Emittance Growth from Modulated Focusing and Bunched Beam

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Based on earlier work with Vladimir Litvinenko and Gang Wang

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RHIC

Outline:

Low Energy RHIC electron Cooling (LEReC) plans

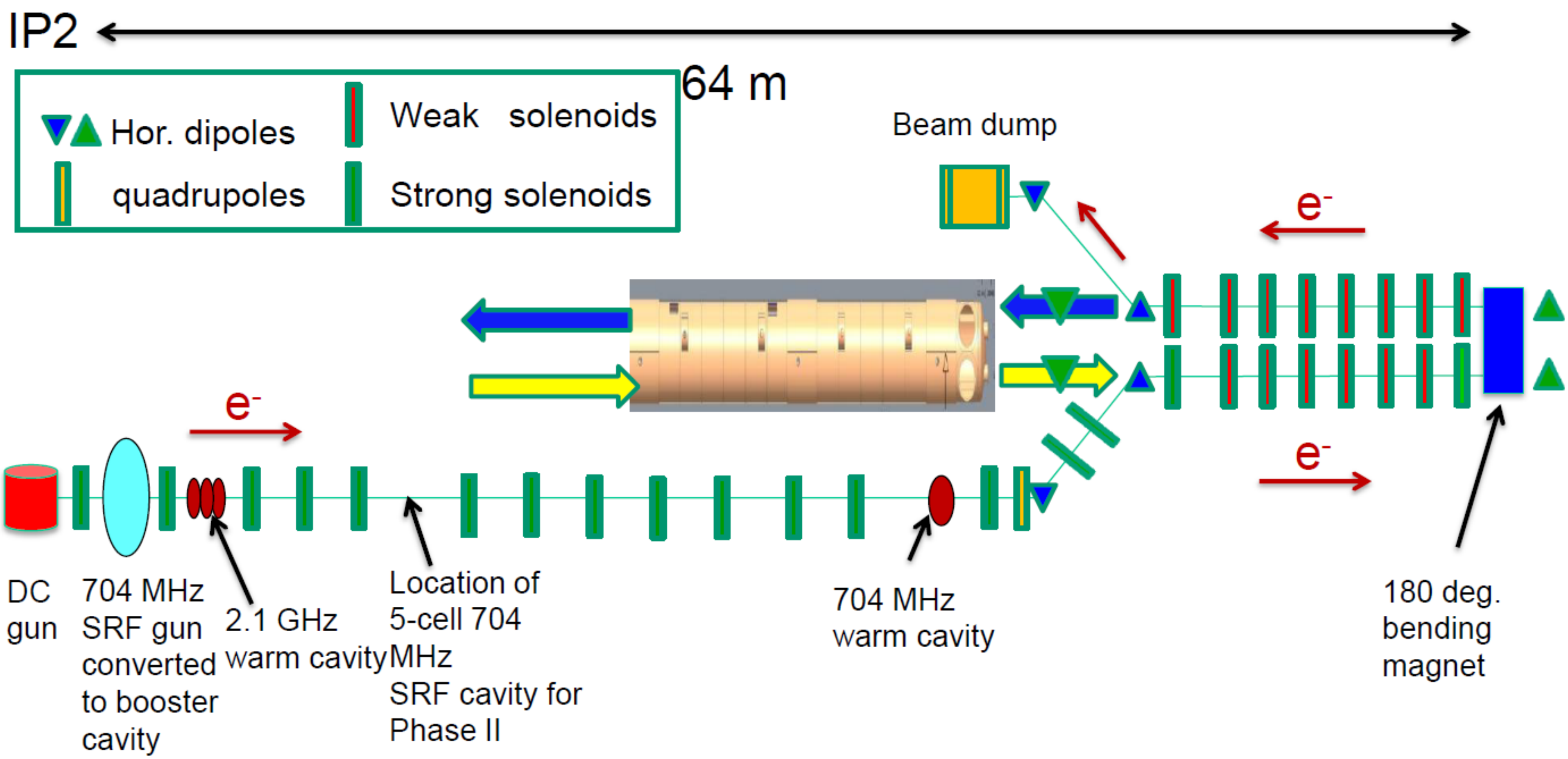
Analytic description of the resonance problem and scaling law

Simulation parameters

General simulation results

Particular results for LEReC

BNL is in the process of implementing low energy bunched beam electron cooling in RHIC. The first phase of the project is illustrated below. Details can be found in Alexei Fedotov's talk.



LEReC beam structure in cooling section
Example for γ= 4.1 (E_{ke}=1.6 MeV)

Ions structure:

