Improvements to Simulations of Microbunched Electron Cooling for the EIC

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MBEC Overview

Hadrons give energy kicks to electrons in modulator

Electron energy modulation is amplified and becomes density modulation, which provides kicks to hadrons in kicker

Hadron delay dependent on transverse and energy offsets – kick received tends to reduce these

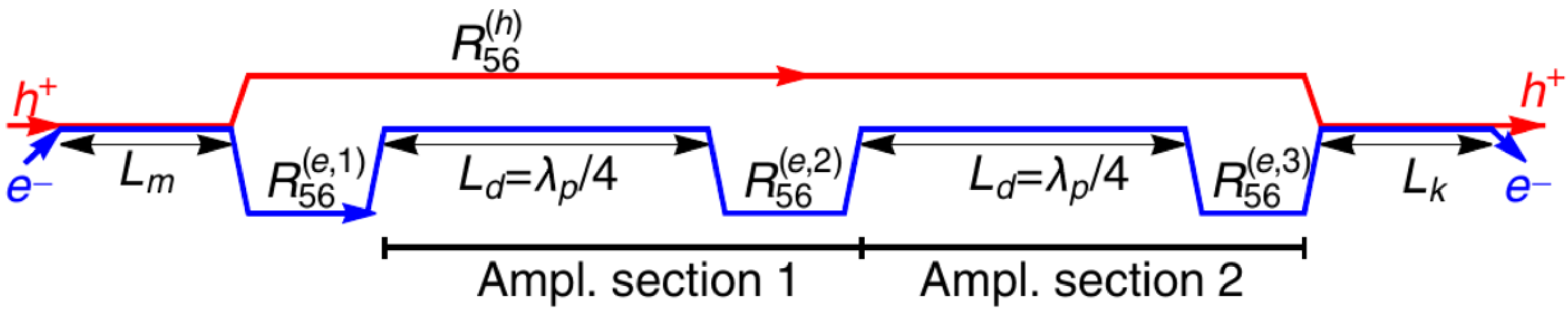

Figure from [5].

Cloud-in-Cell Code

- 1D grid over space established
- Particle charge distributed to neighboring 2 grid points
- Convolve with force function to get effective force at all grid points
- Momentum kick to particle intepolated from neighboring grid points

Kick Details

Macroparticles modeled as 2D charged disks with size equal to respective beam sizes. See [6].

$$F = \frac{4Q_1Q_2\gamma z}{4\pi\epsilon_0\sqrt{\pi}} \int_0^\infty d\lambda\lambda^2 \times \frac{\exp(-\lambda^2\gamma^2 z^2)}{\sqrt{1+2\lambda^2(\Sigma_{x,1}^2+\Sigma_{x,2}^2)}\sqrt{1+2\lambda^2(\Sigma_{y,1}^2+\Sigma_{y,2}^2)}}$$

Resultant momentum kick and position shift:

