A Perturbative Technique for 3D Modeling of the Microbunched Electron Cooling Concept

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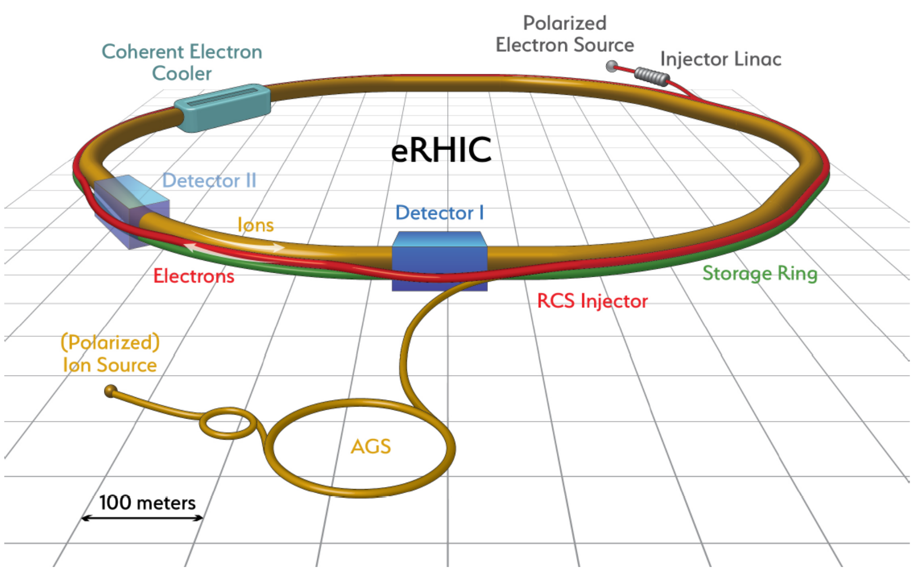
Abstract

Because the efficacy of conventional electron cooling falls off rapidly with energy, reaching the required cooling time at collision energies targeted by the Electron-Ion Collider design can be challenging. A possible solution is offered by cooling schemes that are based on fundamentally different approaches such as microbunched electron cooling (MBEC). Regular PIC simulations in the parameter regime of the EIC cooling system would require a prohibitively large number of particles to resolve the evolution of the ion-imprinted phase space density modulation. We explored a solution to this problem by developing and implementing in the code Warp a computational approach based on two perturbative techniques, the beam-frame δf method and a variant of the distribution difference (DD) technique. To model the dynamics of the ion-seeded modulation in the MBEC chicanes, we developed an approach that combines the DD and quiet start techniques with analysis of correlations between the divergence of pairs of DD trajectories and their location within the e-beam. We have also prototyped in Warp the computation of the time-dependent 3D wakefield in the MBEC kicker.

Motivation – Design of Strong Hadron Cooling Systems

- Polarized electron-ion collider (EIC) at BNL
  - *a high priority for the worldwide nuclear physics community*
- Strong hadron cooling *at relativistic energies*
  - *may be essential for the EIC, but never before demonstrated*

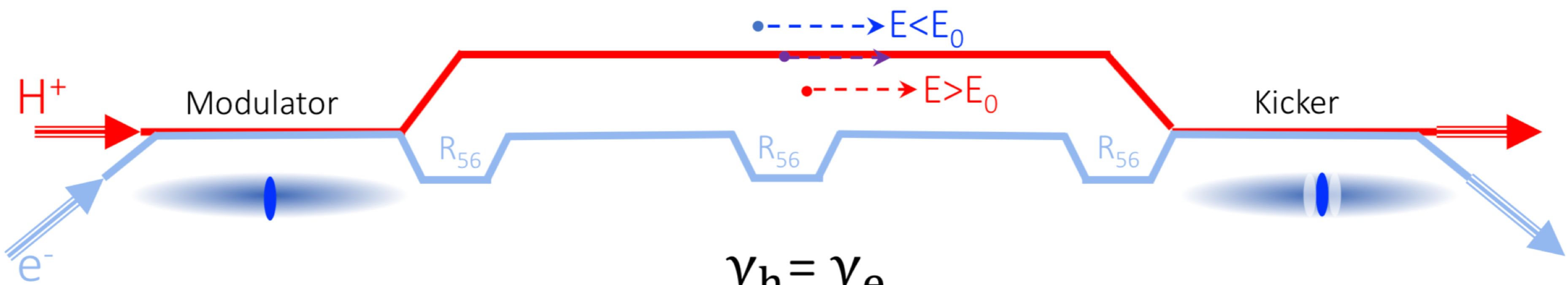
EIC concept from BNL



"Electron-Ion Collider at BNL Conceptual Design Report" (2021)
https://www.bnl.gov/ec/files/EIC_CDR_Final.pdf

