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# PLASMA LENS IN PARAMETRIC RESONANCE IONIZATION COOLING

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*Abstract*

The article presents a concept of a plasma lens which will be generated in a dense hydrogen gas filled RF cavity. The plasma lens will be formed by beam-gas-plasma interactions. An exceptionally strong transverse magnetic and longitudinal RF electric focusing fields will appear in a short length. We consider that it will be integrated into a parametric-resonance muon ionization cooling channel as a strong momentum kicker to mitigate an issue of a large amplitude-dependent time of flight for a large angular distribution.

## INTRODUCTION

A multi-TeV muon collider is a highly demanded machine for studying the Beyond Standard Model (BSM). Table 1 shows a step-approaching COM energy and luminosity of muon collider parameter proposed by the high energy physics theory and accelerator groups [1]. It should be noted that a 14-TeV machine is designed as a site-filler at the Fermilab campus. Besides, a 14 TeV μ+-μ- COM energy is equivalent to a 100 TeV pp collider.

Table 1: Muon Collider Parameter

| Parameter (unit) | 3 TeV | 10 TeV | 14 TeV |
|---|---|---|---|
| Lumi. $L$ ($10^{34}$ cm$^{-2}$s$^{-1}$) | 1.8 | 20 | 40 |
| Num. of μs $N_\mu$ ($10^{12}$) | 2.2 | 1.8 | 1.8 |
| Beam rep. rate (Hz) | 5 | 5 | 5 |
| μ beam power (MW) | 5.3 | 14.4 | 20 |
| B. field in coll. ring (T) | 7 | 10.5 | 10.5 |
| N. L. Emit. $\varepsilon_L$ (MeV m) | 7.5 | 7.5 | 7.5 |
| E. spread (rms) ΔE/E (%) | 0.1 | 0.1 | 0.1 |
| N. T. Emit. $\varepsilon_t$ (μm) | 25 | 25 | 25 |
| Beam size (rms) (mm) | 3.0 | 0.9 | 0.63 |

To reach the desired luminosity, the muon beam size must be a sub-mm scale at a collision point. However, an initial size of the created muon beam is a foot-scale after the charged pions decaying. Ionization cooling is a promising technique to shrink a foot-size muon beam into a sub-mm. A timescale of the cooling should be an order of the average muon lifetime, $\gamma\tau_\mu$. A muon beam is incident into an ionization material and loses a kinetic energy due to ionizing the material. The beam lost energy is compensated by an RF accelerating field. As a result, the beam size continuously shrinks with repeating the process. However, nuclei in the material heat the muon beam via the multiple scattering. To mitigate the beam heating, a low-z element, like Hydrogen, Lithium or Beryllium are used as a cooling material and a beam window. Conventionally, a strong magnetic field is applied in a cooling channel to focus the beam at the material. The beam angular spread and therefore the beam emittance are reduced by ionization cooling. Therefore, to achieve a sub-mm-scale beam size, it requires a 50-Tesla focusing solenoid magnet in the final stage of cooling. Besides the muon beam is decelerated from 120-170 MeV to as an order of 10 MeV in the final cooling stage for applying a strong focusing force. Increasing transmission efficiency of muon beam in a high field solenoid cooling channel is challenging.

Significant progress has been made on a possible solution of the final cooling problem called Parametric-resonance Ionization Cooling [2-4]. Instead of focusing the beam by a strong focusing magnet, it uses a parametric resonant process to generate beam waist points. The beam phase space not evolutes on an elliptical trajectory (left-hand side plot in Fig. 1), but diverges on a hyperbolic function (right-hand side plot in Fig. 1). The main challenge of this solution is prevention of the beam smear at the focal points due to a large phase-space volume of even a pre-cooled muon beam. This can be done by compensating beam aberrations at the focal points or linearizing the beam dynamics between the focal points. The plasma focusing technique described in this article provides much stronger focusing magnetic fields than can be provided by conventional superconducting magnets and is also naturally compatible with and simplifies the complexity of the parametric-resonance ionization cooling approach by making the beam size smaller and therefore reducing the aberrations. The Parametric resonance Ionization Cooling scheme is described in the first half of the article. The plasma lens is presented in the second half of the article.

