

IMPROVEMENTS TO SIMULATIONS OF MICROBUNCHED ELECTRON COOLING FOR THE EIC*

W. F. Bergan[†], Brookhaven National Laboratory, Upton, NY, USA

Abstract

Microbunched electron cooling (MBEC) is a promising new technique for cooling dense hadron beams. It operates by copropagating the hadron beam with a beam of electrons, during which time the hadrons induce an energy modulation on the electrons. This is amplified, turned into a density modulation, and acts back on the hadrons in order to give them energy kicks which tend to reduce their initial energy spread and emittance. We plan to use this technique to cool the proton beams at the Electron-Ion Collider (EIC). In order to better understand the process, we have expanded on our simulation codes of cooling times and saturation effects, allowing us to explore such issues as variable Courant-Snyder parameters within the lattice elements.

MICROBUNCHED ELECTRON COOLING THEORY

In order to cool the dense proton beams in the future Electron-Ion Collider (EIC), we plan to make use of microbunched electron cooling (MBEC) [1]. The theory of MBEC was first developed in [2] and expanded upon in [3–6], and full details can be found therein. The main idea is that the hadrons which one wishes to cool are copropagated with an electron bunch in a straight “modulator” section, where the hadrons induce an energy perturbation in the electron beam. The electrons and hadrons are then separated. The electron beam passes through an amplification section, where its energy perturbation is amplified and transformed into a density perturbation. The hadrons pass through a chicane with non-zero R_{51} , R_{52} , and R_{56} values, so that its delay depends on its initial energy and transverse offsets. In the “kicker” section, the hadrons and electrons again copropagate, and the density perturbations in the electron beam provide energy kicks to the hadrons, with the kick magnitude as a function of hadron delay defined by a wake function. By adjusting the hadron optics appropriately, we can arrange for these kicks to on average cool the hadrons longitudinally and transversely. A diagram of the setup is shown in Fig. 1. Parameters used for this paper are shown in Tab. 1, with the quoted Courant-Snyder parameters evaluated at the center of the appropriate element.

WAKE SENSITIVITY TO VARYING OPTICS

We had noticed that the density perturbations in the electron beam can become comparable to the total electron density, and so saturation effects cause deviations from the lin-

* Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.

[†] wbergan@bnl.gov

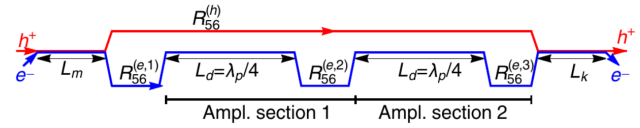


Figure 1: Layout of the MBEC cooler. Figure from [4].

ear cooling theory. To account for this effect, we have developed a one-dimensional simulation code to track hadron and electron macroparticles through the elements of the cooling section using a cloud-in-cell formalism [7] and empirically determine the effective hadron wake function. Details of this simulation code may be found in [8].

Of particular note here is that the previous version of the code made the simplifying assumption that the beam sizes of the electrons and hadrons did not change within the accelerator elements, so that the inter-particle forces only had to be computed once for each element. This is of course not the case in reality, where beta functions of the electrons and hadrons will evolve within the modulator, amplifier straights, and kicker. We assume that the electrons are focused by a simple FODO scheme in each element and that the hadrons experience a drift in the modulator and kicker straights, as illustrated in Fig. 2 and Fig. 3.

We run the simulation code in two separate configurations. First, we run a detailed simulation in which we take 1m steps in the modulator and 10cm steps in the amplifiers and kicker,¹ re-evaluating the beam sizes and electron/electron and electron/hadron interaction functions at each step. We also run a simulation similar to what we had done previously, where we average the beta functions and dispersions across each element for each particle species and use these to create constant beam sizes and interaction functions in each element. We then track the macroparticles in 1m steps. Using 100 random noise seeds, we obtain the effective wake functions from these two methods, as shown in Fig. 4. We see that the use of average Courant-Snyder parameters in an element gives essentially the same result as doing the full detailed tracking.

INSENSITIVITY OF LOCATION OF ENERGY KICK

We have also developed a turn-by-turn code to simulate the multi-turn cooling dynamics in detail. This tracks hadron macroparticles in the bunch through a simplified lattice consisting of the modulator, kicker, and RF section, so that synchrotron motion may be included. Details of the code may be found in [9]. Each turn, each hadron macropar-

¹ A smaller step size was chosen in those elements due to the faster variation in the electron beta functions.

Table 1: Parameters for Longitudinal and Transverse Cooling

Case	275 GeV
Protons per Bunch	6.9e10
Proton Bunch Length (cm)	6
Proton Emittance (x/y) (nm)	11.3 / 1
Proton Fractional Energy Spread	6.8e-4
Electron Normalized Emittance (x/y) (mm-mrad)	2.8 / 2.8
Electron Bunch Charge (nC)	1
Electron Bunch Length (mm)	7
Electron Fractional Energy Spread	1e-4
Horizontal/Vertical Proton Betas in Modulator and Kicker (m)	40 / 60.2
Horizontal/Vertical Electron Betas in Modulator (m)	40 / 25
Horizontal/Vertical Electron Betas in Kicker (m)	4 / 4
Modulator and Kicker Lengths (m)	39
Number of Amplifier Drifts	2
Amplifier Drift Lengths (m)	43
R56 in First Two Electron Chicanes (cm)	0.50
R56 in Third Electron Chicane (cm)	-1.15
R56 in Proton Chicane (cm)	-0.226
Proton Horizontal Phase Advance (rad)	5.446
Proton Horizontal Dispersion in Modulator & Kicker (m)	1.36
Proton Horizontal Dispersion Derivative in Modulator/Kicker	-0.0146 / 0.0146
Electron Betas in Amplifiers (m)	1.00
Horizontal / Longitudinal IBS Times (hours)	2.0 / 2.9
Horizontal / Longitudinal Cooling Times (hours)	1.9 / 3.0

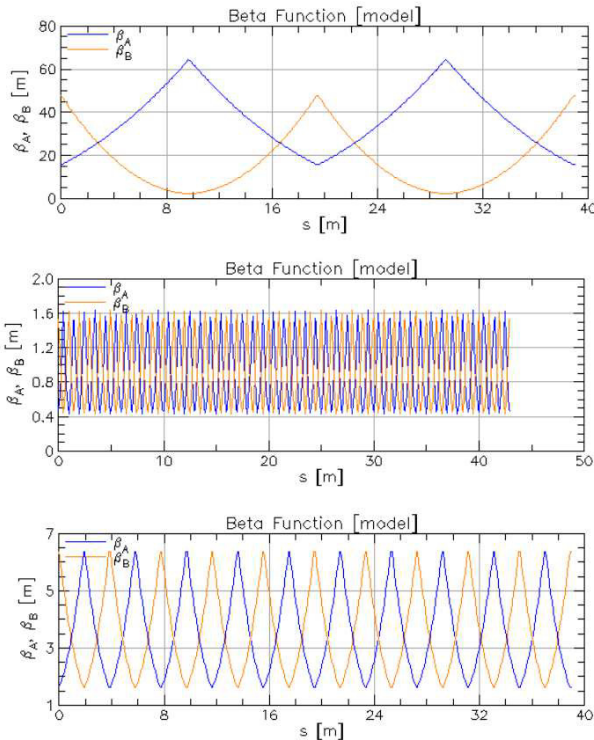


Figure 2: Electron optics in the modulator (top), amplifiers (middle), and kicker (bottom). Dispersions are all near 0.

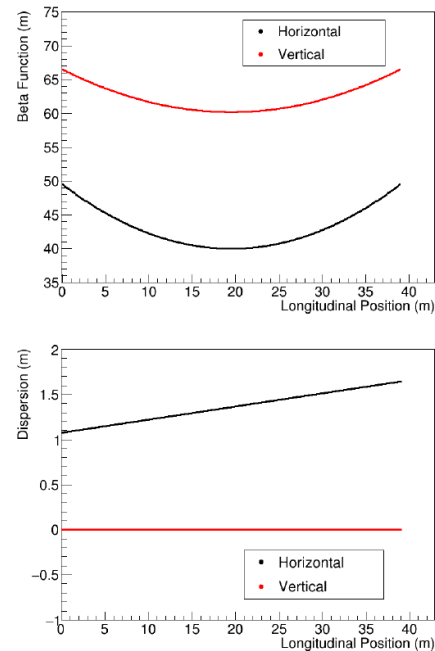


Figure 3: Hadron optics functions in the kicker. The modulator is a mirror image.

ticle receives at the kicker element both a coherent kick, computed from the wake function and the hadron's delay in traveling from modulator to kicker, and a random diffu-

sive kick, with characteristic amplitude set by the size of the wake and local density of electron and hadron beams. Although both kicks are simulated as occurring at the kicker

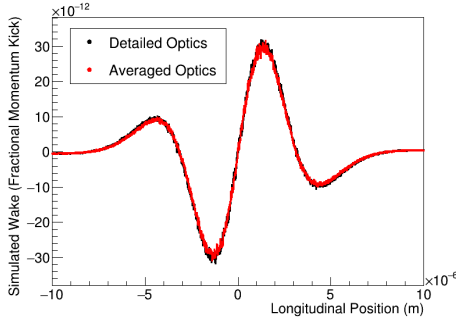


Figure 4: Comparison of effective wakes from the detailed and averaged cloud-in-cell simulations. No significant difference is seen, indicating that re-evaluation of the beam size and interaction functions at each simulation step is not necessary for accurate results.

center, in reality, they will act on the hadron continuously throughout its passage through the kicker. We therefore wish to know what effect this simplification may cause.