120 bunches

f_rep=120x75.8347 kHz=9.1 MHz

N_ion=5e8, I_peak=0.24 A

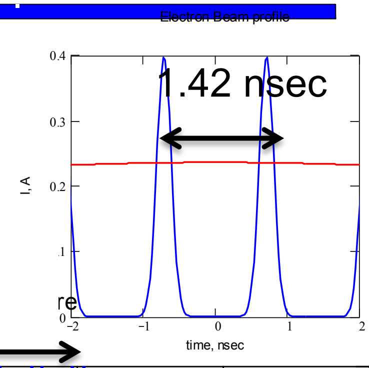
Rms length=3.2 m

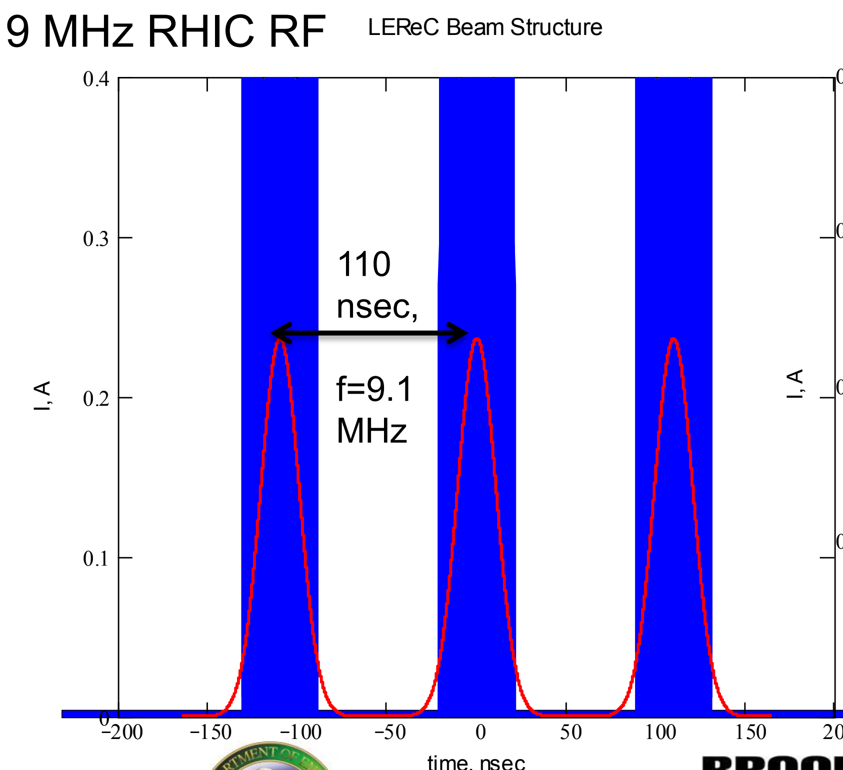
Electrons:

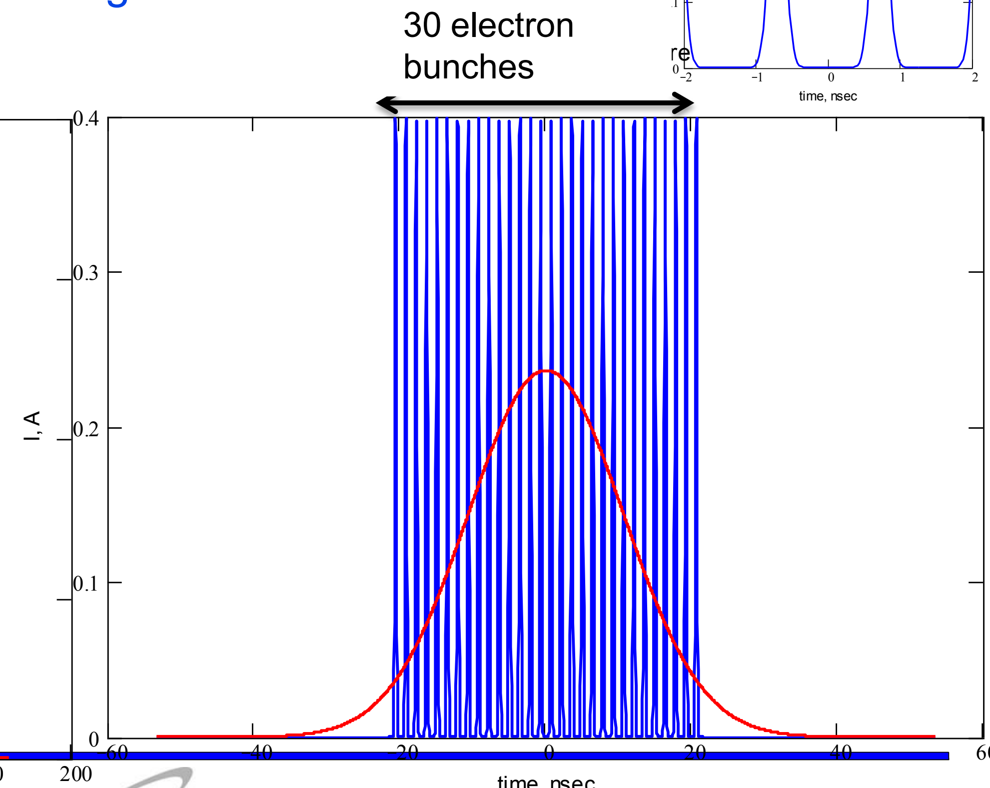
f_SRF=703.5 MHz

Q_e=100 pC, I_peak=0.4 A

Rms length=3 cm







Basic Idea

For RHIC at γ =4 and rms fractional momentum spread σ_p = 5.e-4, the typical longitudinal slip is

$$\sigma_t = T_{rev}\eta\sigma_p = 0.4ns$$

The electron rms bunch length is ≈0.1 ns so a typical ion interacts with an electron bunch no more than twice a synchrotron oscillation. Take it as twice and assume a single electron bunch. After receiving a kick from the electron bunch the ion needs to complete half a synchrotron oscillation before interacting again. Along with the cooling effect the electron bunch has an effective focusing strength so the map for half a synchrotron oscillation is (very approximately)

$$\begin{bmatrix} x_{n+1} \\ p_{n+1} \end{bmatrix} = \begin{bmatrix} \cos(\pi Q_x / Q_s) & \sin(\pi Q_x / Q_s) \\ -\sin(\pi Q_x / Q_s) & \cos(\pi Q_x / Q_s) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta_L k & 1 \end{bmatrix} \begin{bmatrix} x_n \\ p_n \end{bmatrix}$$

When Qx/Qs is near an integer the map is exponentially unstable.

The trace of the matrix is the sum of the eigenvalues

$$2\cos\Psi + \beta_L k \sin\Psi = \lambda + 1/\lambda$$

Assume a weak resonance with $\sin\Psi = \varepsilon$ and $\lambda = 1 + \delta$. Expanding to leading order yields

$$\delta = \sqrt{\beta_L k \varepsilon - \varepsilon^2} \le \frac{|\beta_L k|}{2}$$

There is growth for $0 > \varepsilon > \beta_L k$ so both the width and strength of the growth are proportional to the electron charge per bunch and are very small for our parameters.

Now include longitudinal IBS. This causes the longitudinal action to drift which in turn causes drift in the synchrotron frequency. This leads to particles randomly drifting through the resonances and emittance growth.