- Achieving the EIC design luminosity will likely require cooling the protons at collision energy (γ ~ 300)
- Efficacy of classical electron cooling techniques falls off rapidly with energy, making cooling at collision energy challenging; this provides motivation for the study of alternative approaches such as coherent electron cooling concept and its variants

Microbunched electron cooling (MBEC)



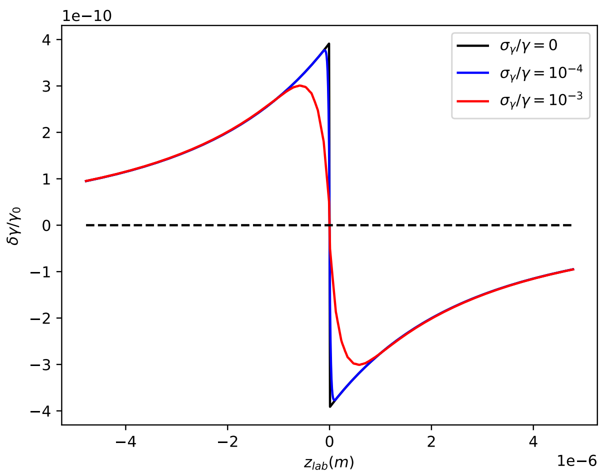
$$\gamma_h = \gamma_e$$

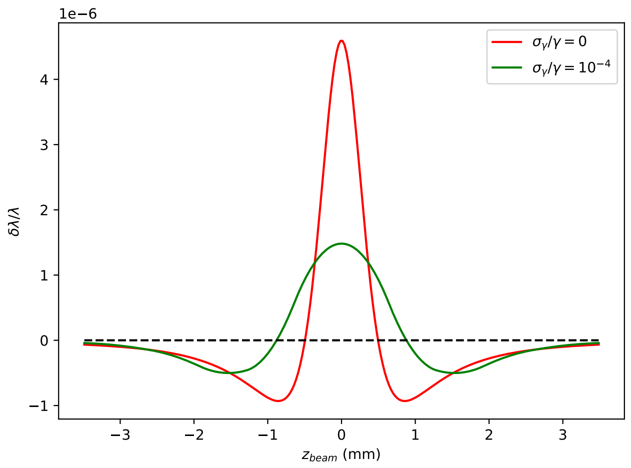
A schematic of the cooling system under study (after D.Ratner, PRL 111, p. 084802 (2013))

- Microbunched electron cooling is a 3-stage scheme:
  - *electron and ion beams co-propagate through the modulator section, each ion imprints an energy modulation on nearby electrons*
  - *in the amplifier, electron energy modulation is converted to a density spike via R_{56} in the chicane(s), protons are phase-shifted in the hadron dispersion section*
  - *the two beams are brought together in the kicker section, protons receive an energy kick (positive or negative) from the amplified electron density spike, so that the energy spread in the proton beam is reduced*

PIC Simulations of Phase Space Modulation Dynamics are Challenging

- *1D* theory predicts exceedingly subtle modulations of the e-beam phase space distribution in the parameter regime of interest





Left: Proton-induced e-beam energy modulation in the modulator section for different values of the e-beam thermal energy spread. Right: Longitudinal density modulation from 1D theory after the first chicane for the thermal energy spread of 10^{-4}

- Due to discreteness noise, standard *3D PIC* is prohibitively expensive for resolving the ion-seeded modulation of the e-beam phase space:
  - *density modulations of ~ 10^{-6} have to be resolved*
  - *energy modulations ~10^{-9} in the presence of thermal energy spread of ~ 10^{-4}*

δf PIC Algorithm Reduces Noise by Representing by Particles Only the Perturbation

- S.E. Parker and W.W. Lee, "A fully nonlinear characteristic method for gyrokinetic simulation," Phys. Fluids B **5**, 78 (1992).
- G. Hu and J.A. Krommes, "Generalized weighting scheme for δf particle-simulation method," Phys. Plasmas **1**, 863 (1994).
- N. Xiang et al., "Low-noise electromagnetic δf particle-in-cell simulation of electron Bernstein waves," Phys. Plasmas **13**, 062111 (2006)