$$\Delta\delta = \frac{F}{\gamma m v^2} dL \qquad \Delta z = R_{56}\delta$$

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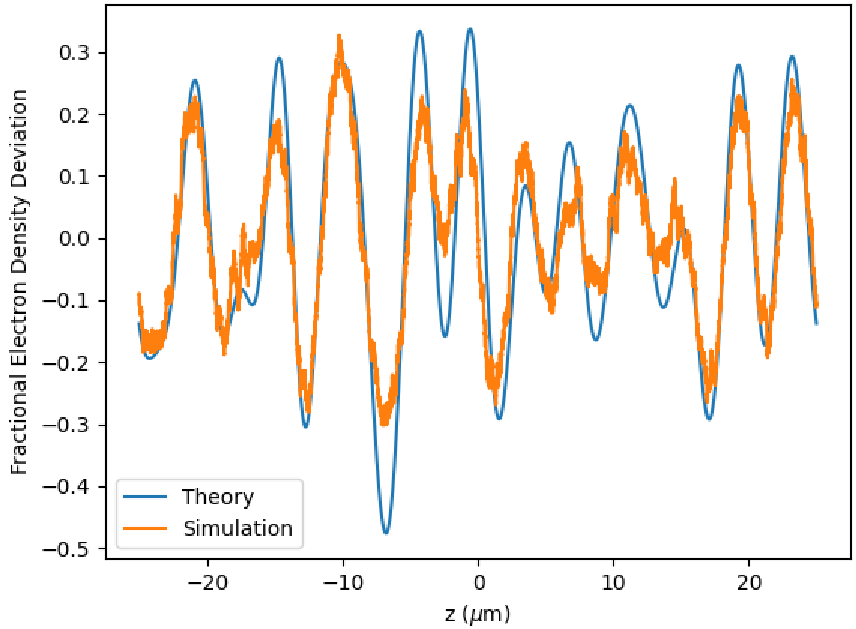
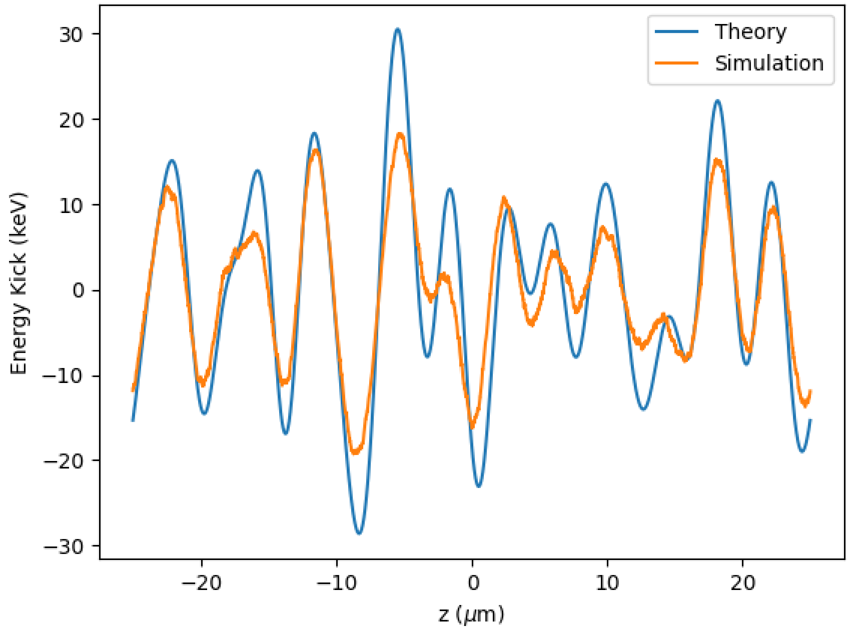
Saturation