More elaborate theory:

$$\frac{d^2\xi}{d\varphi^2} + Q^2\xi = -Q^2\beta_L^{3/2}(\varphi)\frac{\Delta B(\xi,\varphi)}{p/q}$$

The force is space charge from the electrons

$$E + v\times B = \frac{Z_0 I_e(\omega_0 t - \varphi)x}{4\pi\beta\gamma^2\sigma_x^2} \to v\Delta B$$

Where $\omega_0 t - \varphi = \omega_0\tau(\varphi)$ which oscillates at Q_s.

Neglect the variation in ξ during the space charge kick

$$\Delta\frac{d\xi}{d\varphi} = -Q\frac{qZ_0 I_e(\omega_0\tau(\varphi))}{4\pi\beta^3\gamma^2\sigma_x^2 E_T}\xi\int_{-l/2}^{l/2} ds\,\beta_L(s) = -Q\xi F(\omega_0\tau)$$

The kick is localized at ϕ =0 so using a tune greater than ½ just complicates matters. Hence introduce

$$\hat{Q} = |Q - \mathrm{nint}(Q)|$$

So, $$\frac{d^2\xi}{d\varphi^2}+\hat{Q}^2\xi=-\hat{Q}\xi F(\omega_0\tau)\delta_p(\varphi)\approx-\frac{\hat{Q}\xi F(\omega_0\tau)}{2\pi}$$

The motion in τ is modulated at Q_s so we expect resonances of the form $2\hat{Q}-mQ_s=\varepsilon\approx 0$

This is a standard problem, see Landau's Mechanics pg 80.

One has unstable motion of the form exp(s ϕ) with $2s=\sqrt{\hat{c}^2-\varepsilon^2}$

For a single centered electron bunch we have

$$\hat{c}=\frac{qZ_0\hat{I}_e l\langle\beta_L\rangle}{8\pi^2\beta^3\gamma^2E_T\sigma_x^2}\exp\left(-\frac{\hat{\tau}^2}{4\sigma_t^2}\right)I_{m/2}\left(\frac{\hat{\tau}^2}{4\sigma_t^2}\right)even(m)$$

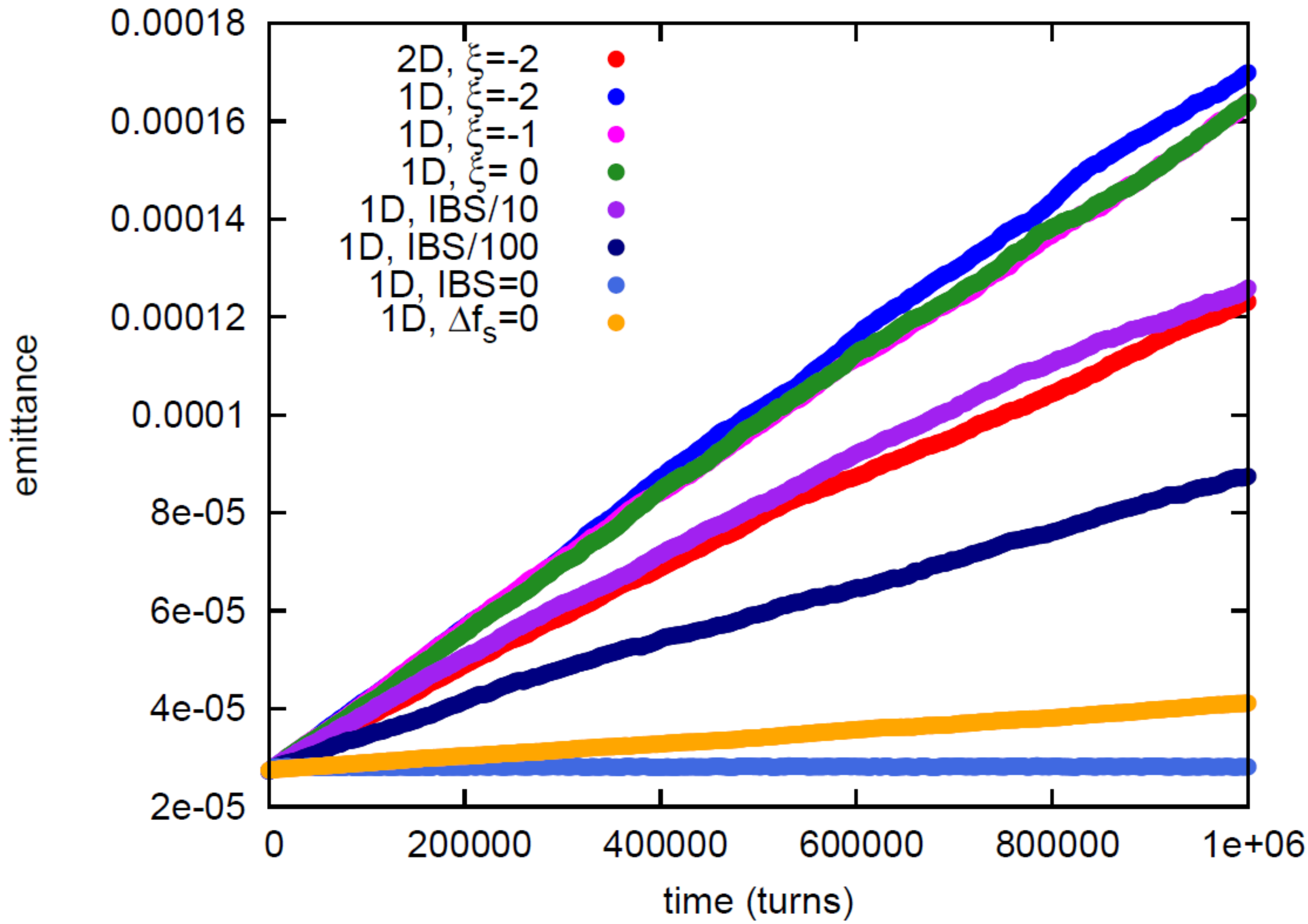
The fraction of time spent in resonance is proportional to the resonance width , which is proportional to electron bunch charge. While on resonance the growth rate is proportional to electron bunch charge too. Hence the emittance growth rate is proportional to the square of the electron bunch charge. The earlier model satisfies the same scaling law.

