ISBN: 978-3-95450-243-1 ISSN: 2226-0374 doi:10.18429/JACoW-COOL2021-S803

WIGGLER ENHANCED PLASMA AMPLIFIER FOR COHERENT ELECTRON COOLING*

G. Stupakov†, SLAC National Accelerator Laboratory, Menlo Park, CA, USA
A. Zholents, Argonne National Laboratory, Argonne, IL, USA

Abstract

Coherent electron cooling [1] using a plasma-cascade amplifier (PCA) [2] can provide much faster cooling rates of hadrons than the conventional microwave stochastic cooling due to an extremely wide bandwidth of a pickup modulator, a kicker, and the amplifier. A PCA creates unstable plasma oscillations using modulation of the plasma frequency by varying the transverse beam size along the beam line with strong field solenoids. In this work we propose an alternative approach to the problem: the plasma frequency is modulated in a sequence of wiggler magnets separated by drifts or chicanes. This approach has the promise of obtaining a compact amplifier due to a more efficient modulation of the plasma frequency, although it requires separation of the hadron and electron orbits in the amplifier region to synchronize their time of flight through the cooling system.

INTRODUCTION

Coherent electron cooling [1] (CEC) can provide much faster cooling rates of hadrons than the conventional microwave stochastic cooling due to a wide bandwidth of the pickup modulator, the kicker, and the amplifier. While the original idea of coherent cooling relied on a free electron laser as an amplifier, which has a relatively narrow bandwidth [3], a later development of the idea involved a broadband amplifier based on the microbunching instability [4–6]. This approach is known under the acronym of MBEC (microbunched electron cooling). More recently, in Ref. [2], the idea of a plasma cascade amplifier (PCA) was proposed that conceptually has an even broader bandwidth than MBEC. The PCA creates unstable plasma oscillations using modulation of the plasma frequency by varying the transverse beam size along the beam line with strong-field solenoids. Unfortunately, the PCA length with several amplification sections can become prohibitively long because the plasma wavelength increases with the Lorentz gamma factor as $\gamma^{3/2}$. In this work we propose an alternative approach to the problem: the plasma frequency is modulated in a sequence of wiggler magnets separated by drifts or small chicanes. We refer to this scheme as the *wiggler enhanced plasma amplifier*, or WEPA. This approach has the promise of obtaining a compact amplifier due to a more efficient modulation of the plasma frequency.

* This work supported by the Director, Office of Science, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contracts No. DE-AC02-06CH11357 and DE-AC02-76SF00515.
† stupakov@slac.stanford.edu

PLASMA OSCILLATIONS IN A RELATIVISTIC BEAM

We begin with a derivation of the frequency of plasma oscillations when a relativistic beam propagates inside a wiggler or in a drift. In our derivation, we neglect the particle energy spread assuming a cold beam. Our interest here is in short-wavelength plasma perturbations (with wavelength in the micron range, or even shorter) so we can treat the beam in local approximation assuming that its linear density n_0 is constant and neglecting its dependence on the longitudinal coordinate z. We consider the linear density perturbation $\Delta n(s,z)$ and the relative energy perturbation $\Delta\eta(s,z) = \Delta E(s,z)/E_0$, where z is the longitudinal coordinate in the beam relative to a reference particle, and s is the path length along the beam line. These quantities are Fourier transformed over the coordinate z: $\Delta\hat{n}_k(s) = \int_{-\infty}^{\infty} \Delta n(s,z) e^{-ikz} dz$ and $\Delta\hat{\eta}_k(s) = \int_{-\infty}^{\infty} \Delta\eta(s,z) e^{-ikz} dz$.

In a drift, we have the linearized continuity equation for cold plasma,

$$\frac{\partial \Delta n(s,z)}{\partial s} + \frac{1}{\gamma^2}\frac{\partial}{\partial z} n_0 \Delta\eta(s,z) = 0. \tag{1}$$

Applying the Fourier transform to this equation yields

$$\frac{d\delta\hat{n}_k}{ds} = -\frac{ikl}{\gamma^2} n_0 \Delta\hat{\eta}_k. \tag{2}$$

In a wiggler, particles move along the s coordinate with a smaller longitudinal velocity corresponding to the *longitudinal* gamma factor

$$\gamma_z = \frac{1}{\sqrt{1 - v_z^2/c^2}} = \frac{\gamma}{\sqrt{1 + K^2/2}}, \tag{3}$$

where $K = eB\lambda_w/2\pi mc^2$ is the wiggler parameter with B the amplitude magnetic field in the wiggler and λ_w the wiggler period. Here we assume a plane wiggler. Correspondingly, in a wiggler, we replace γ^2 by γ_z^2 in the continuity equation Eq. (2),

$$\frac{d\Delta\hat{n}_k}{ds} = -ik\frac{1}{\gamma_z^2} n_0 \Delta\hat{\eta}_k. \tag{4}$$