A simple analysis shows that, in the absence of dipoles, the effect of an energy kick on a proton's transverse action is independent of the location within the kicker where said kick takes place.

A proton's action is given by

$$J = \frac{1}{2} [\beta(x' - D'\delta)^2 + 2\alpha(x - D\delta)(x' - D'\delta) + \gamma(x - D\delta)^2] \quad (1)$$

where δ is the proton's fractional momentum deviation, and the other variables are standard Courant-Snyder parameters, dispersions, and phase-space coordinates. A small energy kick gives a change in action

$$\Delta J = [-D'\beta(x' - D'\delta) - D\alpha(x' - D'\delta) - D'\alpha(x - D\delta) - D\gamma(x - D\delta)]\Delta\delta \quad (2)$$

This may be rewritten as

$$\Delta J = [-\vec{x}^T \mathbf{B} \vec{D} + \mathcal{H}\delta]\Delta\delta \quad (3)$$

where $\mathcal{H}$ is the usual dispersion invariant and

$$\begin{aligned} \vec{x} &= \begin{bmatrix} x \\ x' \end{bmatrix} \\ \vec{D} &= \begin{bmatrix} D \\ D' \end{bmatrix} \\ \mathbf{B} &= \begin{bmatrix} \gamma & \alpha \\ \alpha & \beta \end{bmatrix} \end{aligned} \quad (4)$$

Under the operation of a transfer matrix $\mathbf{M}$, which does not include any dipoles, it can be shown that

$$\begin{aligned} \mathcal{H} &\rightarrow \mathcal{H} \\ \vec{x} &\rightarrow \mathbf{M}\vec{x} \\ \vec{D} &\rightarrow \mathbf{M}\vec{D} \\ \mathbf{B} &\rightarrow (\mathbf{M}^T)^{-1} \mathbf{B} \mathbf{M}^{-1} \end{aligned} \quad (5)$$

It immediately follows that, to leading order in $\Delta\delta$, Eqn. 3 will be invariant as we move along the kicker.

Similarly, we may examine the effect on diffusion. Taking the quadratic term in the change in action, we see that

$$\Delta J = [\beta D'^2 + 2\alpha D D' + \gamma D^2]\Delta\delta^2 = \mathcal{H}\Delta\delta^2 \quad (6)$$

which is again invariant as we move through the kicker.

Therefore, the effect of both the cooling and diffusive energy kicks on the hadron's transverse action will be the same wherever the hadron is within the kicker, and so it is safe to take the full kick as happening at the center in our cooling simulations. This is borne out by simulations.

CONCLUSION

We have expanded on our simulations of the MBEC process to incorporate realistic optics into the modulator, kicker, and amplifier sections, and have verified that this addition does not significantly change the results of the basic MBEC theory.

ACKNOWLEDGEMENT

We would like to thank Chris Mayes for his assistance in the implementation of this code in Python. We would also like to thank Erdong Wang, Steve Peggs, Panos Baxevanis, Mike Balskiewicz, and Gennady Stupakov for many useful discussions.

REFERENCES

- [1] Electron-ion collider at Brookhaven National Laboratory, conceptual design report 2021, https://www.bnl.gov/EC/files/EIC_CDR_Final.pdf
- [2] D. Ratner, "Microbunched electron cooling for high-energy hadron beams", *Phys. Rev. Lett.*, vol. 111, p. 084802, Aug. 2013.
- [3] G. Stupakov, "Cooling rate for microbunched electron cooling without amplification", *Phys. Rev. Accel. Beams*, vol. 21, p. 114402, Nov. 2018.
- [4] G. Stupakov and P. Baxevanis, "Microbunched electron cooling with amplification cascades", *Phys. Rev. Accel. Beams*, vol. 22, p. 034401, Mar. 2019.
- [5] P. Baxevanis and G. Stupakov, "Transverse dynamics considerations for microbunched electron cooling", *Phys. Rev. Accel. Beams*, vol. 22, p. 081003, Aug. 2019.
- [6] P. Baxevanis and G. Stupakov, "Hadron beam evolution in microbunched electron cooling", *Phys. Rev. Accel. Beams*, vol. 23, p. 111001, Nov. 2020.

- [7] R.W Hockney and J.W Eastwood, *Computer simulation using particles*. New York, NY, USA: Taylor & Francis, 1988.
- [8] W. F. Bergan, "Plasma simulations for an MBEC cooler for the EIC", presented at IPAC'21, Campinas, Brazil, May 2021, paper TUPAB180, pp. 1823-1826.
- [9] W. F. Bergan, P. Baxevanis, M. Blaskiewicz, E. Wang, and G. Stupakov, "Design of an MBEC cooler for the EIC", presented at IPAC'21, Campinas, Brazil, May 2021, paper TUPAB179, pp. 1819-1822.