Simulation model

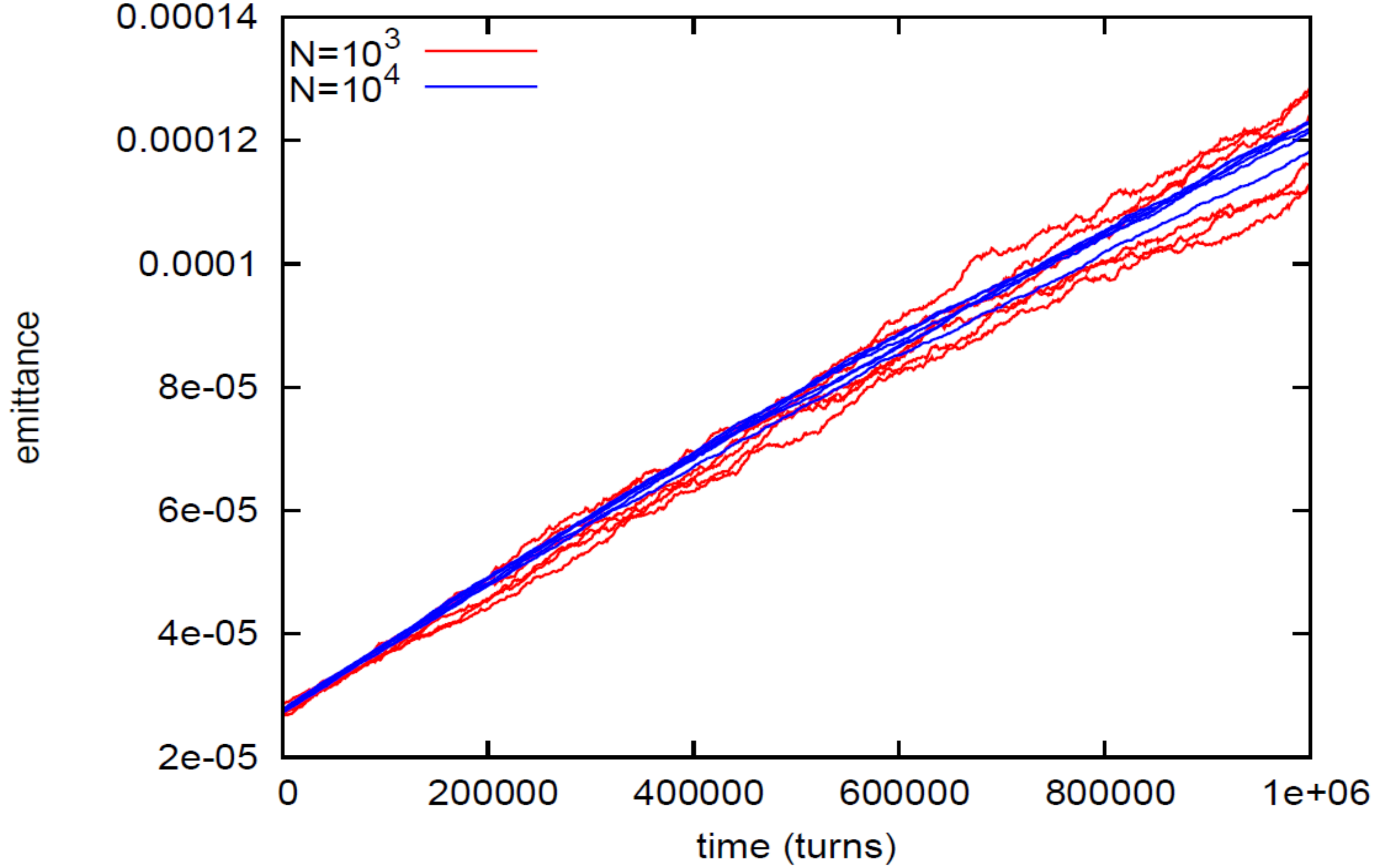
The model includes

1) Nonlinear RF, tunes,chromaticity, dQx/dJx, coupling
2) Ion-electron space charge once per turn as a thin lens. Assumes a long 3D Gaussian and ignores longitudinal kicks.
3) IBS implemented as Langevin kicks including diffusion (Piwinski) and damping (Zenkevich). Only longitudinal IBS is implemented since the smooth lattice rates have cooling (the wrong sign) for transverse.
4) Thin lens electron cooling assuming that the electron velocity distribution is a single temperature Gaussian in the rest frame of the bunch. (Now have $T_{\|} \neq T_{\perp}$ but no fundamental difference.)
5) Ion-ion space charge 10 times per turn (now use $\Delta\Psi(z,J_x+J_y)$). Assumes a long bunch with no coherent dynamics so that the longitudinal profile is a 1000 turn one pole average. Transverse is Gaussian.