- δf is a perturbative technique: $f(t, \boldsymbol{x}, \boldsymbol{v}) = f_0(t, \boldsymbol{x}, \boldsymbol{v}) + \delta f(t, \boldsymbol{x}, \boldsymbol{v})$
  - $f_0 (t, \boldsymbol{x}, \boldsymbol{v})$ is assumed continuous and analytically known
  - f_0 is constant along the characteristics of the unperturbed motion
  - weight is defined by $w = (f - f_0) / f$
  - convenient to choose $f(t=0) = f_0(t=0)$, so that $w(t=0) = 0$
- δf PIC: $\delta f(t, \boldsymbol{x}, \boldsymbol{v})$ is represented by variable-weight particles
  - half-step staggered δf particle velocity and coordinate updates
  - velocity updated using the Boris push
  - in continuous time, weights are evolving according to

$$\frac{dw}{dt} = -\frac{1}{f_0}(1-w)\frac{df_0}{dt} = -(1-w)\left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{x}} + \frac{q}{m}(\vec{E}_{total} + \vec{v}\times\vec{B}_{total})\frac{\partial}{\partial \vec{v}}\right)\ln(f_0)$$

  - 2nd order accurate discretization of the weight updates

$$w^n = w^{n-1/2} + \frac{dt}{2}\frac{q}{m}\left(\mathbf{E}_{pert}(\mathbf{x}^n, t_n) + \mathbf{v}^{n-1/2}\times\mathbf{B}_{pert}(\mathbf{x}^n, t_n)\right)\frac{\partial}{\partial \mathbf{v}}\ln f_0(\mathbf{x}^n, \mathbf{v}^{n-1/2}, t_n)$$

$$w^{n+1/2} = w^n + \frac{dt}{2}\frac{q}{m}\left(\mathbf{E}_{pert}(\mathbf{x}^n, t_n) + \mathbf{v}^{n+1/2}\times\mathbf{B}_{pert}(\mathbf{x}^n, t_n)\right)\frac{\partial}{\partial \mathbf{v}}\ln f_0(\mathbf{x}^n, \mathbf{v}^{n+1/2}, t_n)$$

Using the δf Algorithm Formulated for the Beam Frame with Time-Dependent f_0 in the Modulator

- Convenient to work in the beam frame
  - *non-relativistic dynamics and electrostatic field solve*
  - *$f_0 = f^{(x)} f^{(y)} f^{(z)}$ given in terms of Twiss parameters and beam-frame variables:*

$$f_0^{(x)}(s,x,x') = C \exp\left(- \frac{\hat{\gamma}_x(s)x^2 + 2\hat{\alpha}_x(s)xx' + \hat{\beta}_x(s)x'^2}{2\epsilon_x} \right)$$

$$\frac{dw}{dt} = -\frac{1}{f_0}(1-w)\frac{df_0}{dt} = -(1-w)\left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{x}} + \frac{q}{m}(\vec{E}_{total} + \vec{v}\times\vec{B}_{total})\frac{\partial}{\partial \vec{v}}\right)\ln(f_0)$$

Beam frame:

$$\frac{\partial}{\partial t_b}\ln f_0^{(x)} = \gamma_0\beta_0 c\frac{\partial}{\partial s}\left[-\frac{1}{2\epsilon_x}\left(\hat{\gamma}_x(s)x^2 + \frac{2\hat{\alpha}_x(s)}{\gamma_0\beta_0 c}xv_{bx} + \frac{\hat{\beta}_x(s)}{(\gamma_0\beta_0 c)^2}v_{bx}^2\right)\right]$$

$$= -\frac{\gamma_0\beta_0 c}{\epsilon_x}\left[\frac{\hat{\alpha}_x(s)}{\hat{\beta}_x(s)}\left(\frac{\partial\hat{\alpha}_x(s)}{\partial s} + \hat{\gamma}_x(s)\right)x^2 + \frac{1}{\gamma_0\beta_0 c}\frac{\partial\hat{\alpha}_x(s)}{\partial s}xv_{bx} - \frac{\hat{\alpha}_x(s)}{(\gamma_0\beta_0 c)^2}v_{bx}^2\right] ,$$

$$\frac{\partial}{\partial x}\ln f_0^{(x)} = -\frac{1}{\epsilon_x}\left[\hat{\gamma}_x(s)x + \frac{\hat{\alpha}_x(s)}{\gamma_0\beta_0 c}v_{bx}\right] ,$$

$$\frac{\partial}{\partial v_{bx}}\ln f_0^{(x)} = -\frac{1}{\epsilon_x}\left[\frac{\hat{\alpha}_x(s)}{\gamma_0\beta_0 c}x + \frac{\hat{\beta}_x(s)}{(\gamma_0\beta_0 c)^2}v_{bx}\right] ,$$

$$\frac{\partial}{\partial v_{bz}}\ln f_0^{(z)} = -\frac{v_{bz} - < v_{bz} > (s)}{\sigma_{v_{bz}}^2}$$

- Weights stay constant in bends if space charge is negligible
  - *arguments of f_0 are transformed with the cumulative transport matrix for the chicane*