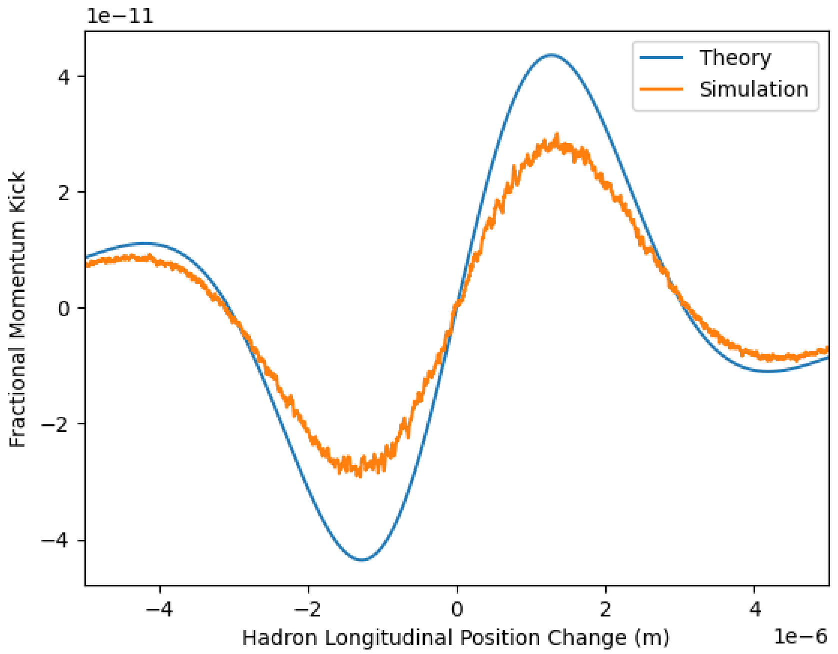




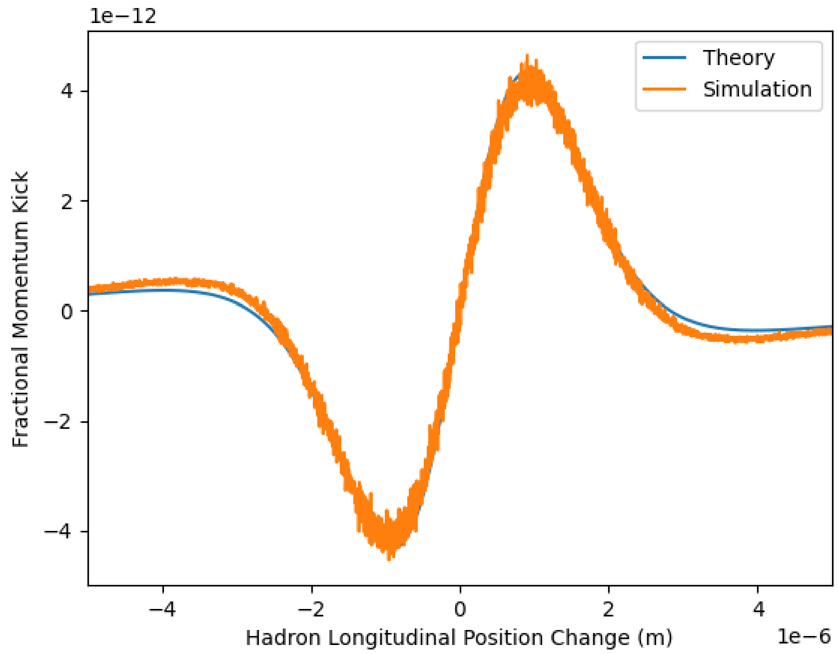
Modulations in electron beam are significant fraction of total electron density – linear theory of [4-7] is inadequate.

Effective Wake

Run simulations with same random electron/proton noise, but with additional proton macroparticle at origin. Difference in kick-per-proton at kicker gives an effective wake. Average over 100 seeds takes ~1 hour.



Nominal 275 GeV cooler parameters



Only 1 amplifier (low saturation) and no plasma oscillations in modulator/kicker in order to benchmark simulation

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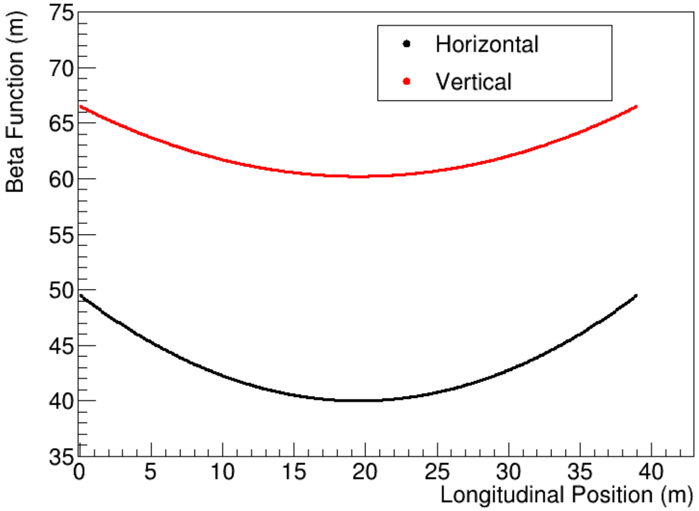
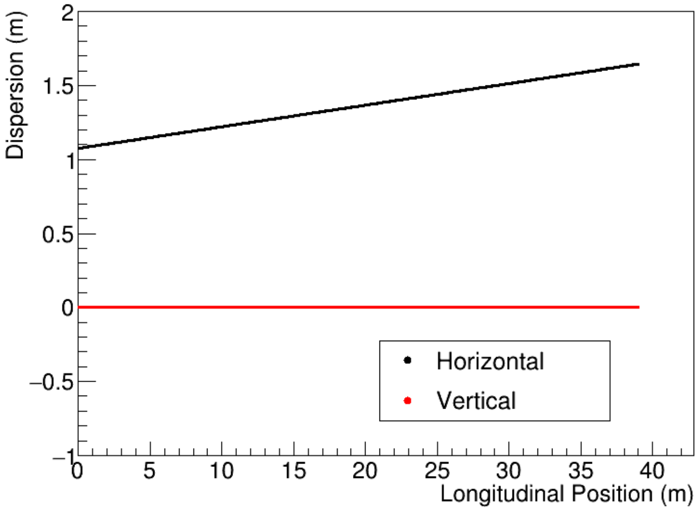
Need for Detailed Transverse Considerations