Initially neglect the cooling force. The growth rate depends weakly on dimensionality, very weakly on chromaticity, and weakly on IBS



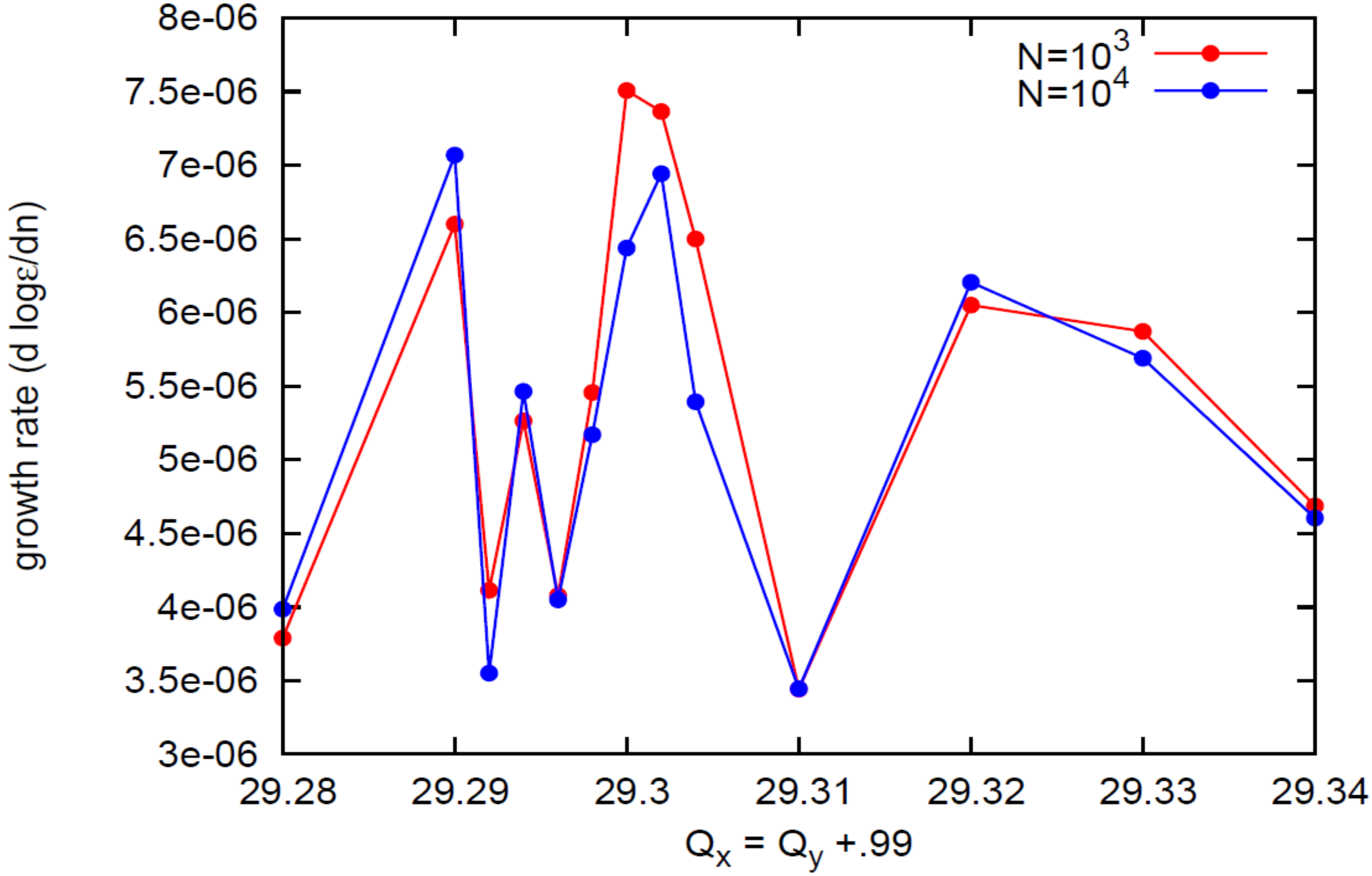
Effect of different random seeds and the number of simulation particles. There is a well defined growth rate that depends on coarse grained parameters.



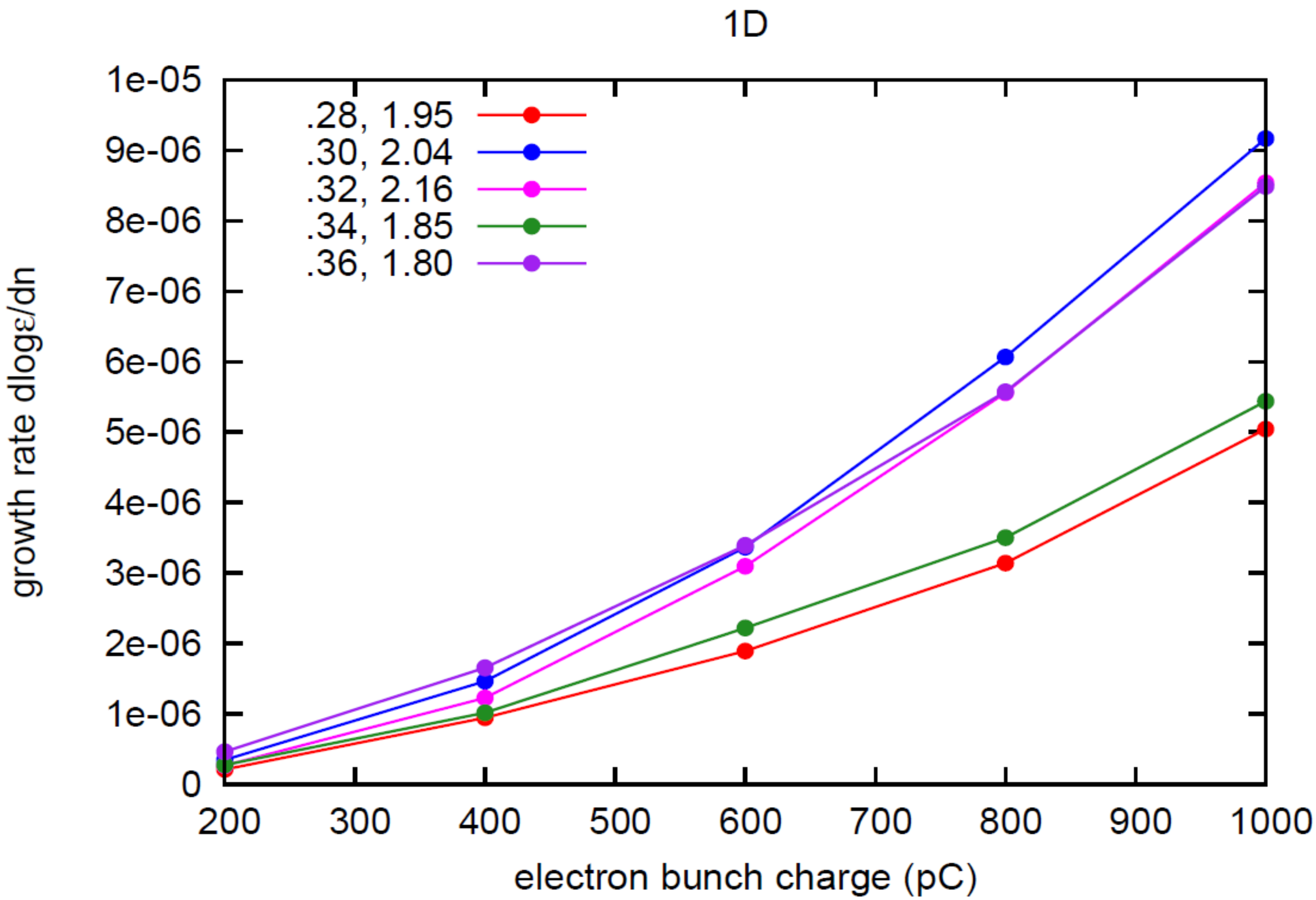
General features of the resonant interaction with IBS (no cooling):