Capabilities of the Warp Code

- D.P. Grote, A. Friedman, J.-L. Vay, I. Haber, "The WARP Code: Modeling High Intensity Ion Beams," AIP Conf. Proc. **749**, 55 (2005).
- J.-L. Vay, D.P. Grote, R.H. Cohen and A. Friedman, "Novel methods in the Particle-In-Cell accelerator Code-Framework Warp," Comput. Sci. & Disc. **5**, 014019 (2012).
- Open source, https://bitbucket.org/berkeleylab/warp

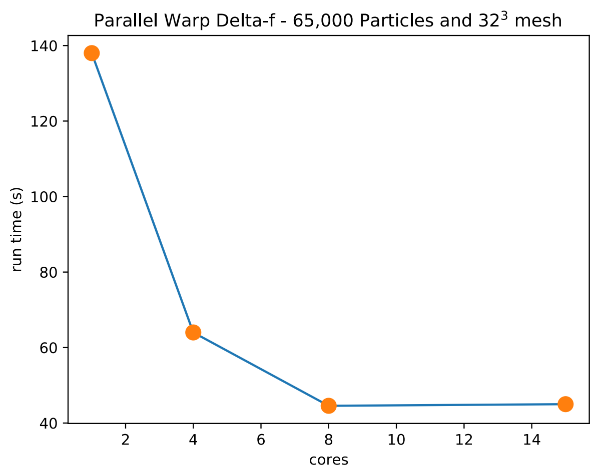
- Warp is used for many applications:
  - *non-neutral plasmas in traps*
  - *stray "electron clouds" in accelerators*
  - *laser-based acceleration*
  - *focusing of ion beams produced when short-pulse lasers irradiate foil targets*

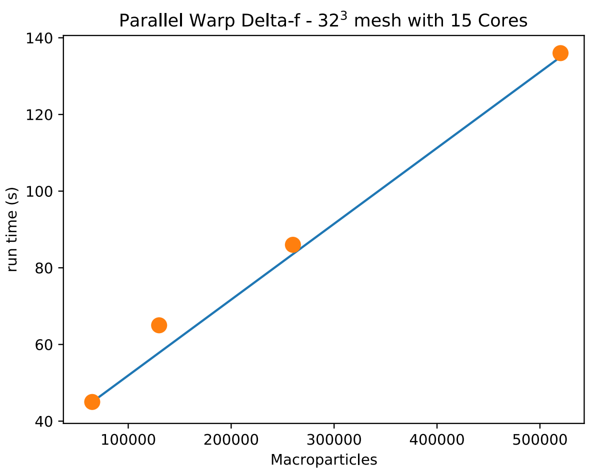
- Novel algorithms implemented in Warp include:
  - *an interactive Python-Fortran-C structure that enables scripted and interactive user "steering" of runs*
  - *a variety of geometries (3-D x, y, z; 2-D r, z; 2-D x,y)*
  - *a cut-cell representation for internal boundaries with electrostatic PIC*
  - *adaptive mesh refinement, including space-charge-limited flow from curved surfaces*
  - *models for particle interactions with gas and walls*
  - *a "drift-Lorentz" mover for rapid tracking through regions of strong and weak magnetic field*
  - *a Lorentz-boosted frame formulation with a Lorentz-invariant modification of the Boris mover*
  - *an electromagnetic solver with tunable dispersion and stride-based digital filtering*

- Warp is available as an open-source code under a BSD license.

We Implemented the δf PIC Algorithm in Warp

- Implementation through Warp's Python wrapper
  - *Two versions of f_0 calculation available*
    - No space charge or external fields, only Twiss parameters propagated in drift
    - Space charge envelope calculation in a drift, based on current and initial Twiss parameter
  - *Two options for describing perturbing field*
    - Analytic calculation of an ion field – calculated directly from particle positions (field passed to Warp to apply suitable kick at each time step)
    - Meshed field – provide pre-calculated field on a mesh to Warp. Faster evaluation, in theory, but getting δf to converge to the analytic calculation can require very high resolution mesh
  - *Scaling tested up through single workstation load*





Strong and weak scaling results from initial testing of the parallelized δf PIC in Warp