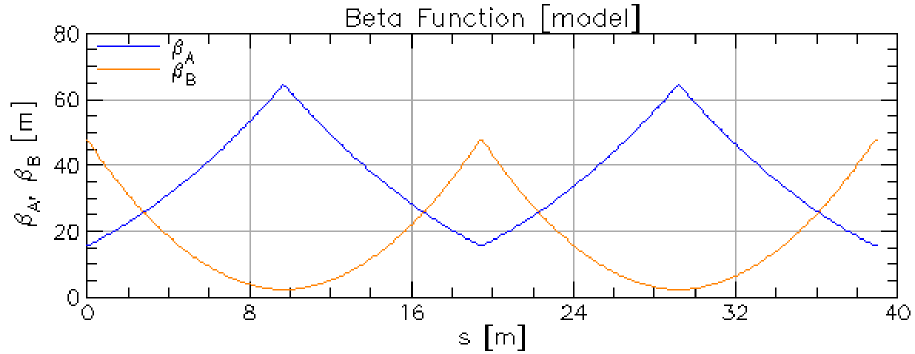
- Existing theory [4-7] assumes constant electron and hadron beam sizes in each element for both wake computation and finding effect on transverse actions
- Prior simulations [8-9] used these same assumptions
- Real accelerators have varying optics parameters
- How important is this?

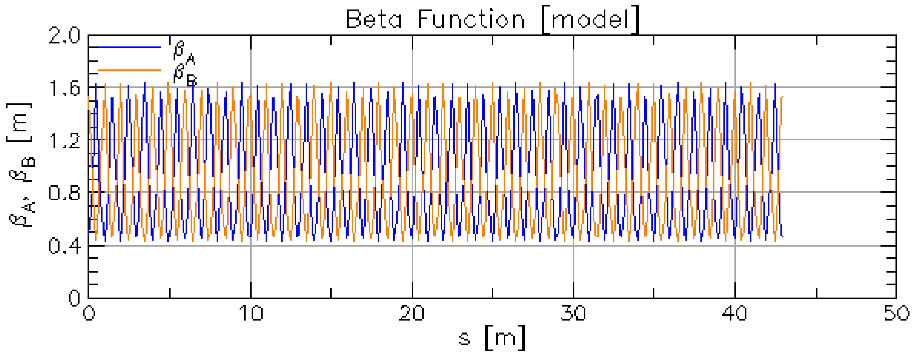
Proton optics in kicker
(Modulator is a mirror image)

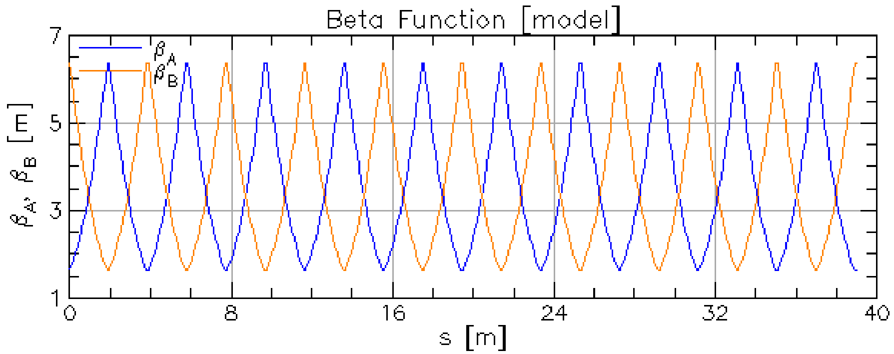




Electron betas in: modulator (top), each amplifier (middle), and kicker (bottom)
(Dispersions are all 0)







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Simulation Details

- For detailed optics, evaluate beam size at each step: 1m steps in modulator, 10cm steps in amplifiers and kicker (where electron betas change more rapidly)

- For averaged optics, just use average e-/p+ beta functions and dispersion in each element to get beam size – use 1m steps everywhere