The equation for $\Delta\hat{\eta}_k$ describes the energy exchange between the particles due to the Coulomb interaction. Here we will accept the model of Ref. [5] in which the interaction between the particles of the beam is replaced by the Coulomb interaction of charged disks (or slices of the beam) with a Gaussian surface charge distribution. Then the equation for

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

ISBN: 978-3-95450-243-1 ISSN: 2226-0374 doi:10.18429/JACoW-COOL2021-S803

$\Delta\hat{\eta}_k$ can be recovered from Eqs. (54) and (55) of Ref. [5],

$$\frac{d\Delta\hat{\eta}_k}{ds} = \zeta(k)\Delta\hat{n}_k, \tag{5}$$

with

$$\zeta(k) = -\frac{2ir_e}{\gamma\gamma_z\sigma_\perp} H\left(\frac{k\sigma_\perp}{\gamma_z}\right), \tag{6}$$

where r_e is the classical electron radius, $\sigma_\perp$ is the rms size of the plasma in the wiggler (we assume an axisymmetric beam), and the function H is defined in Ref. [5]. Equation (6) is written for the wiggler; in free space one has to replace γ_z by γ. Combining Eqs. (4) and (5) we obtain

$$\frac{d^2\Delta\hat{n}_k}{ds^2} = -c^{-2}\omega_p^2(k)\Delta\hat{n}_k, \tag{7}$$

in which the plasma frequency ω_p is defined by

$$\omega_p(k) = \sqrt{\frac{r_e n_0 c^2}{\sigma_\perp^2\gamma\gamma_z^2}} F(\kappa_p), \tag{8}$$

where $F(\kappa_p) = \sqrt{2\kappa_p H(\kappa_p)}$ with $\kappa_p = k\sigma_\perp/\gamma_z$. The plot of the function F is shown in Fig. 1; for large values of its argument F tends to 1.

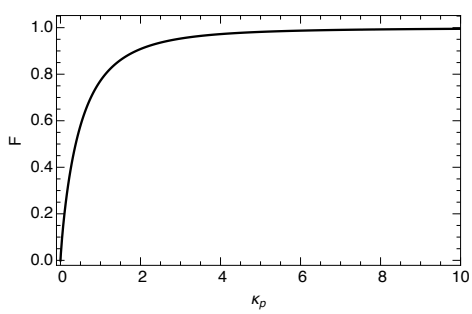

Figure 1: Plot of function $F(\kappa_p)$.

To illustrate the typical numerical values, for the electron beam with the peak current $I_e = 100$ A, $\sigma_\perp = 100\ \mu$m, and $\gamma_z = \gamma = 300$, the plasma period measured in units of length is $\lambda_p = 2\pi c/\omega_p \approx 45$ m, for $2\pi/k < 10\ \mu$m (in this region the function F is approximately equal to 1).

PERIODIC SYSTEM OF WIGGLERS AND DRIFTS

We now consider a periodic sequence of wigglers separated by drift sections. One cell of such a system of length $2L = l_w + l_d$ is shown in Fig 2. Our goal here is to calculate how an initial density perturbation $\Delta\hat{n}_k$ is amplified after the passage through the cell. For this, one has to consider the evolution of the column vector $(\Delta\hat{n}_k, \Delta\hat{\eta}_k)^T$. Noting that from Eq. (4) it follows that $\Delta\hat{\eta}_k \propto \gamma_z^2 d\Delta\hat{n}_k/ds$, it is more convenient to track the evolution of the vector

$$\left(\Delta\hat{n}_k, \frac{\gamma_z^2}{\gamma^2}\frac{d\Delta\hat{n}_k}{ds}\right)^T, \tag{9}$$

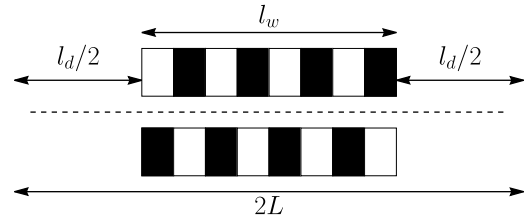

Figure 2: One cell of a periodic system of wigglers and drifts. The wiggler length is l_w and the drift length is l_d; the total cell length is $2L$.

where inside the wiggler γ_z is given by Eq. (3), and in the drift $\gamma_z = \gamma$.