The growth rate depends on the tune in a chaotic way.

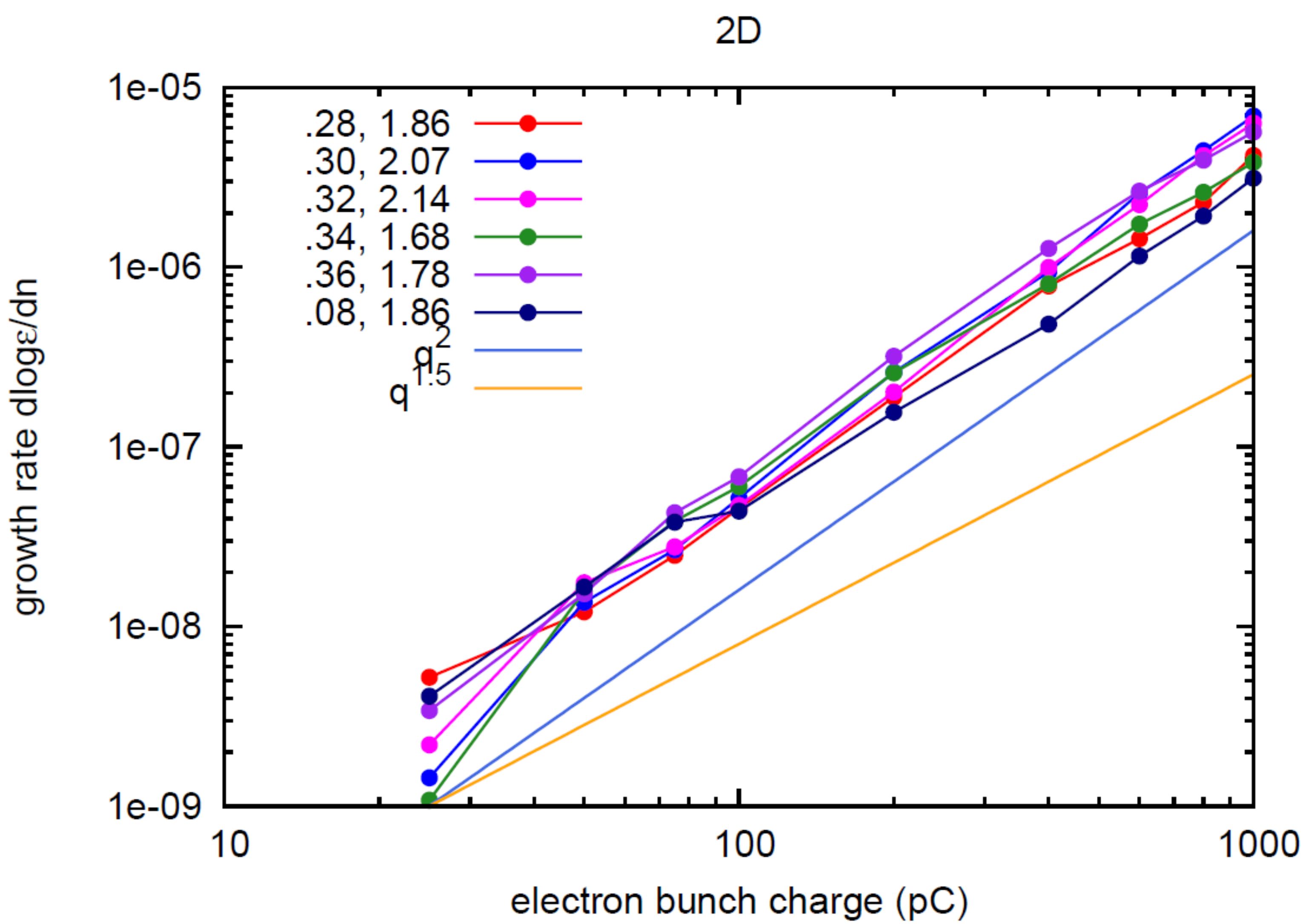
Very near the integer things tend to diverge.



Scaling of emittance growth rate with intensity for a variety of tunes. The power law fits are close to 2.