Benchmarking of Warp δf PIC Simulations of the MBEC Modulator

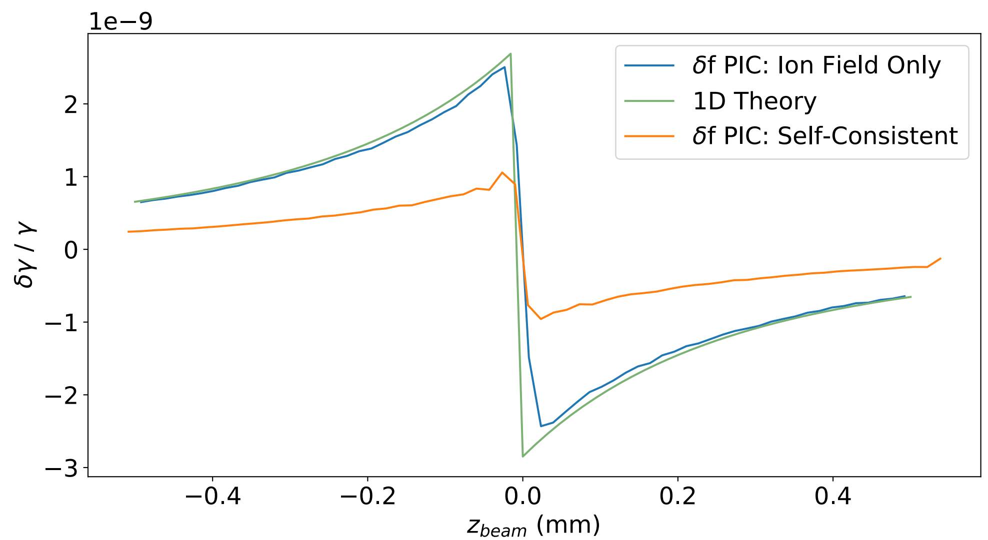
- Both density and energy modulation are imprinted in the electron beam by the hadrons
- Of primary interest (and more difficult to resolve) is the correlated *energy modulation* at the end of the modulator
- 1D theory (G. Stupakov, PRAB 21, 114402 (2018)) for a cold e-beam predicts that relative energy modulation in the modulator is given by

$$\frac{\delta\gamma}{\gamma} = -\frac{Zr_eL_m}{\gamma\sigma_r^2}\Phi\left(\frac{\gamma z_{lab}}{\sigma_r}\right) \quad \text{with} \quad \Phi(x) = \frac{1}{2}\left[\frac{x}{|x|} - \frac{x\sqrt{\pi}}{2}\exp\left(\frac{1}{4}x^2\right) erfc\left(\frac{1}{2}|x|\right)\right]$$

Comparison shown at right is for

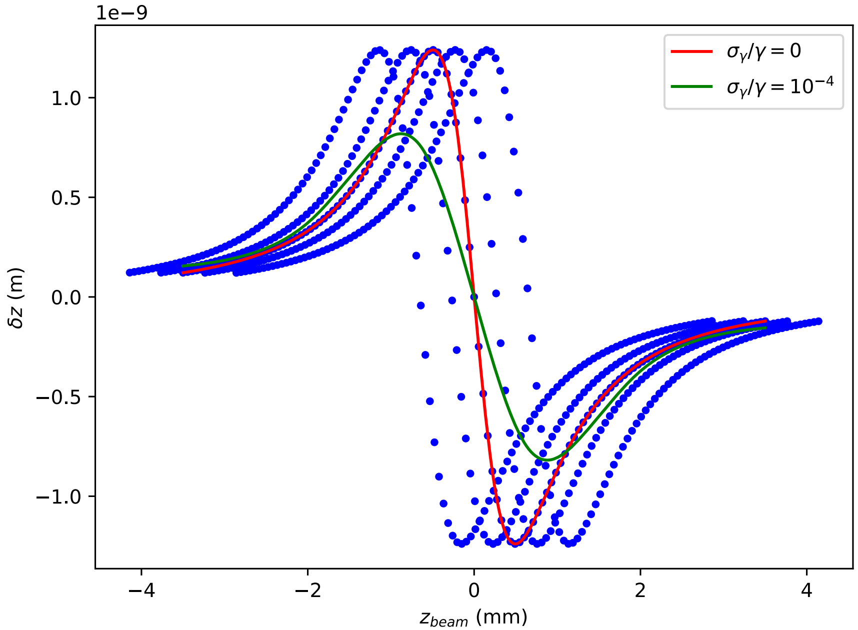
$$L_m = 40\ m,\ \gamma = 313,\ Z = 10,\ \sigma_r = .12$$

Right: Ion-imprinted relative energy modulation at the end of the modulator from 3D δf-PIC simulation (with and without space charge due to electron density modulation), compared with the cold-beam 1D theory prediction



Fast computation of the density modulation

- Approach similar to distribution difference in that pairs of tracer particles are used:
  - *unperturbed dynamics in the modulator for one particle, perturbed for the other*
  - *pairwise-identical coordinates and thermal v at entrance to the chicane, with one particle carrying an integrated over the trajectory velocity modulation*
  - *a quiet-start distribution arranged for the chicane simulation can be described as a "quiet finish" for the modulator*
  - *velocity modulation causes the particles in each pair to diverge by a pair-specific δz after the chicane*
- In contrast to distribution difference technique, the difference between the histograms of perturbed and unperturbed distributions is not computed:
  - *instead, use the correlations between δz and tracer particle z coordinate after the chicane*



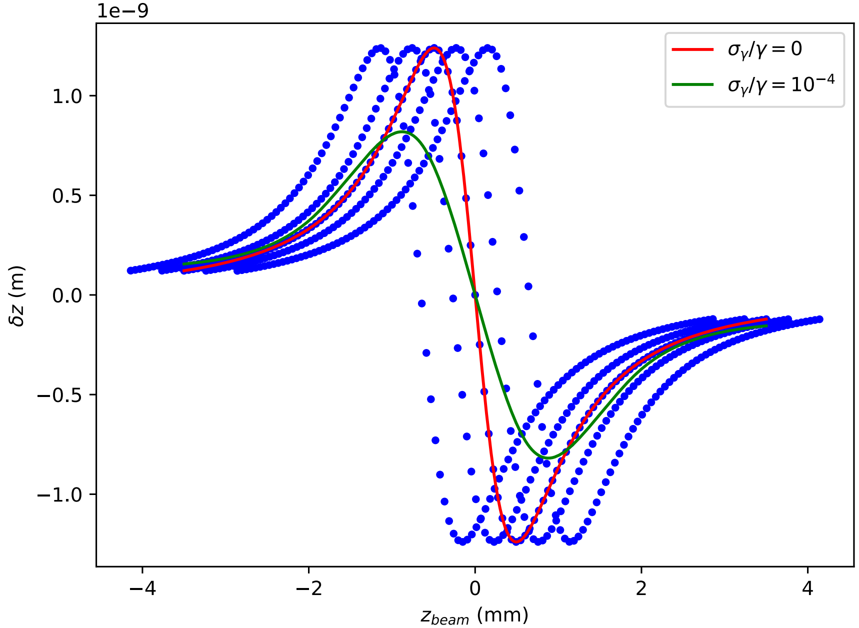
A correlation between the perturbed/unperturbed particle-pair divergence δz and particle position after the chicane for the $\delta\gamma/\gamma = 10^{-4}$ case. Pairs in quintuplets differ by thermal velocity.

For comparison with G. Stupakov, PRAB 21, 114402 (2018), we used a round, transversely cold beam with

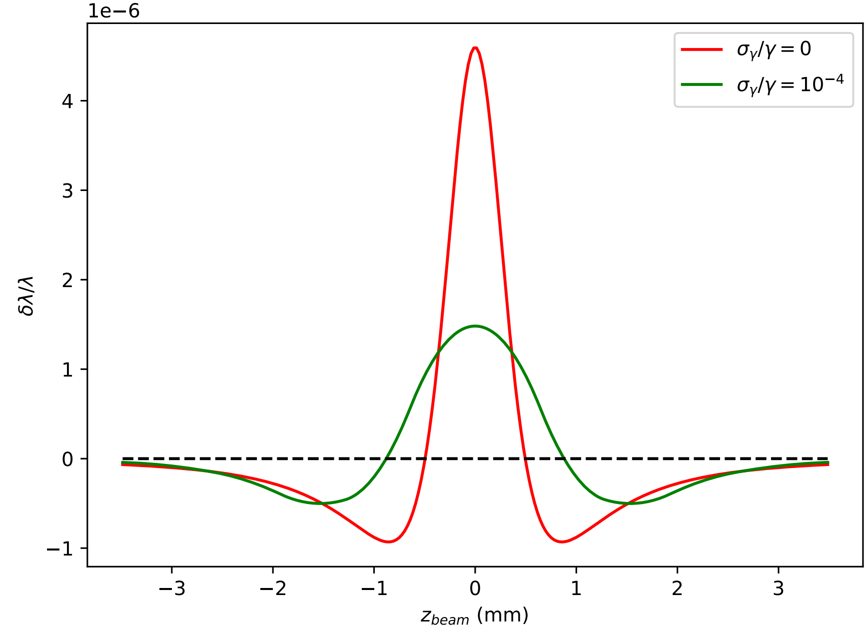
$$\gamma_0 = 313, \quad \sigma_\gamma/\gamma = 1. \times 10^{-4}$$
$$\sigma_x = 0.7\ mm, \quad R_{56} = 1.4\ cm$$

Data shown above for impact parameter b = $1\sigma_x$

Fast computation of the density modulation (cont-d)



A correlation between the perturbed/unperturbed particle-pair divergence δz and particle position after the chicane for the $\delta\gamma/\gamma = 10^{-4}$ case. Each pair in a quintuplet has different thermal velocity.