## PARAMETRIC-RESONANCE IONIZATION COOLING CONCEPT

The limit on the minimum achievable emittances in muon ionization cooling comes from the equilibrium between the cooling process and multiple Coulomb scattering in the absorber material. The concept of Parametric-resonance Ionization Cooling (PIC) is to push this limit by an order of magnitude in each transverse dimension by focusing the muon beam very strongly in both planes at thin absorber plates. This creates a large angular spread of the beam at the absorber locations, which is then cooled to its equilibrium value resulting in greatly reduced transverse emittances. Achieving adequately strong focusing using conventional magnetic optics would require unrealistically strong magnetic fields. Instead, PIC relies on a resonant process to provide the necessary focusing. A half-integer parametric resonance is induced in a cooling channel, causing focusing of the beam with the period of the channel's free oscillations.

The resonant perturbation changes the particles' phase-space trajectories at periodic locations along the channel from their normal elliptical shapes to hyperbolic ones as shown in Fig. 1. Thus, at certain periodic focal positions, the beam becomes progressively narrower in $x$ and wider in $x'$ as it passes down the channel. Without damping, the

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beam dynamics are not stable because the beam envelope grows with every period as illustrated in Fig. 2. Placing energy absorbers at the focal points stabilizes the beam motion by limiting the beam's angular divergence at those points through the usual ionization cooling mechanism. These dynamics then result in a strong reduction of the beam spot size at the absorber locations leading to transverse beam emittances that are an order of magnitude smaller than without the resonance. The longitudinal emittance is maintained constant against energy struggling by emittance exchange occurring due to dispersion or its slope at the locations of wedge or flat absorbers.

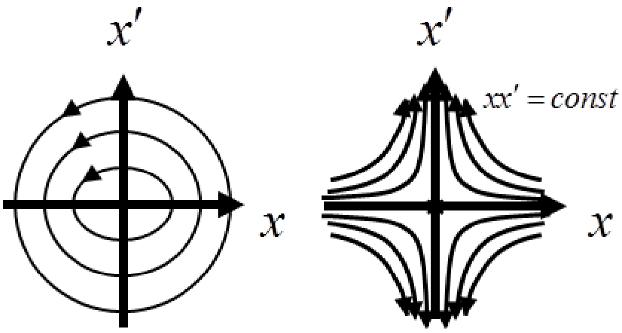


Figure 1: Parametric-resonance Ionization Cooling.

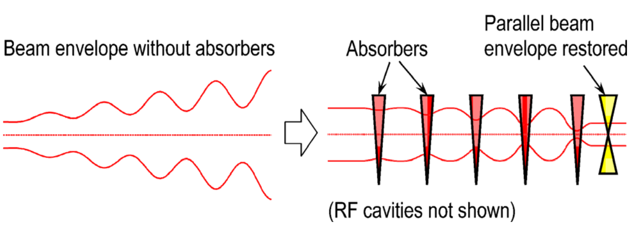


Figure 2: Stabilizing effect of ionization cooling energy absorbers in a channel with a half-integer resonance.

The normalized equilibrium transverse emittance achievable in PIC is given by [5]

$$\varepsilon_{\perp}^{n} = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w\,,$$

where $\beta = v/c$ is the relativistic factor, $Z$ is the absorber material's atomic number, $m_e$ and $m_\mu$ are the electron and muon masses, respectively, and $w$ is the average absorber thickness in the beam direction. The equilibrium beam size $\sigma_a$ and angular spread $\theta_a$ at the absorber and the equilibrium momentum spread $\Delta p/p$ are given by [5]

$$\sigma_a^2 = \frac{1}{8}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}w^2\,,$$

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu},$$

$$\left(\frac{\Delta p}{p}\right)^2 = \frac{3}{8}\frac{(\gamma^2+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}\frac{1}{\log}$$

where $\gamma$ is the muon relativistic energy factor and log is the Coulomb logarithm of ionization energy loss for fast particles. The expected PIC parameters following from the equations above for a 250 MeV/c muon beam are summarized in Table 2. Note that the absorbers are thicker at the beginning of the channel in order to produce a higher cooling rate of an initially large-emittance beam. As the beam cools propagating down the channel, the absorber thickness is gradually reduced in order to reach the minimum practical transverse emittance. Since the cooling rate gets lower for thinner absorbers, the minimum practical absorber thickness is determined by the practically acceptable beam loss due to muon decay.