Inside the wiggler the plasma frequency is constant, and using Eq. (7) it easy to find that the transformation of the vector in Eq. (9) over length s is given by the matrix

$$M_w(s,\omega_p) = \begin{pmatrix} \cos(\omega_p s/c) & \frac{\gamma^2 c}{\gamma_z^2\omega_p}\sin(\omega_p s/c) \\ -\frac{\gamma_z^2\omega_p}{\gamma^2 c}\sin(\omega_p s/c) & \cos(\omega_p s/c) \end{pmatrix}. \tag{10}$$

The same matrix can be used in the drift if we set $\gamma_z = \gamma$ in Eqs. (10) and (8). However, to simplify our analysis we assume that the plasma frequency in the drift is so small compared with the plasma frequency in the wiggler that it can be neglected. The matrix M in this case can be obtained from Eq. (10) as the limit $\omega_p \to 0$, which gives

$$M_d(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}. \tag{11}$$

The matrix for the cell shown in Fig. 2 is then given by

$$M_{\text{cell}} = M_d(gL)\cdot M_w(2(1-g)L,\omega_p)\cdot M_d(gL), \tag{12}$$

where $g = l_d/2L$ is the fraction of the length occupied by the drifts.

GAIN IN THE WEPA AMPLIFIER

Let G be an eigenvalue of the one-cell matrix

$$M_{\text{cell}} - GI = 0, \tag{13}$$

where I is the unit 2×2 matrix. Because M_{cell} is symplectic, it has two eigenvalues of G with either $|G| = 1$, or one of the eigenvalues has $|G| > 1$. We define the *gain* $|G|$ in one cell as the maximum value of the two eigenvalues, $|G| \geq 1$.

In our analysis we will use the undulator parameter $K = 2$ that corresponds to $\gamma_z/\gamma = 0.58$. Figure 3 shows the plot of the gain as a function of the parameter g and the ratio of the half cell length L to the plasma period $\lambda_p = 2\pi c/\omega_p$ inside the wiggler. As we see from this plot, higher values of $|G|$ are attained for larger values of g. We choose $g = 0.9$ for further analysis, which means that the wiggler occupies 10% of the length of the cell. In order to obtain the gain factor as a function of the wavenumber k, we need to specify the

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

ISBN: 978-3-95450-243-1 ISSN: 2226-0374 doi:10.18429/JACoW-COOL2021-S803

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

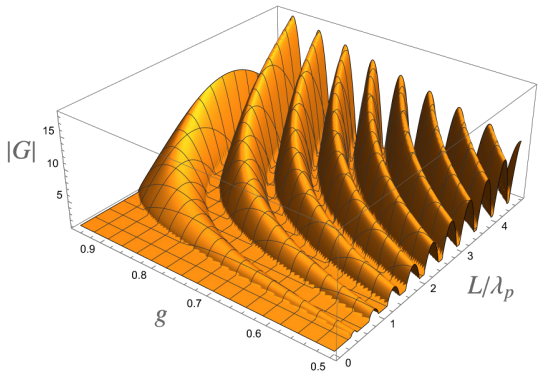

Figure 3: Gain for one cell as a function of g and L/λ_p.

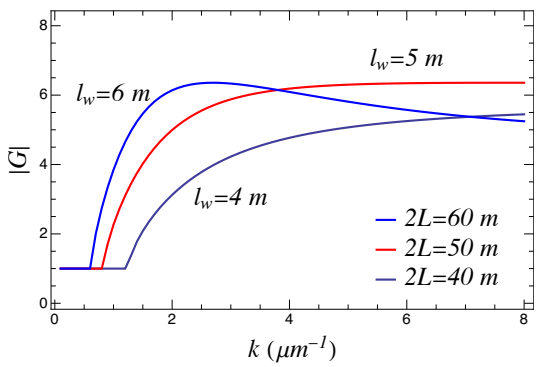

Figure 4: Gain per one cell for several values of l_w. Since we assume that l_w is equal to 0.1 of the cell length, the total length of the cell is ten times larger than l_w (indicated in the figure).

beam parameter for which we choose the values $I_e = 150$ A and $\sigma_\perp = 70$ μm. The gain profiles for several values of the length of the wiggler l_w are shown in Fig. 4. Note that formally, in the model of a cold beam, for these three cases the gain remains constant (non zero) when $k \to \infty$. If we choose $l_w = 5$ m as a reference case, then we see that at large values of k the gain function is approximately equal to 6. With four such cells the amplification factor will be $6^4 \approx 1300$, which is a large enough number to make it promising for use in a CEC cooling system.

Wigglers (and chicanes, see below) slow down the electrons, which should be carefully synchronized with hadrons in the modulator and the kicker of the cooling system. This means that the hadrons' path in the cooler should be modified (relative to the straight beam line) such that their time of flight between the modulator and the kicker is equal to that of the electrons.