Longitudinal density modulation for warm and cold e-beam computed from the δz-z correlation data

- At each z, compute the distribution-averaged ⟨δz(z)⟩:
  - *horizontal extent of quintuplets shows the extent of washing-out due to the finite beam temperature, which causes the warm-beam result (green) to differ from the cold-beam result (red)*

Longitudinal relative density modulation is calculated as

$$\frac{\delta\lambda}{\lambda} = -\frac{d}{dz}\langle\delta z(z)\rangle$$

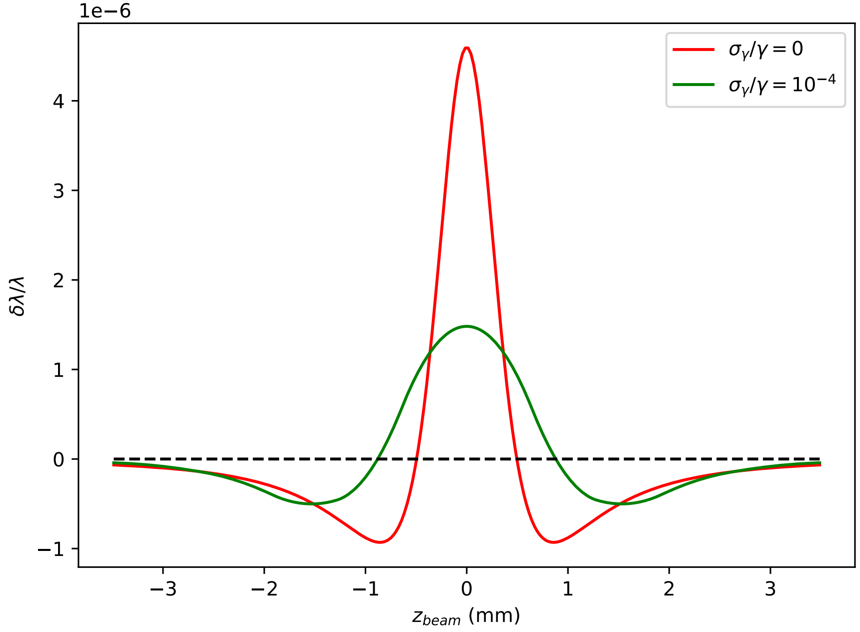
In general, $\lambda = \lambda(x,y)$; here, assume $\lambda = I_e/(ec)$

Results shown above are for impact parameter b = $1\sigma_x$

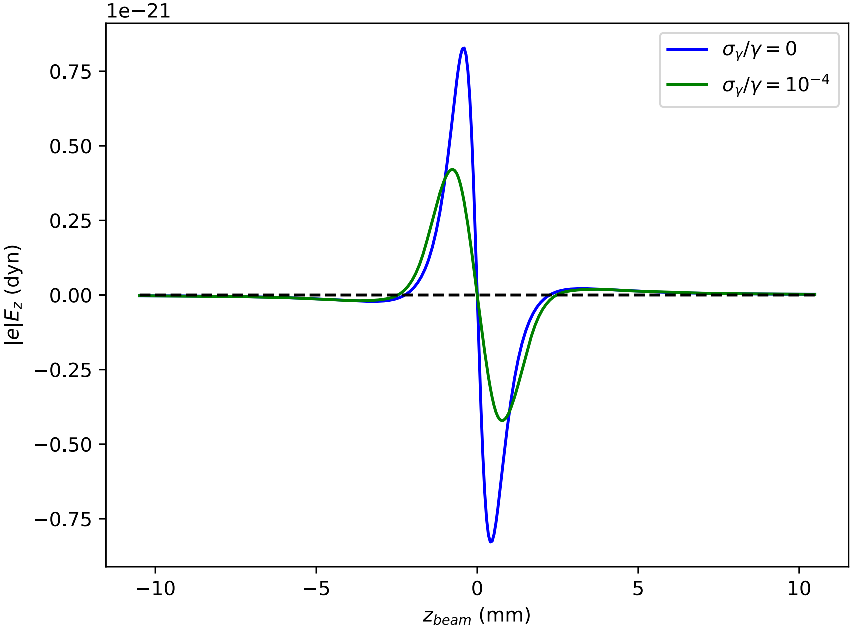
$$\gamma_0 = 313, \quad \sigma_\gamma/\gamma = 1. \times 10^{-4}$$

$$\sigma_x = 0.7\ mm, \quad R_{56} = 1.4\ cm$$

On-axis wake computation and benchmarking



Longitudinal density modulation for warm and cold e-beam computed from the δz-z correlation data



On-axis longitudinal wakefield in the kicker computed from the density modulation shown at left.

