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A PERTURBATIVE TECHNIQUE FOR 3D MODELING OF THE MICROBUNCHED ELECTRON COOLING CONCEPT*

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Abstract

Because the efficacy of conventional electron cooling falls off rapidly with energy, reaching the required cooling time at collision energies targeted by the Electron-Ion Collider (EIC) design [1] can be challenging. A possible solution is offered by cooling schemes that are based on fundamentally different approaches such as microbunched electron cooling (MBEC) [2]. Regular particle-in-cell (PIC) simulations in the parameter regime of the EIC cooling system would require a prohibitively large number of particles to resolve the evolution of the ion-imprinted phase space density modulation. We explored a solution to this problem by developing and implementing in the code Warp a computational approach based on two perturbative techniques, the beam-frame δf method and a variant of the distribution difference (DD) technique. To model the dynamics of the ion-seeded modulation in the MBEC chicanes, we developed an approach that combines the DD and quiet start techniques with analysis of correlations between the divergence of pairs of DD trajectories and their location within the e-beam. We have also prototyped in Warp the computation of the time-dependent 3D wakefield in the MBEC kicker.

δf SIMULATION OF THE MODULATOR SECTION

We have prototyped the δf algorithm [3–5] for modeling the ion-induced modulation dynamics in the modulator section of the EIC cooler, assuming a single-chicane MBEC layout [2, 6] for this initial investigation. In the δf-PIC approach, the phase space density f of the electron beam is decomposed into the sum of (i) the background distribution f_0, assumed to be an analytically known function of the phase space variables and time, and (ii) the perturbation δf which is represented by variable-weight macroparticles whose weights w and phase space coordinated evolve in response to the perturbing influence (e.g., the ion) and the background. Two key aspects of the dynamics in the modulator are the interaction of the δf particles with the ion and the space charge forces associated with the e-beam density modulation. It is therefore advantageous to work in the beam frame where the relevant dynamics are non-relativistic and an electrostatic field solver can be used. We have developed a hybrid formulation of the δf algorithm, where the phase space coordinates and weights of the δf particles evolve in the beam frame, while the background distribution f_0, whose gradients enter the evolution equations, is given in terms of the *beam-frame* phase space coordinates and the Twiss parameters of the e-beam specified in the lab frame. Such formulation allows us to include in simulations the effects of focusing quadrupoles on the electron beam dynamics. This is important for modeling the actual cooler lattice that includes multiple quadrupole focusing sections, because the transverse-to-longitudinal coupling in the quads can perturb the electrons' longitudinal momentum modulation imprinted by the ion.

We take $f_0(s)$ to mean the background e-beam distribution evolving in response to the external focusing magnet fields as well as (for non-emittance-dominated beams) the bulk space charge self-force. The growth of the perturbation $\delta f(s)$ is then due to the beam-frame Coulomb field of the ion and the Coulomb field of the ion-induced electron density perturbation, the latter expected to be a small fraction of the field of the ion in the parameter regime of the EIC cooler modulator section. We employ an approximation that f_0 is separable in x, y, and the longitudinal phase space coordinates:

$$f_0(s;x,x',y,y',z,v_z) = f_0^{(x)}(s,x,x')f_0^{(y)}(s,y,y')f_0^{(z)}(s,z,v_z) \tag{1}$$

with $f_0^{(x)}$ and $f_0^{(y)}$ bi-Gaussian in their respective trace spaces, and $f_0^{(z)}$ constant in z and Maxwellian in v_z, *i.e.*,

$$f_0^{(x)}(s,x,x') = C\exp\left(-\frac{\hat{\gamma}_x(s)x^2 + 2\hat{\alpha}_x(s)xx' + \hat{\beta}_x(s)x'^2}{2\epsilon_x}\right) \tag{2}$$

and similarly for the y trace space.

With the above assumptions, we obtain the δf particle weight evolution equation:

$$\frac{dw}{dt} = -\frac{1}{f_0}(1-w)\frac{df_0}{dt} = -(1-w)\left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{x}} + \frac{q}{m}(\vec{E}_{total} + \vec{v}\times\vec{B}_{total})\frac{\partial}{\partial \vec{v}}\right)\ln(f_0). \tag{3}$$