Table 2: Expected PIC Parameters

| **Parameter with unit** | **Initial** | **Final** |
|---|---|---|
| Beam momentum, $p$ MeV/c | 250 | 250 |
| Num. of particles per bunch, $N_b$ $10^{10}$ | 1 | 1 |
| Be (z = 4) absorber thickness, $w$ mm | 20 | 2 |
| Nor. Tran. Emit. (rms), $\varepsilon_x = \varepsilon_y$ µm | 230 | 23 |
| Beam size at abs., $\sigma_a = \sigma_x = \sigma_y$ mm | 0.7 | 0.1 |
| Ang. spr. at abs., $\theta_a = \theta_x = \theta_y$ mrad | 130 | 130 |
| Momentum spread (rms), $\Delta p/p$ % | 2 | 2 |
| Bunch length (rms), $\sigma_z$ mm | 10 | 10 |

A PIC cooling channel can be implemented using alternating transverse focusing. To provide focusing of the beam in both horizontal and vertical planes simultaneously, the horizontal oscillation period $\lambda_x$ must be equal to or be a low-integer multiple of the vertical oscillation period $\lambda_y$. Emittance exchange also requires $\lambda_x$ and $\lambda_y$ to be low integer multiples of the dispersion period $\lambda_D$. Thus, a cooling channel with alternating focusing optics must have correlated values of $\lambda_x$, $\lambda_y$ and $\lambda_D$:

$$\lambda_x = n\lambda_y = m\lambda_D\,,$$

where $n$ and $m$ are integers. An optics example satisfying the above equation with $\lambda_x = 2\lambda_y = 4\lambda_D$ is illustrated in Fig. 3. The correlated optics design allows one to excite parametric resonances independently in the two planes using periodically placed quadrupoles.

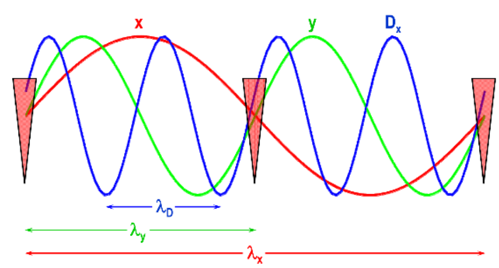


Figure 3: Particle's horizontal $x$ and vertical $y$ betatron trajectories and horizontal dispersion $D_x$ for the $\lambda_x = 2\lambda_y = 4\lambda_D$ case of correlated optics.

There have been several correlated-optics designs of PIC cooling channels with increasingly improved performance:

1. Epicyclic channel consisting of lumped elements [2],
2. Epicyclic channel design using continuous fields [2],
3. Twin helix channel design using continuous fields [2, 3,4],

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4. Skew parametric-resonance ionization cooling channel [2,3].

Cooling in a PIC channel was successfully demonstrated with stochastic effects ignored. For example, Fig. 4 shows evolution of the three 2D emittances along a twin-helix channel where the correlated optics conditions have been tuned and parametric resonances have been excited in both transverse planes at appropriate growth rates.

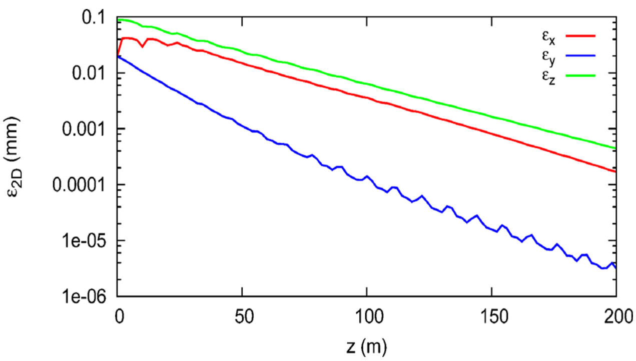


Figure 4: Evolution of the horizontal $\varepsilon_x$, vertical $\varepsilon_y$, and longitudinal $\varepsilon_z$ 2D emittances along a twin-helix channel ignoring stochastic effects.