SHORT CELLS WITH CHICANES

Unfortunately, the cell length of $\sim$ 50 m in the system considered in the previous section is relatively large. We can considerably shorten it if we notice that the drift matrix, Eq. (11), is physically equivalent to the transport matrix element R_{56} equal to its ballistic value, $R_{56} = l_d/\gamma^2$. This element converts the energy perturbation $\Delta\hat{\eta}_k$ in the plasma oscillation into a density perturbation $\Delta\hat{n}_k$. For $l_d = 45$ m and $\gamma = 300$, we find $R_{56} = 0.52$ mm. Such a value of R_{56} can be easily provided by a weak chicane (of length $\sim$ 1 m), which would drastically shorten the length of the WEPA amplification cell. Two cells of such a modified WEPA amplifier are shown in Fig. 5.

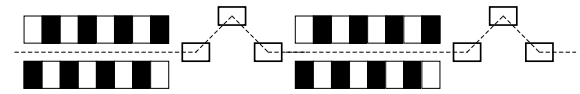

Figure 5: Two cells of a WEPA amplifier in which drifts are replaced by small chicanes.

EFFECTS OF BEAM ENERGY SPREAD AND CSR WAKE FIELDS

In our previous analysis we ignored the energy spread in the beam. A finite energy spread leads to Landau damping of plasma oscillation with very short wavelengths. This effect can also be explained as smearing of the density perturbations in a drift or a chicane due to the fact that particles with different energies are shifted in the longitudinal direction of the beam in proportion to $R_{56}\eta$. The suppression of the amplitude of the plasma oscillations is given by the factor [7]

$$\exp\left[-\frac{1}{2}k^2 R_{56}^2 \sigma_\eta^2\right], \tag{14}$$

where σ_η is the rms relative energy spread in the beam. As an example, let us take $\sigma_\eta = 2 \times 10^{-4}$ and $R_{56} = 0.5$ mm. With these parameters, the suppression of the amplification becomes noticeable at $k \sim \sqrt{2}/R_{56}\sigma_\eta \approx 13.5$ μm^{-1} (this corresponds to the frequency 640 THz).

Another effect that was ignored in our analysis is the coherent synchrotron radiation (CSR) wake field inside the wigglers. This wake field becomes more important in wigglers with a large value of K and hence a stronger magnetic field. Adding to space charge forces, this wake field will modify the property of the plasma oscillations. In the limit when this wake field dominates the space charge, it might lead to a different kind of instability as studied in Refs. [8,9].

SUMMARY

In this paper we extended the PCA concept for the case when the variation of the plasma frequency in the beam is achieved with the help of wiggler magnets. A periodic variation of this frequency leads to the parametric instability of plasma oscillations in the beam and can be used as an amplifier for the coherent cooling of hadrons beams. This system has an excellent frequency bandwidth that can be considerably larger than that of MBEC. We plan to include the CSR effects in future analysis of the WEPA cooling system.

ACKNOWLEDGEMENTS

The authors thank members of M. Blaskiewicz's group for useful discussions of this work.

ISBN: 978-3-95450-243-1 ISSN: 2226-0374 doi:10.18429/JACoW-COOL2021-S803

REFERENCES

[1] V. N. Litvinenko and Ya. S. Derbenev, "Coherent electron cooling", *Phys. Rev. Lett.*, vol. 102. p. 114801, 2009.

[2] V. N. Litvinenko *et* al., "Plasma-cascade instability", *Phys. Rev. Accel. Beams*, vol 24, p. 014402, Jan 2021.

[3] G. Stupakov and M. S. Zolotorev, "Comment on "coherent electron cooling"", *Phys. Rev. Lett.*, vol 110, p. 269503, Jun 2013; V. N. Litvinenko and Ya. S. Derbenev, "Litvinenko and Derbenev Reply", *Phys. Rev. Lett.*, vol. 110, p. 269504, Jun 2013.

[4] D. Ratner, "Microbunched electron cooling for high-energy hadron beams", *Phys. Rev. Lett.*, vol 111, p. 084802, Aug 2013.

[5] G. Stupakov, "Cooling rate for microbunched electron cooling without amplification", *Phys. Rev. Accel. Beams*, vol 21, p. 114402, Nov 2018.

[6] G. Stupakov and P. Baxevanis, "Microbunched electron cooling with amplification cascades", *Phys. Rev. Accel. Beams*, vol. 22, p. 034401, Mar 2019.

[7] E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, "Klystron instability of a relativistic electron beam in a bunch compressor", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 490, no. 1, p. 1, 2002.

[8] G. Stupakov and S. Heifets, "Beam instability and microbunching due to coherent synchrotron radiation", *Phys. Rev. ST Accel. Beams*, vol. 5, p. 054402, 2002.

[9] S. Heifets, G. Stupakov, and S. Krinsky, "Coherent synchrotron radiation instability in a bunch compressor", *Phys. Rev. ST Accel. Beams*, vol. 5, p. 064401, 2002.

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI