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COMPARISON OF AVAILABLE MODELS OF ELECTRON COOLING AND THEIR IMPLEMENTATIONS

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Abstract

Modelling of the electron cooling process is complex and challenging. The simulation needs to include elements like ions, plasma of electrons, the thermal effects of electrons and the influence of the magnetic field. In this work, the performance of three available tools, namely RF-Track [1], Betacool [2], and JSPEC [3], are discussed taking into account only the cooling and neglecting any heating effect. The friction force and cooling times are studied in a wide range of different parameters presenting the main behaviour of the available models together with the limitations of particular simulation codes. Furthermore, a qualitative comparison with experimental data is performed.

INTRODUCTION

The study is focused on the dependence of the friction force and of the cooling time on crucial parameters. A short introduction of the analysis is presented in this paper, while details can be found in [4].

Several simulation codes and models of electron cooling implementation have been used for this analysis. The key aspect of each simulation software and model are presented in the following.

RF-Track

RF-Track [1] is a tracking code developed at CERN. Here, electron cooling is modelled on the basis of the description given in Ref. [5], in which the force is expressed as the sum of unmagnetized and magnetised components:

$$\vec{F} = -\frac{4\pi n_e K^2}{\mu}\left\{F_{\text{unmagnetized}} + F_{\text{magnetized}}\right\}. \tag{1}$$

The unmagnetized part is implemented as:

$$F_{\text{unmagnetized}} = L_F \iiint \left[\frac{\vec{U}}{U^3}\right] f(\vec{v}_e)d\vec{v}_e \tag{2}$$

whereas three different versions of $F_{\text{magnetized}}$ have been implemented due to ambiguities in the description provided in [5]:

RF-Track A

$$L_M \iiint \left[\frac{U_{B\perp}^2}{U_B^5}\left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2}\left(1-\frac{U_{B\parallel}^2}{U_{B\perp}^2}\right) f(v_e)dv_e\right)\right] \tag{3}$$

RF-Track B

$$L_M \int \left[\frac{U_{B\perp}^2}{U_B^5}\left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2}\left(1-\frac{U_{B\parallel}^2}{U_{B\perp}^2}\right)\right)\right] f(v_{e\parallel})dv_{e\parallel} \tag{4}$$

RF-Track C

$$\int \left[L_A\frac{\vec{U}_B}{U_B^3} + L_{M2}\frac{U_{B\perp}^2}{U_B^5}\left(\vec{U}_{B\parallel}^5 + \frac{\vec{U}_{B\perp}}{2}\left(1-\frac{U_{B\parallel}^2}{U_{B\perp}^2}\right)\right)\right] f(v_{e\parallel})dv_{e\parallel} \tag{5}$$

with L being the so called Coulomb logarithms:

$$L_F = \frac{1}{2}\log\left(1+\frac{r_F^2}{r_{\text{min}}^2}\right),\quad L_M = \log\frac{r_{\text{max}}}{r_L},$$

$$L_{M2} = \log\frac{r_{\text{max}}}{r_F},\quad L_A = \frac{1}{2}\log\left(1+\frac{r_L^2}{r_F^2}\right)$$

based on the following impact parameters:

$$r_L = \frac{\sqrt{V_{e\perp}^2+\Delta_{e\perp}^2}}{\omega_e},\quad r_F = \frac{\sqrt{U_{B\parallel}^2+\Delta_{e\parallel}^2}}{\omega_e},$$

$$r_{\text{min}} = \frac{K}{\mu(U^2+\Delta_e^2/3)},\quad r_{\text{max}} = \min\left(r_a, \lambda_D\sqrt{1+\frac{U^2}{\Delta_e^2/3}}, U\Delta t\right)$$

where $\vec{U}$ is the velocity difference between ions and mean electron velocity, and Δ_e is the electron temperature. Detailed meaning of all symbols is provided in [4].