To proceed to cooling simulations including stochastic effects, compensation of beam aberrations is required. This was verified by running a first-order simulation with stochastic effects included. Aberrations from one absorber to another must be compensated to a degree where they are small compared to the beam size at the absorber. Since the equilibrium angular spread is on the order of a hundred milliradians, the angle-dependent aberrations must be precisely compensated over the angular range of a few hundred milliradians. This is perhaps the most challenging aspect of the PIC channel design.

Alternating transverse focusing allows one to use multipole fields to correct aberrations. However, multipole fields in combination with correlated optics introduce another serious problem, namely, non-linear resonances causing loss of dynamical stability. For example, multiple octupole families are needed in a cooling channel to compensate spherical aberrations. However, with our choice of betatron tunes of $\nu_x = 0.25$ and $\nu_y = 0.5$, any octupole periodicity $m$ causes resonances in both planes. Dispersion further complicates the resonance structure. Selecting different betatron tunes does not help; as long as the betatron periods are integer multiples of the channel period as required for the correlated optics, multipole fields will tend to cause non-linear resonances. This makes it difficult to find a set of multipoles sufficient for aberration compensation that does not cause beam instabilities.

The Skew PIC concept was developed to overcome this issue. Skew PIC introduces coupling into a cooling channel in such a way that the periodic focusing is preserved, but the canonical betatron tunes are shifted from the resonant values that caused issues in the PIC channel. The beam is azimuthally rotated between consecutive focal points due to coupling, and this moves the betatron motion away from nonlinear resonances. This reduces the dimensionality of the aberration compensation problem to just the radial dimension, therefore reducing the number of required compensating multipoles. In addition, coupling equates the parametric resonance rates in the two transverse dimensions, thus requiring only one resonance harmonic, and equipartitions the cooling decrements in the two transverse dimensions.

A skew PIC channel still requires a large dynamic aperture to accommodate the large beam angular spread at the focal points, or equivalently large beam size between the focal points. A significant progress has been made on dynamic aperture optimization of the skew PIC channel using higher-order multipoles. It has been demonstrated [6] that an optimized channel with realistic magnetic fields can accommodate a beam with an angular divergence of 120 mrad at the focal points and a momentum spread of 1%. No particles were lost after tracking through 300 channel periods. However, this angular acceptance is still about a factor of two too small given the equilibrium rms angular spread of about 130 mrad at absorbers.

A short 3-dimensional momentum kicker is considered in the skew PIC which can mitigate the amplitude-dependent time of flight for a large angular spread beam. A dense hydrogen gas filled RF cavity will be the solution for this problem because the cavity can excite extremely high RF gradients (> 40 MV/m) in a strong magnetic field and the plasma lens is formed in the gas volume and induce a high magnetic field gradient (> 300 T/m) in a mm-scale volume. The plasma lens is discussed in the next section.

## PLASMA DYNAMICS TO FORM PLASMA LENS IN GAS-FILLED RF CAVITY

The proposed plasma lens in a gas-filled RF cavity is induced in a dense hydrogen gas due to beam-gas-plasma interactions. Figure 5 shows a diagram of the interactions in the cavity. As the first step, a muon beam incident into the cavity ionizes hydrogen molecule (beam-gas interactions in Fig. 5). Because the gas density is an order of $10^{21}$ cm$^{-3}$, the number of electron-ion pairs in the gas by ionization is an order of 1,000/cm per muon. In the next step, because the gas density is so high, ionized electrons are quickly thermalized (gas-plasma interactions). The estimated mean thermalization time is $\tau_{therm} \sim 1/\zeta_e \cdot \nu_e$ = 1-20 ps [7] where $\zeta_e$ is an energy damping ratio (typically 0.1 to 0.005) and $\nu_e$ is a mean collision frequency of electrons in hydrogen gas (typically $10^{13}$ Hz). When the plasma reaches to an equilibrium condition, a mean electron velocity becomes a drift velocity $v_d$ in $\tau_{therm}$ (gas-plasma interactions). $v_d$ was measured in the past as a function of $E_{rf}/p_{gas}$, where $E_{rf}$ is a peak RF field gradient and $p_{gas}$ is a gas pressure at STP. The range of measured $E_{rf}/p_{gas}$ is for ionization cooling. $v_d$ is typically an order of $10^6$ cm/s in the cavity [8,9].