- For approximate comparison to Fig. 4 of G. Stupakov, PRAB 21, 114402 (2018), assume $\lambda(z)$ as above and Gaussian transverse distribution
  - *wake computed by convolution shown above*
  - *Warp-based 3D computation as previously reported, except using positron-like macroparticles to sample negative δλ regions*
  - *in general, $\lambda = \lambda(x,y)$; here, assumed $\lambda = I_e/(ec)$*

For a transversely cold beam, initial wake in the kicker is the same as time-averaged (longitudinal spreading-out is negligible)

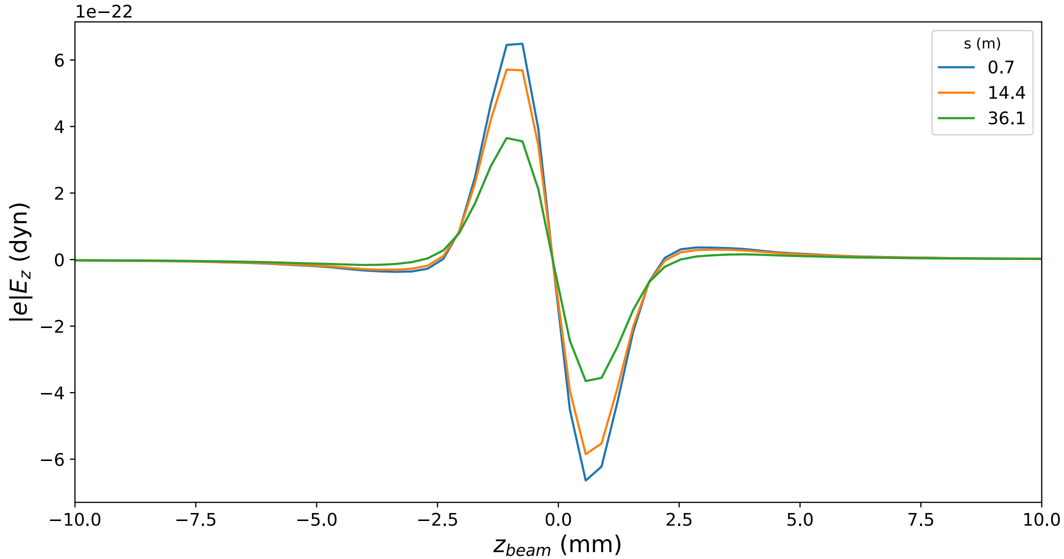
Location of the wake peaks is in agreement with the reference 1D result, peak values differ by ~16%

Results shown above are for impact parameter b = 1σ_x

$$\gamma_0 = 313, \quad \sigma_\gamma/\gamma = 1. \times 10^{-4}$$
$$\sigma_x = 0.7\ mm, \quad R_{56} = 1.4\ cm$$

3D wake computation with the Warp code

- Computation of the 3D, time-dependent wakefield generated in the kicker (or an amplifier cascade drift) by the density modulation out of the chicane is implemented in Warp
  - *assume local unperturbed e-beam density constant over the spatial scales on which the density modulation develops, leave out the background field*
  - *sample positive $\delta\lambda$ regions by electron macroparticles, using positron-like macroparticles to sample negative $\delta\lambda$ regions*
  - *solve the Poisson equation on a 3D grid in the beam frame as the beam traverses the kicker and evolves, so as to get the 3 time-dependent E-field components*



On-axis longitudinal wakefield in the kicker computed in Warp for 3 values of s into the kicker for a beam expanding in the transverse plane (a subset of the 3D wake result).

Summary / the big picture

- Modulator simulations:
  - beam frame, non-relativistic dynamics and electrostatic field solve
  - prototyped Warp-based 3D δf computation of space charge effects in the presence of transverse focusing, stationary or moving ion
  - prototyped a "quiet finish" technique to assemble a quiet-start distribution for the chicane simulations, with macroparticles carrying the accumulated modulation information computed by integration over their trajectories in the modulator
  - arbitrary time-dependent, 3D perturbing influence can be accommodated
- Conversion of energy modulation into a density modulation in a chicane:
  - prototyped an approach to computing the density modulation from correlation between perturbed/unperturbed particle position difference and the tracer particles' positions in coordinate space
- 3D wake computation in the kicker (or amplifier drift sections):
  - prototyped a Warp-based computation from the 3D distribution of macroparticles sampling the density modulation produced in a chicane
  - time-dependent wake E-field computed in this way can be used as a perturbing agent for computing the energy modulation that will go into the next chicane (if any), similar to modeling the modulator section
- The prototyped requisite "building blocks" can now be combined and extended beyond modeling single-chicane configurations for a fully 3D modeling of the MBEC cooling systems

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