BETACOOL

Betacool [2] is a widely used code for simulating beam dynamics developed at JINR. It includes a broad-range of effects that can be used and few models of electron cooling:

Parkhomchuk It is the simplest and commonly used model described by the following semi-empirical formula:

$$F = -4\frac{Z^2e^4n_e}{m}\log\left(\frac{b_{\text{max}}+b_{\text{min}}+\rho_c}{b_{\text{min}}+\rho_c}\right)\frac{\vec{U}}{(U^2+v_{\text{eff}}^2)^{3/2}} \tag{6}$$

with impact parameters:

$$b_{\text{max}} = \frac{v_i}{1/\tau_{\text{flight}}+\omega_p},\quad b_{\text{min}} = \frac{Ze^2}{m}\frac{1}{U^2+v_{\text{eff}}},\quad \rho_c = \frac{cmv_\perp}{eB}$$

A key parameter of this model is v_{eff}, the effective velocity, which is a tuning parameter that can be used to take into account magnetic field line perturbations and other imperfections and it can help to fit the simulation to actual results.

Debrenev-Skrinsky-Meskov It assumes three types of collisions – fast, adiabatic and magnetized, depending on the value of impact parameter with respect to the Larmor radius (r_L). The role of those interactions depends on the relative velocities of electrons and ions, which defines three regions of velocities and impact parameters. This model allows choosing between asymptotic and numerical approaches [2]. After first short tests the asymptotic version was discarded because of nonphysical discontinuities, and therefore not mentioned in this work.

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Toepffer This is a model of binary collisions assuming three types of interactions – fast collisions at impact parameters less than the radius of electron rotation, collisions with "tight" helices and collisions with "stretched" helices. Details on the mathematical form of the force can be found in [6]. According to the model description in the Betacool user manual and source code, this model is the one that resembles most what is implemented in RF-Track.

JSPEC

JSPEC [3] is a package developed by Jefferson Lab (JLab) for electron cooling simulations. At least three major version of this code has been found and used in this work: JSPEC and JSPEC2 [3] originally by JLab, and RadiaSoft's version [7,8]. All versions implement the Parkhomchuk model, which was used in this study and for which all versions gave comparable results.

FORCE COMPARISON

The force on a single ion was computed as a function of velocity difference between ion and mean electron beam velocity. In the case of the longitudinal force, the scan was performed over the longitudinal velocity difference while the transverse one was set to zero. In the case of the transverse force, it was the other way round. Parameters used in these simulations were chosen to match the parameters of LEIR (CERN):

- Ions: A:208, Q=+54, K_0=862.68 MeV, #10000
- Electrons: uniform distribution, I=0.6 A, $T_\perp$=0.1 eV, $T_\parallel$=0.01 eV, r=25 mm, $K_0(e^-)$=2.3 keV
- E-Cooler: L=2.5 m, B=0.075 T

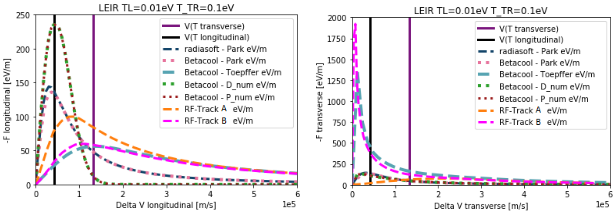

Figure 1: Longitudinal (left), and transverse (right) force on single ion as a function mean electron and single ion velocity difference for all considered models.

Figure 1 shows the cooling forces for each model. The velocities corresponding to electron temperatures are indicated by vertical lines, calculated as $v = c\sqrt{\frac{T}{m_e}}$. Those plots highlight considerable differences and similarities between the different models. The transverse force of RF-Track A is much weaker in comparison to all other models. It suggests that RF-Track A does not consider all physical phenomena. The Toepffler model from Betacool and RF-Track B behave similarly and were considered as the closest to expectation. RF-Track C, not shown in Fig. 1, has a behavior similar to RF-Track B.