A beam-induced gas plasma is polarized by an electric field due to the space charge of beam (beam-plasma interactions). The space charge is neutralized by the polarization. A time constant of the space charge neutralization is

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given $\tau_{SCN} \sim \frac{r_b}{2(n_p/n_b)v_d}$, where $r_b$ is an RMS beam spot size, and $n_p/n_b$ (typically 1,000) is the ratio of the plasma and the beam, respectively [7]. The estimated time is $\tau_{SCN} \sim$ 50 ps at $r_b$ = 1 mm. It should be noted that the space charge neutralization time is simulated in WARP, which is $\tau_{SCN} \sim$ 15 ps. The beam bunch length is 50-200 ps. It will suggest that the space charge is neutralized during beam passing in the gas-filled cavity. It should also be noted that the space charge neutralization occurs with both signs by polarizing plasma.

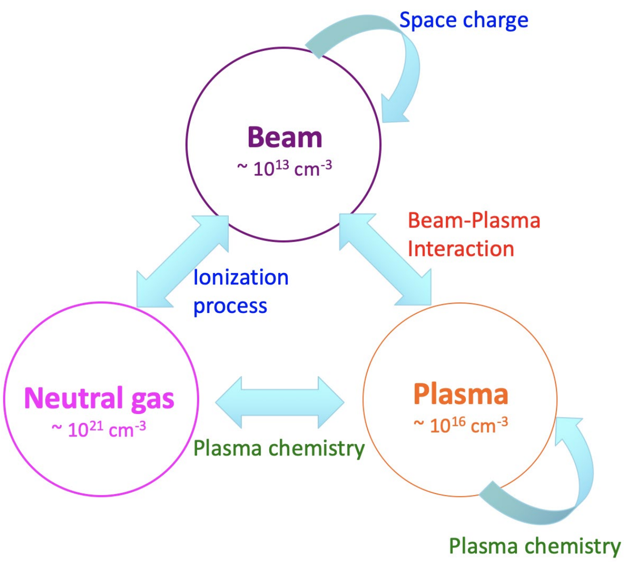


Figure 5: Diagram of beam-gas-plasma interactions.

Because the space charge is neutralized while the axial magnetic field still remains in the beam volume, the magnetic field focuses the beam. Figure 6 shows the estimated azimuthal magnetic fields with $r_b$ = 2 mm (blue) and 1 mm (orange) [7]. The bunch length is 10 mm and the number of particles is $10^{12}$ per bunch. The magnetic field gradient is 87 (blue) and 326 (orange) T/m, respectively. This field will be used for focusing beam in the transverse direction.

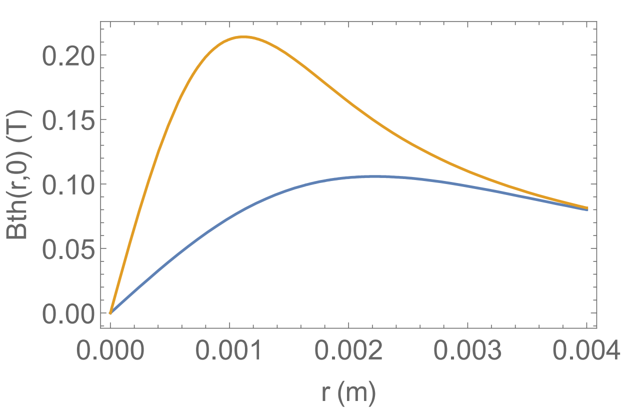


Figure 6: Estimated magnetic field induced by muon beam.

Figure 7 shows a preliminary simulation result of a plasma lens [7]. On the left-hand plot, there is no hydrogen gas in the beam volume. The size of muon beam, especially a tail grows due to a space charge. On the other hand, on the right-hand plot, a muon beam interacts with hydrogen gas and creates an electron-ion pair. For simplicity, the beam does not lose its kinetic energy by ionization, instead, the beam energy is immediately restored in simulation. There is no RF field so that there is no longitudinal focusing. Nevertheless, the beam size is shrunken, especially a tail due to the plasma lens effect.