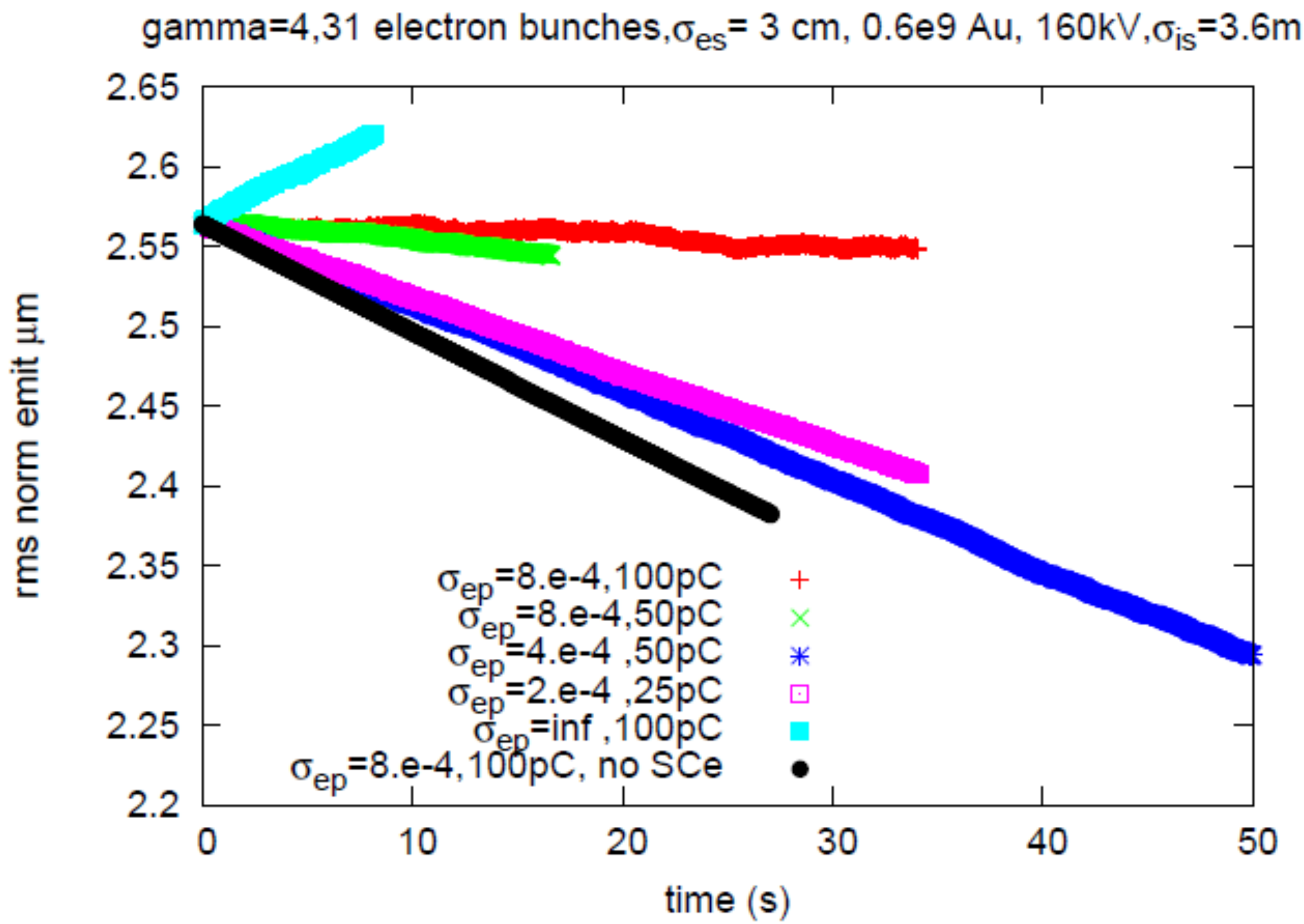


For 2D growth rates the average slope is around 2.