FORCE SCAN

To study the behaviour of the models and their performance limits, a set of simulations were performed over a wide range of parameters. Figure 2 presents the peak value of the force as a function of magnetic field and transverse electron temperature.

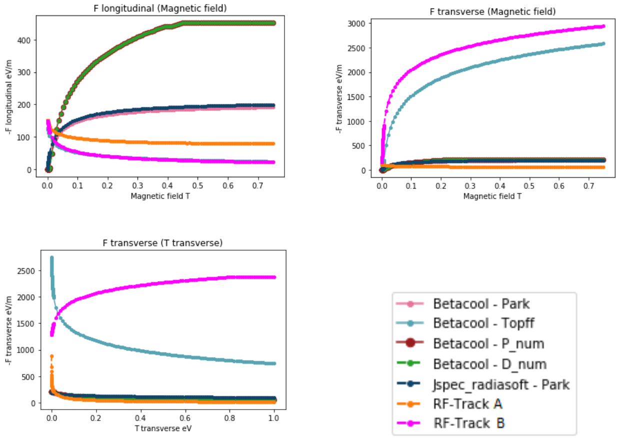

Figure 2: Friction force peak value as a function of electron cooler solenoid field and transverse electron temperature.

Figure 2 (top-left) presents the longitudinal force scan as a function of the solenoid field. There are two separate groups of models: for one the maximum value of force increases with the magnetic field, for the second one it decreases. There is no straightforward reason to decide which behaviour is correct. In this plot the RF-Track B and Toepffer model strongly agree with each other. However, one should note that by setting the transverse velocity difference to zero in Eq. (4) for RF-Track B one removes the magnetized component of the force.

Figure 2 (top-right) shows the dependence of the transverse force on the magnetic field. Here the Toepffer model and RF-Track B have much stronger dependence (up to a factor 10) than the other models.

The scan of transverse force as a function of transverse temperature is presented in Fig. 2 (bottom-left). Although the RF-Track B seems to fit the Toepffer model implemented in Betacool quite well in all previous scans, the behavior of RF-Track B presented on this plot is opposite with respect to all other models, including RF-Track A.

COOLING TIME SCAN

In order to study the influence of the main cooler parameters on the cooling rate, a set of tracking simulations was performed and results are presented in Fig. 3. The scans were performed by changing one of the considered parameters. Cooling time (τ) was defined as the time to reach 20% of the initial emittance, i.e. $\epsilon(\tau) = \epsilon_0/5$.

For the studied parameters, the cooling time is generally longer in the case of Betacool, but for magnetic fields higher than about 0.5 T. RF-Track A shows an unexpected strong dependence on transverse temperature. The scan of the

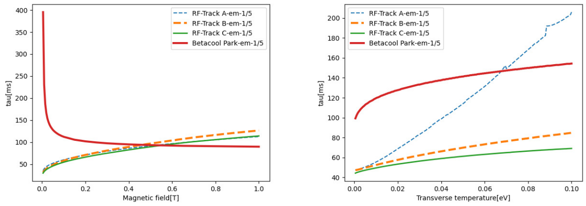

Figure 3: Transverse cooling time as a function of transverse electron temperature (Left) and solenoid field (Right).

magnetic field shows an opposite trend between models, which is compatible with the force analysis in the previous section.

EXPERIMENTAL RESULTS

A set of simulations was performed to reproduce experimental results of cooling time measurements obtained in 1996 on LEAR [9]. Those measurements present the influence of lattice functions at the e-cooler location on the cooling time. Four different machine configurations were measured with lattice functions presented in Table 1. The remaining parameters were specified based on the data from the paper [9]: $\epsilon_0 = 10$, $dp = 0.25‰$, $I = 350\,\mathrm{mA}$, $B = 0.06\,\mathrm{T}$, $L = 1.5\,\mathrm{m}$. As the electron temperatures were not clearly defined, temperatures of $T_\perp$=0.1 eV, $T_\|$=0.1 eV were chosen. In these studies the cooling time is defined as: $\epsilon(\tau) = \epsilon_0/10$ as in [9].