Because we work in the beam frame, the gradients of the f_0 have to be expressed in terms of the beam frame variables (labeled by "b"), and the beam-frame time $t_b = s/\gamma_0\beta_0 c$ is used to parametrize the dynamics. The partial derivatives of f_0 that enter Eq. (3) are given by

$$\frac{\partial}{\partial t_b}\ln f_0^{(x)} = -\frac{\gamma_0\beta_0 c}{\epsilon_x}\left[\frac{\hat{\alpha}_x(s)}{\hat{\beta}_x(s)}\left(\frac{\partial\hat{\alpha}_x(s)}{\partial s} + \hat{\gamma}_x(s)\right)x^2 + \frac{1}{\gamma_0\beta_0 c}\frac{\partial\hat{\alpha}_x(s)}{\partial s}xv_{bx} - \frac{\hat{\alpha}_x(s)}{(\gamma_0\beta_0 c)^2}v_{bx}^2\right], \tag{4}$$

$$\frac{\partial}{\partial x}\ln f_0^{(x)} = -\frac{1}{\epsilon_x}\left[\hat{\gamma}_x(s)x + \frac{\hat{\alpha}_x(s)}{\gamma_0\beta_0 c}v_{bx}\right], \tag{5}$$

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$$\frac{\partial}{\partial v_{bx}} \ln f_0^{(x)} = -\frac{1}{\epsilon_x}\left[\frac{\hat{\alpha}_x(s)}{\gamma_0\beta_0 c}x + \frac{\hat{\beta}_x(s)}{(\gamma_0\beta_0 c)^2}v_{bx}\right], \quad (6)$$

and similarly for the y trace space. For the longitudinal phase space, one only needs the derivative with respect to velocity:

$$\frac{\partial}{\partial v_{bz}} \ln f_0^{(z)} = -\frac{v_{bz} - \langle v_{bz}\rangle(s)}{\sigma^2_{v_{bz}}(s)} . \quad (7)$$

After implementing the discrete-time version of the above equations in Python and performing initial testing, the δf algorithm has been implemented in the particle-in-cell code Warp [7–9]. Warp's use of a Python layer to setup and interface with Fortran code that performs the computationally intensive simulations simplifies the adding of new features such as δf. The δf algorithm was implemented on the Python level using vectorized operations over the particle and field data (Warp's particle and field interfaces at the Python level inherit from NumPy) to ensure that the code still maintains reasonable performance. Most of the algorithm is trivially parallelizable with the exception of the weight calculation which does require some MPI communication between ranks holding particle data to compute the ensemble quantities. The parallel performance of the resulting code was examined on RadiaSoft's internal servers and found to be quite good, in terms of both weak and strong scaling.

To use δf PIC in Warp there is a base `DriftWeightUpdate` that allows users to input initial conditions for the electron bunch and set ion charge, position, and velocity. This class handles update of the background beam properties and field calculations during each step. Field calculation can either be performed from an analytic calculation for the ion's field or by introducing an ion macroparticle into the simulation that deposits to the grid during the PIC cycle. A second, inherited class `SpaceChargeDriftWeightUpdate` is also provided that solves the coupled envelope evolution including a space charge term and can be used for a coasting beam with space charge.

Because the beam-frame transit time in the MBEC modulator is a small but non-negligible fraction of the plasma period, the ion induces both energy and density modulation in the electron phase space distribution, the energy modulation being of primary interest. Using the modulator and beam parameters considered at the time, we performed $3D$ Warp δf simulations of the modulator and compared the energy modulation results to the predictions of 1D theory for the same parameters, calculated in [6]. Shown in Fig. 1, the agreement is quite good, with the simulation showing slightly lower peak modulation, which is probably due to the finite electron temperature in our simulation as opposed to a cold electron beam assumed by a theoretical 1D model.

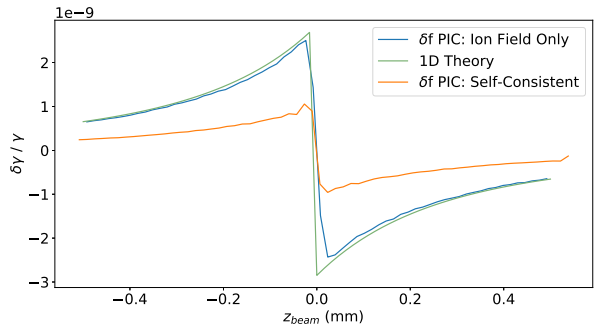

Figure 1: Comparison of energy modulation from a δf PIC simulation and 1D theory.

