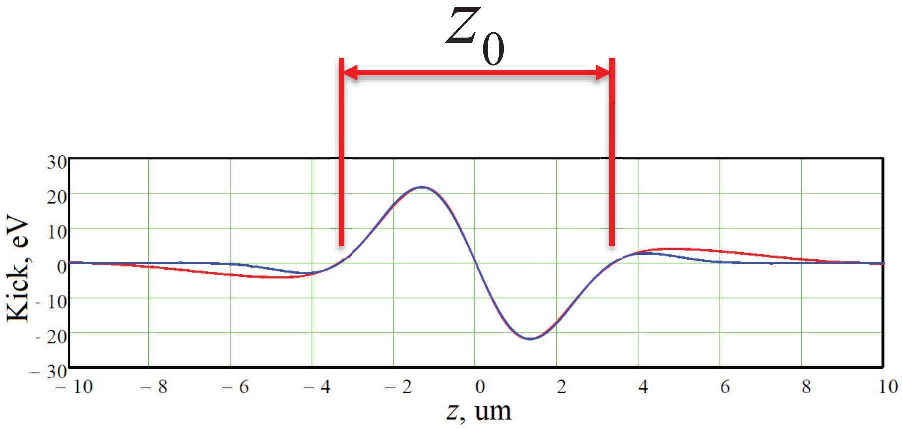
$$\frac{1}{\tau_c} = \lambda\left(1 + \frac{{z_0}^2}{2n^2{\sigma_0}^2}\right)^{-3/2} \exp\left(-\frac{\pi^2}{2n^2 + {z_0}^2/{\sigma_0}^2}\right), \qquad (5)$$

where n is the cooling range, such that $nR_{56}\delta_p = z_0/2$ and λ is the small-amplitude cooling rate,

$$\lambda = \frac{\pi f_0 V_0}{\delta_p n E_0}\frac{\sigma_{ez}}{\sqrt{2}\sigma_{pz}}. \qquad (6)$$

$$nR_{56}\sigma_p = \frac{z_0}{2}$$





$$V(z) = V_0 \sin\left(2\pi \frac{z}{z_0}\right) \exp\left(-\frac{z^2}{\sigma_0^2}\right)$$

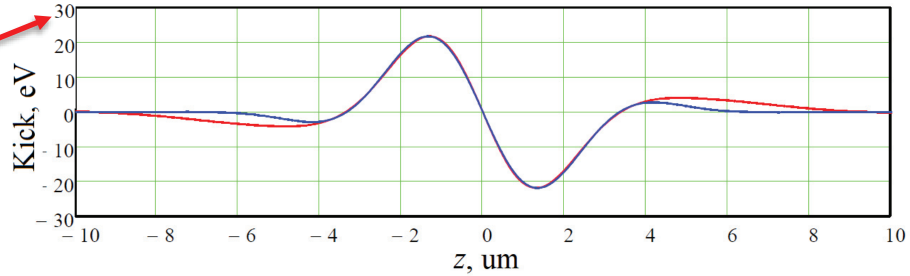
- The minimum cooling time (~60 minutes) is achieved at n ~ 3.7
  - Corresponds to proton R56 = 1.3 mm
  - Note: L/γ^2 = ~$100/290^2$ = 1.2 mm

Cooling and diffusion at optimal gain

$$\frac{1}{\epsilon_L}\frac{d\epsilon_L}{dt} = -\frac{1}{\tau_c} + \frac{\tilde{D}\beta}{\epsilon_L}$$

- For an optimal gain: $$\frac{1}{\tau_c} = \frac{2\tilde{D}\beta}{\epsilon_L}.$$

$$V_{opt} \approx 150 \text{ V}$$



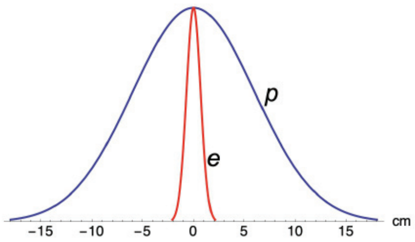
- With this kick, the cooling time becomes 24 minutes…

Optimal cooling time

$$\frac{1}{t_c} \sim \frac{\Delta f}{CNh/\sigma_z} \sim 0.5 \text{ min}^{-1}$$

$\Delta f = 40$ THz, C =3834 m, Nh=6.9×10^{10}, σ_z=6 cm

From previous talk by G. Stupakov



The electron bunch length is shorter than the hadron one. Hadrons with large synchrotron amplitudes spend a fraction of time inside the electron beam.

- Cooling time should be increased by the ratio of bunch lengths (proton/electron)
- The resulting cooling time is then ~20 min

$$\delta E_{rms} = E_0\sqrt{D_0 T} \approx 26\ \mathrm{keV}.$$

$$\frac{dE_z}{dz'} = 4\pi e n_e(z'),$$ -- 1D model

$$\frac{\sqrt{\langle n_e'^2\rangle}}{n_{e0}'} \approx \frac{kV_0\sigma_{ez}\sigma_{ex}\sigma_{ey}}{2e^2L_k}\frac{\sqrt{N_p}}{N_e}\sqrt{\frac{\sqrt{2\pi}\sigma_0}{\sigma_{pz}}} \approx 0.15.$$

- A 3D model gives ~18% relative rms density fluctuations for the proposed wake with $V_0 = 28\ \mathrm{V}$

- Contributions from electrons (shot noise only) are similar or 25% combined. It's desirable to have at least 2.5-sigma range, or 64%

- The proposed wake model allows to estimate cooling, diffusion and modulation analytically. Having such analytic expressions should allow for better understanding of the CEC mechanism, and for a quicker analysis and optimization of main system parameters.
- Even though we have used the wake calculated for the MBEC amplification scheme, our analysis can be easily applied to other coherent cooling techniques, for example, to the PCA concept, as long as the wake (kick) function is available.