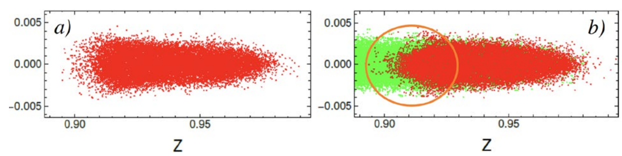


Figure 7: Preliminary simulation result of plasma lens. A red point is muon, a green point is electron. a). No hydrogen gas in the beam volume, while b) with hydrogen gas. The beam is focused at the tail (orange circle) due to the plasma lens.

As the last discussion, the maximum available RF gradient in the gas filled RF test cell is presented. Figure 8 shows the observed RF gradient as a function of the hydrogen gas density [10]. One conclusive result is that gas can greatly suppress the electric breakdown even an extremely high RF gradient is excited in the cavity in a strong magnetic field. The gas-filled RF can excite 40 MV/m or higher field gradients when the gas density > 0.0045 g/cm$^3$. The experiment validates a simple model. A dense gas increases an electrical resistance between two electrodes and suppresses the dark current flow. It suggests that the gas-filled RF cavity functions under strong magnetic fields and severe radiation environments. Also, we observed that no conditioning is needed in the gas-filled RF test cell. The breakdown probability is independent of surface condition of the electrodes. It may suggest that the gas-filled RF cavity removes the resonant frequency dependence on the maximum available field gradient expressed by Kilpatrick's limit. Further investigations are needed.

It should also be noted that the plasma density of the RF electric breakdown was measured in the gas-filled RF test cell. It is an order of $10^{19}$ cm$^3$. The value is three orders of magnitude lower than the beam-induced plasma density. It suggests that the beam-induced plasma does not induce the breakdown in the cavity. It is also experimentally validated. Instead, the beam-induced plasma consumes the RF power in the cavity. It is a considerable issue to design the cooling channel with the gas-filled RF cavity. Further investigations are needed.

Because the gas-filled RF cavity can excite extremely high RF gradients, the length of gas-filled RF cavity can be very short, a few mm. Besides, the focusing field gradient of the plasma lens can be > 360 T/m. It will be an ideal momentum kicker, $F \cdot \delta t = \delta p$ in the transverse direction by using the plasma lens and in the longitudinal direction by the high RF gradient. Further analysis is needed to apply the plasma lens in PIC. Possible study is given in the following list:

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- Validate the concept of plasma lens in numerical simulation. We need a numerical simulation to study beam-gas-plasma interactions.
- Manipulate the plasma lens parameter. Plasma density, temperature, electron drift velocity, etc can be controlled by adjusting $E_{rf}/p_{gas}$ , and applying plasma chemistry. For example, a small electronegative dopant immediately reduces the electron plasma density and plasma temperature. Those were experimentally demonstrated in the gas-filled RF test cell. A new experiment is needed to measure the fast plasma signal.
- Optimize the plasma lens parameter and RF field gradient for PIC. Because the plasma lens is formed by the beam itself, the field gradient may not be linear. It may require correction fields to maximize the cooling performance. Those studies can be made in numerical simulations.

We have developed a parallel processing plasma simulation code, SPACE for the simulation study [11]

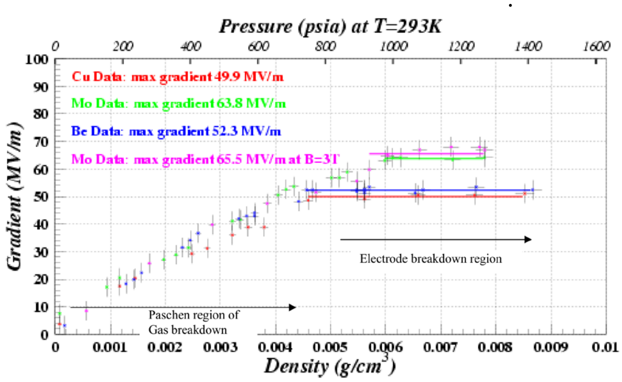


Figure 8: Measured maximum available RF gradient in the gas-filled RF test cell.

## CONCLUSION

We propose a parametric resonance ionization cooling for the final muon ionization cooling channel. We found that solving the amplitude-dependent time of flight for a large angular spread beam is the key to make PIC for the real cooling channel. To mitigate the issue, we propose to integrate the plasma lens which is formed in the hydrogen gas-filled RF cavity. The design is still conceptual.

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