COMPUTATION OF THE DENSITY MODULATION AND THE WAKEFIELD

A perturbative approach to modeling the conversion of the energy modulation into a density modulation in an MBEC amplifier chicane that we prototyped is based on a combination of the quiet start and a variant of the DD techniques. Similar to other DD variants, pairs of electron macroparticles are used, with one particle in each pair not affected by the source(s) of the perturbation and the other interacting with the perturbing agent. The quiet-start aspect of the approach is in arranging such pairs to smoothly sample the distribution in position and *thermal* velocity space at the entrance to the chicane. Both particles in each pair arrive at their positions at the entrance to the chicane by moving along the unperturbed trajectories, but one particle in each pair keeps track of the accumulated velocity modulation (on top of its thermal velocity) due to the interaction with the ion in the modulator. R_{56} of the chicane converts this velocity modulation into a difference δz in longitudinal positions of the perturbed and unperturbed pair-particles after the chicane, resulting in a density modulation. This is illustrated here for the case of a transversely cold e-beam with longitudinal thermal momentum distribution sampled by 5 points (quintuples of blue dots in Fig. 2) and the initial conditions uniformly sampling a line of "impact parameter" $b = 1.0 \times \sigma_x$ parallel to the z axis, the ion being at rest at the coordinate system origin. (We use $\sigma_x = 0.7$ mm, $\sigma_\gamma/\gamma = 1. \times 10^{-4}$, $\gamma_0 = 313$, and $R_{56} = 1.4$ cm, the same values as in [6].) The average value of δz at each z after the chicane, $\langle\delta z(z)\rangle$ (the green line in Fig. 2, with a hypothetical $\sigma_\gamma/\gamma = 0$ case shown for comparison in red) can then be used to calculate the relative modulation of the line density of the electrons:

$$\delta\lambda/\lambda_0 = -d/dz(\langle\delta z(z)\rangle) . \quad (8)$$

The result is shown in Fig. 3. For a round Gaussian beam with the lab-frame linear number density $\lambda_0 = 6.25 \times 10^{11}$ m^{-1} corresponding to the $I_e = 30$ A and other parameters as above, this density modulation would result in a longitudinal on-axis wakefield $E_z(z)$ shown in Fig. 4. An actual calculation (as distinct from the simplified illustrative example here) should partition the e-beam in transverse coordinate space

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and use $\lambda_0(x, y)$ appropriate for the actual transverse density profile of the e-beam to compute $\delta\lambda(x, y, z)$, which is then used as the source for computing the wake field. We note that the technique described in this section can be used with an arbitrary time-dependent, 3D source of perturbation, *e.g.*, a moving ion and an electron density modulation developing in the modulator in response to the ion.

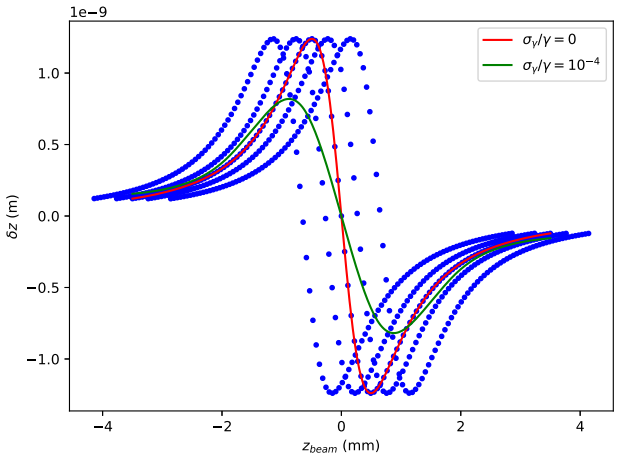

Figure 2: Calculation of $\langle\delta z(z)\rangle$ after the chicane (see text).

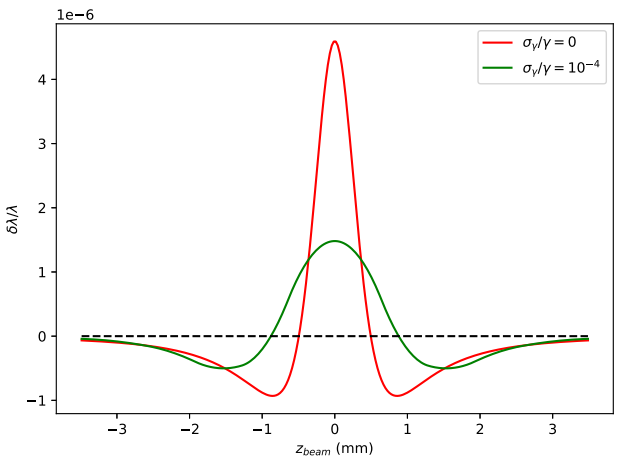

Figure 3: Longitudinal relative density modulation after the first chicane (beam frame z).