- Take average of 100 runs of the plasma simulation

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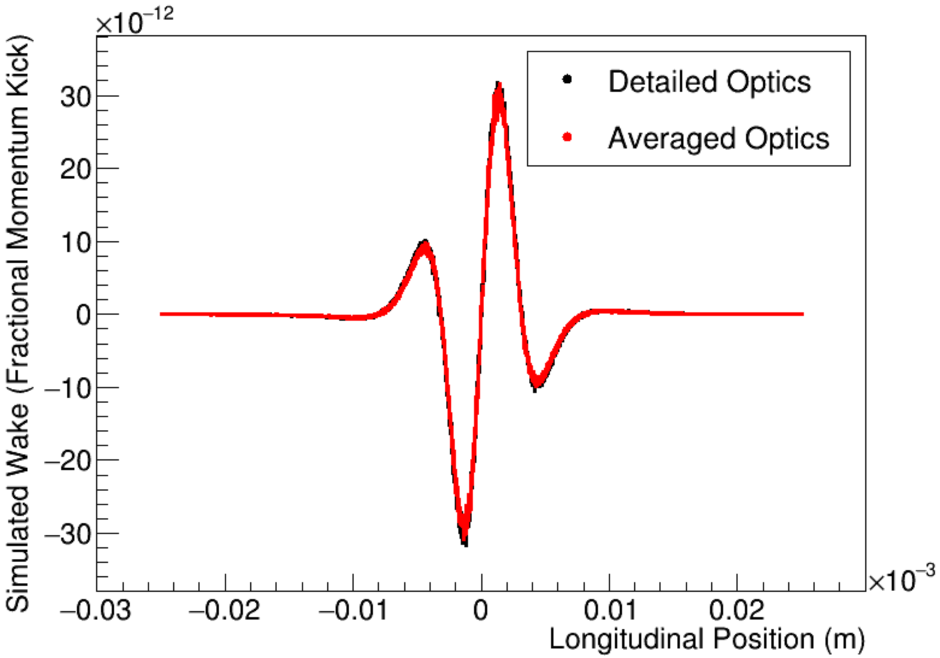
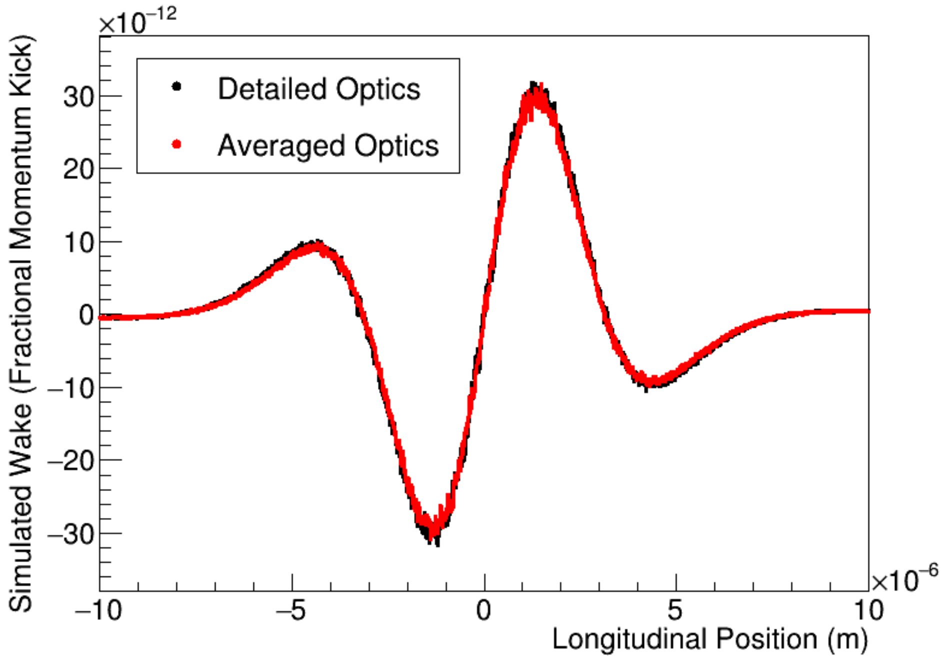
Wake Comparison





Zoomed-in version
of plot on left.

No significant difference observed!

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Effect of Realistic Optics on Transverse Cooling

- Energy kicks in the kicker cause change in transverse action of a hadron, dependent on optics
- Previously, took full kick to happen at kicker center
- If optics evolve throughout the kicker, how does this change the picture?

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Effect of Realistic Optics on Transverse Cooling

$$J = \frac{1}{2}[\beta(x' - D'\delta)^2 + 2\alpha(x - D\delta)(x' - D'\delta) + \gamma(x - D\delta)^2]$$

$$\Delta J = [-(D'\beta x' + D\alpha x' + D'\alpha x + D\gamma x) + (\beta D'^2 + 2\alpha DD' + \gamma D^2)\delta]\Delta\delta$$

$$\Delta J = [-\vec{x}^T \boldsymbol{B}\vec{D} + \mathcal{H}\delta]\Delta\delta$$ where:

$\mathcal{H}$ is the dispersion invariant, $\vec{x} = \begin{bmatrix} x \\ x' \end{bmatrix}$, $\vec{D} = \begin{bmatrix} D \\ D' \end{bmatrix}$, and $\boldsymbol{B} = \begin{bmatrix} \gamma & \alpha \\ \alpha & \beta \end{bmatrix}$

In the absence of dipoles, these transform as:

$\mathcal{H} \to \mathcal{H}$, $\vec{x} \to \boldsymbol{M}\vec{x}$, $\vec{D} \to \boldsymbol{M}\vec{D}$, and $\boldsymbol{B} \to (\boldsymbol{M}^T)^{-1}\boldsymbol{B}\boldsymbol{M}^{-1}$

ΔJ is invariant to first order in $\Delta\delta$. We can treat full kick as happening at kicker center for getting change to transverse optics.

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Conclusions

- Realistic optics can be simulated, and don’t pose a fundamental challenge to the basic MBEC theory

Acknowledgments and References

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[4] G. Stupakov, "Cooling rate for microbunched electron cooling without amplification", *Phys. Rev. Accel. Beams*, vol. 21, p. 114402, Nov. 2018.

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[6] P. Baxevanis and G. Stupakov, "Transverse dynamics considerations for microbunched electron cooling", *Phys. Rev. Accel. Beams*, vol. 22, p. 081003, Aug. 2019.

[7] P. Baxevanis and G. Stupakov, "Hadron beam evolution in microbunched electron cooling", *Phys. Rev. Accel. Beams*, vol. 23, p. 111001, Nov. 2020.

[8] W. F. Bergan, P. Baxevanis, M. Blaskiewicz, E. Wang, and G. Stupakov, "Design of an MBEC cooler for the EIC", presented at IPAC'21, Campinas, Brazil, May 2021, paper TUPAB179, pp. 1819-1822.

[9] W. F. Bergan, "Plasma simulations for an MBEC cooler for the EIC", presented at IPAC'21, Campinas, Brazil, May 2021, paper TUPAB180, pp. 1823-1826.

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Parameters

Table 1: Parameters for Longitudinal and Transverse Cooling

Case	**100 GeV**	**275 GeV**
Protons per Bunch	6.9e10	6.9e10
Proton Bunch Length (cm)	7	6
Proton Emittance (x/y) (nm)	30 / 2.7	11.3 / 1
Proton Fractional Energy Spread	9.7e-4	6.8e-4
Electron Normalized Emittance (x/y) (mm-mrad)	2.8 / 2.8	2.8 / 2.8
Electron Bunch Charge (nC)	1	1
Electron Bunch Length (mm)	14	7
Electron Peak Current (A)	8.5	17
Electron Fractional Energy Spread	1e-4	1e-4
Horizontal/Vertical Proton Betas in Modulator (m)	40 / 44	40 / 60.2
Horizontal/Vertical Electron Betas in Modulator (m)	40 / 35	40 / 25
Horizontal/Vertical Proton Betas in Kicker (m)	40 / 44	40 / 60.2
Horizontal/Vertical Electron Betas in Kicker (m)	10 / 10	4 / 4
Modulator Length (m)	39	39
Number of Amplifier Drifts	2	2
Amplifier Drift Lengths (m)	43	43
Kicker Length (m)	39	39
R56 in First Two Electron Chicanes (cm)	1.86	0.50
R56 in Third Electron Chicane (cm)	-4.82	-1.15
R56 in Proton Chicane (cm)	-0.635	-0.226
Proton Horizontal Phase Advance (rad)	5.055	5.446
Proton Horizontal Dispersion in Modulator & Kicker (m)	1.108	1.36
Proton Horizontal Dispersion Derivative in Modulator/Kicker	-0.0177 / 0.0177	-0.0146 / 0.0146
Electron Betas in Amplifiers (m)	5.45	1.00
Horizontal / Longitudinal IBS Times (hours)	2.0 / 2.5	2.0 / 2.9
Horizontal / Longitudinal Cooling Times (hours)	1.8 / 2.3	1.9 / 3.0