The obtained results are presented in Fig. 4. The shape of the dependence of the cooling time on the optics function is comparable. However, the cooling time scales between simulations and measurement are considerably different, especially for Betacool which predicts a factor of about 5-10 slower cooling. This discrepancy can be due to several reasons, for example the absence of heating effects in all simulations; the use of a different definition of beam emittance: in the measurements most likely the emittance was measured from a Gaussian fit of the transverse beam profile, while in simulations this would not be possible due to the un-physical generation of a dense core, and clearly the different model of the cooling force used in the two codes.

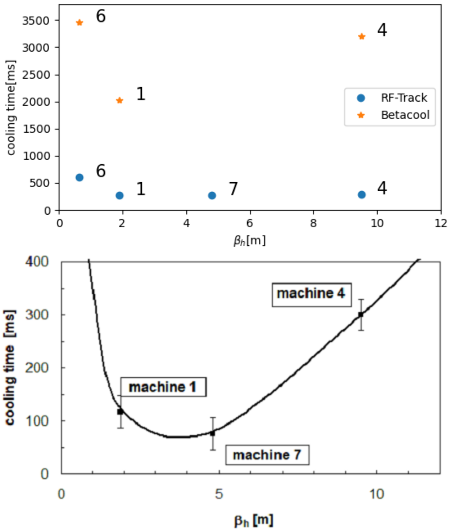

Figure 4: Top: Simulated cooling times for each machine. Bottom: experimental results [9]

Table 1: Lattice functions of measured LEAR machines.

Machine	1	4	6	7
β_H [m]	1.9	9.5	0.65	4.8
β_V [m]	6.4	10.5	5.5	5.0
D[m]	3.6	0	0	5.0

Figure 5 (left) presents the evolution of the momentum distribution in time in the form of a waterfall plot for RF-Track C simulation including e^- space-charge and ions dispersion effects for LEIR e-cooler parameters (but still neglecting any heating effects). This can be qualitatively compared to a measurement taken during the recent recommissioning of the Antiproton Decelerator (AD) [10], shown in Fig. 5 (right). Despite the different ions and e^- parameters (in AD I_{e^-}=2.4 A, $K_0(e^-)$=25.5 keV), it is interesting to observe that both graphs show the same behaviour: the energy of the whole beam increases and then slowly tends to a central edge. This behavior was tracked back to e^- space-charge effects in the simulations.

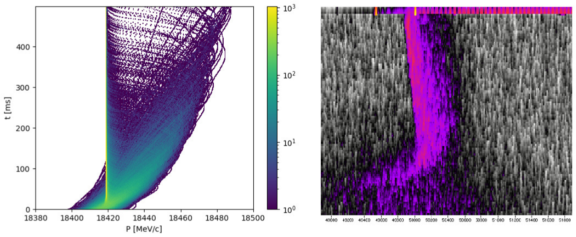

Figure 5: Beam momentum distribution evolution in time simulated with RF-Track C (left) and measured in AD (right) [10].

CONCLUSION

Different models and implementations of electron cooling were compared and tested for a wide range of parameters. Despite the substantial differences observed, it is believed that the latest implementation of e-cooling in RF-Track, model "C", is suitable for future studies. For the presented studies all heating effects (e.g. intra-beam scattering) were neglected, therefore the final distributions of ions include non-physically dense cores. Still, qualitative comparisons with previous experiments at LEAR and observations at AD show that the underlying physics is captured.

The next step will be to consider heating effects, like space-charge and intra-beam scattering, and to compare simulations with new measurements such to increase the predictive power on the e-cooling process in all CERN e-coolers. The different behavior observed between RF-Track and Betacool as a function of e-cooler magnetic field is puzzling. This could be an additional topic of study with experimental measurements in the future.

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