In order to compute the $3D$, time-dependent wakefield generated in the kicker (or an amplifier cascade drift) by the evolving density modulation out of the chicane, we implemented in Warp the following approach. We assume that the local unperturbed background e-beam density is constant over the spatial scales on which the density modulation develops, leaving out the background E-field. Regions of positive $\delta\lambda(x, y, z)$ are then sampled by electron macroparticles, and positron-like macroparticles are used to sample negative $\delta\lambda$ regions. These are used as a source term, as the

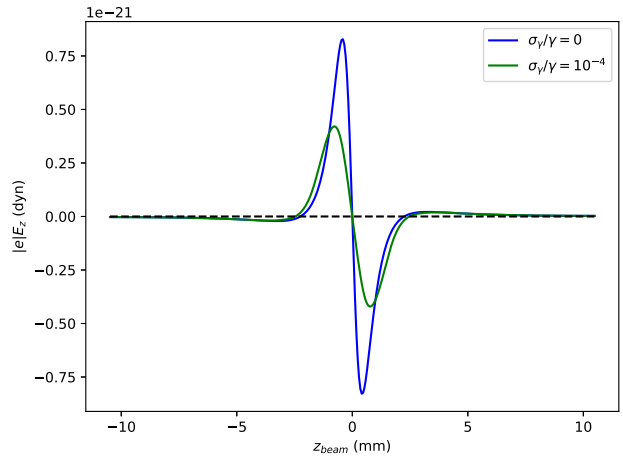

Figure 4: On-axis wake field for the longitudinal charge modulation as in Fig. 3 and a Gaussian transverse density profile (beam frame z).

beam traverses the kicker and evolves, for solving the Poisson equation in Warp in the beam frame on a $3D$ Cartesian grid, so as to find the 3 time-dependent E-field components on the grid. For a transversely cold beam we benchmarked the implementation against the $1D$ longitudinal wake result in [6]. Fig. 5 shows an example of the wake computation in Warp with this approach for the parameters of benchmarking simulations as above, except allowing the beam to freely expand in the 40-meter long kicker. The figure shows snapshots of the beam-frame on-axis $|e|E_z(z)$ at times corresponding to s = 0.7, 14.4, and 36.1 m into the kicker.

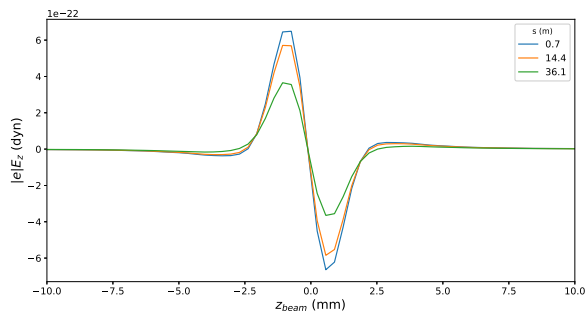

Figure 5: On-axis longitudinal wake computed in Warp at 3 positions in the kicker, for a transversely expanding beam.

REFERENCES

[1] Electron-Ion Collider at BNL. Conceptual Design Report. https://www.bnl.gov/ec/files/EIC_CDR_Final.pdf

[2] D. Ratner, *"Microbunched Electron Cooling for High-Energy Hadron Beams," Phys. Rev. Lett.* **111**, *084802,* 2013.

[3] S.E. Parker and W.W. Lee, *"A fully nonlinear characteristic method for gyrokinetic simulation," Phys. Fluids B* **5**, *78* 1992.

ISBN: 978-3-95450-243-1 ISSN: 2226-0374 doi:10.18429/JACoW-COOL2021-P2009

[4] G. Hu and J.A. Krommes, *"Generalized weighting scheme for δf particle-simulation method," Phys. Plasmas* **1**, *863,* 1994.

[5] N. Xiang *et al.*, *"Low-noise electromagnetic δf particle-in-cell simulation of electron Bernstein waves," Phys. Plasmas* **13**, *062111,* 2006.

[6] G. Stupakov, *"Cooling rate for microbunched electron cooling without amplification," Phys. Rev. AB* **21**, *114402,* 2018.

[7] Warp, https://bitbucket.org/berkeleylab/warp

[8] D.P. Grote, A. Friedman, J.-L. Vay, I. Haber, *"The WARP Code: Modeling High Intensity Ion Beams," AIP Conf. Proc.* **749**, *55,* 2005.

[9] J.-L. Vay, D.P. Grote, R.H. Cohen and A. Friedman, *"Novel methods in the Particle-In-Cell accelerator Code-Framework Warp," Comput. Sci. Disc.* **5**, *014019,* 2012.