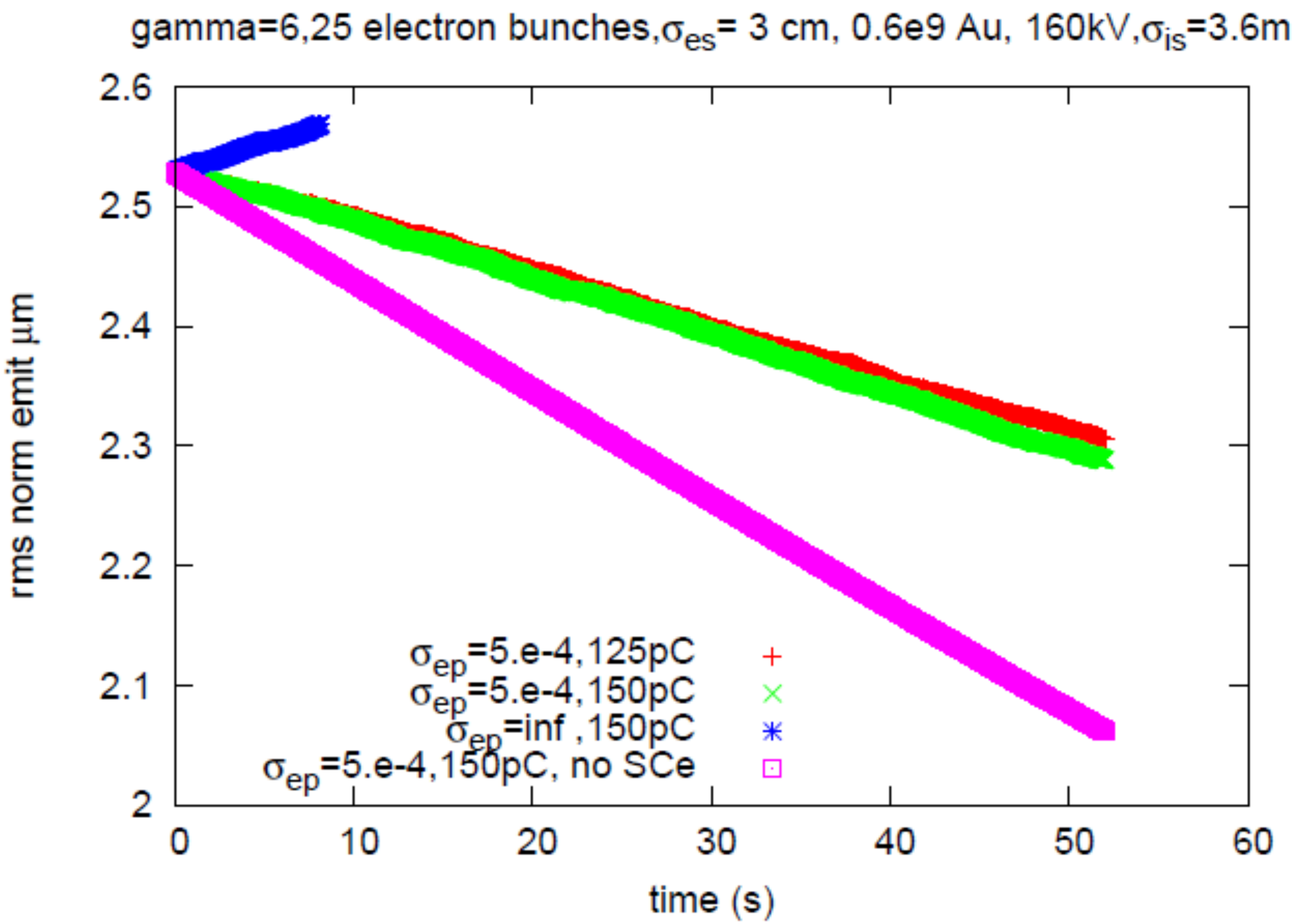


Now include cooling and other relevant parameters for LEReC.

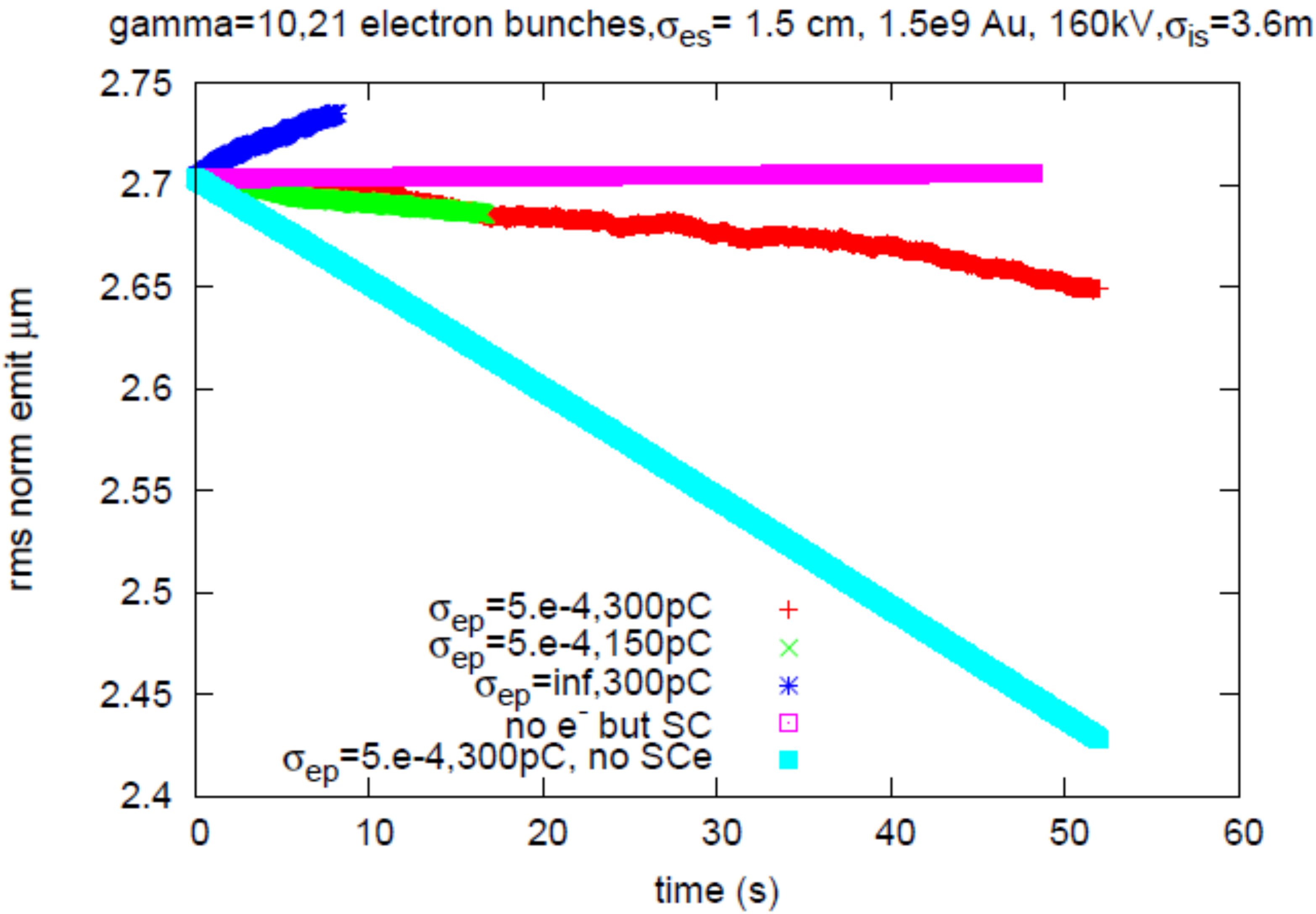
For lowest energy we need cold electrons but not too many.



Intermediate energy OK with planned parameters



Top energy will benefit from longer electron bunches



Conclusions:

- Using bunched beams for electron cooling can lead to dynamically generated emittance growth.
- The required ingredients for the process are

1) Electron bunches that are of comparable length to the rms longitudinal slip per turn of the ions

2) Variation in synchrotron frequency with amplitude

3) Longitudinal Intra-Beam Scattering (IBS) is required but there is a weak dependence on IBS rates

- Low Energy RHIC electron Cooling (LEReC) operates in a parameter regime where these effects are clearly present but it is possible to adjust parameters and still have adequate cooling.