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Optical Stochastic Cooling at IOTA ring

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Fermilab

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Contents

- Basics of Optical Stochastic Cooling
- Optical Stochastic Cooling at IOTA ring
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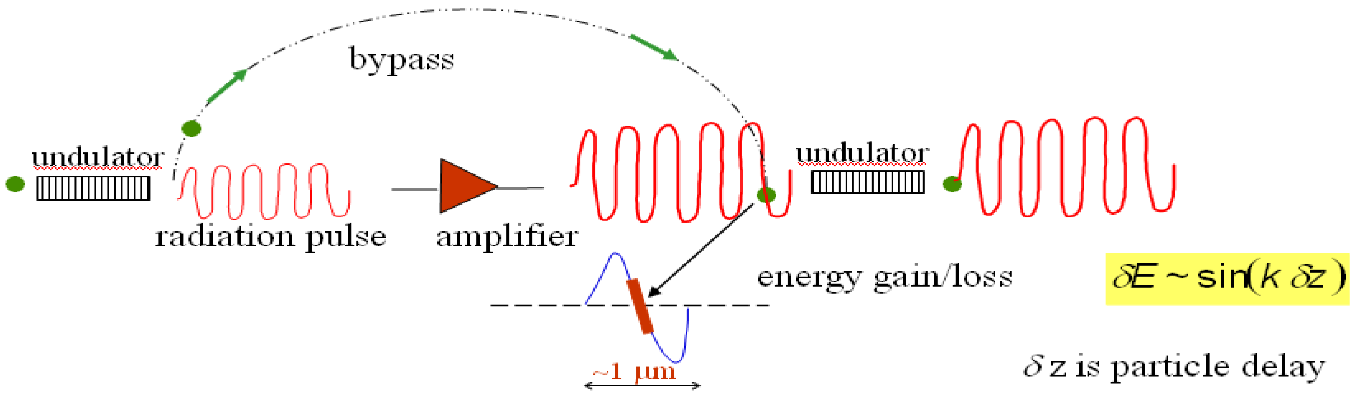
- Conclusions

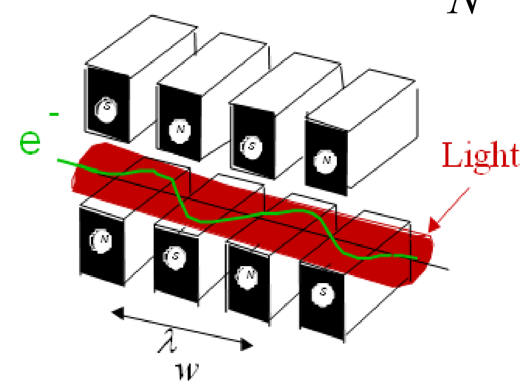
Principles of Optical Stochastic Cooling

- OSC - suggested by Zolotorev, Zholents and Mikhailichenko (1994)
- OSC obeys the same principles as the microwave stochastic cooling, but exploits the superior bandwidth of OA ~10^{13}-10^{14} Hz



- At optimum the cooling rate of stochastic cooling for continuous beam can be estimated as: $\lambda_{\max} f_0 \approx \dfrac{W}{N}$

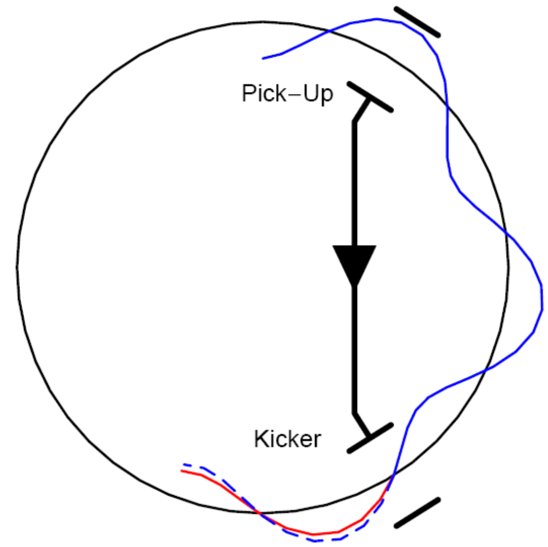
  Or dimensionless damping rate: $\lambda_{\max} \approx 1/N_{sample}$
  - Potential gain in damping rates: $10^3 \div 10^4$
- Pickup and kicker must operate at the optical frequencies (same as an optical amplifier)
  - Undulators suggested for pickups & kickers



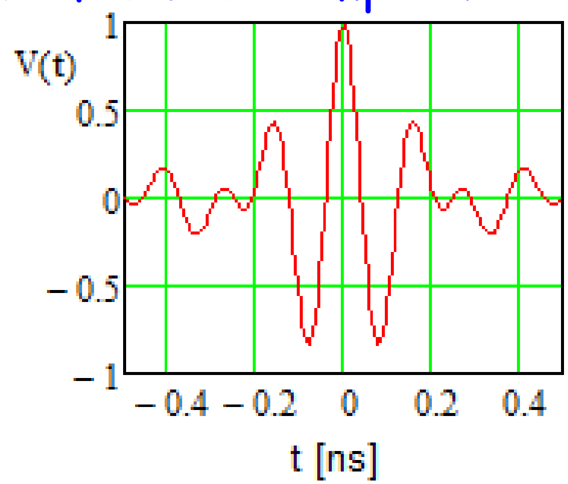
- Slow particles do not radiate at optical frequencies
  - OSC can operate only with ultra-relativistic particles

Optimal Gain in OSC

- Three types of stochastic cooling (microwave)
  - Transverse (differential signal is difference of signals of two sides of a pickup)
  - Longitudinal Palmer cooling (same as above)
  - Longitudinal filter cooling (signal is difference of signals from two different turns)



- Suppression of diffusion in all 3 cases – zero signal for zero amplitude
- Longitudinal transient time cooling (ToF)
  - Also tested experimentally
  - No subtractions => more diffusion
  - Only method to be used at optical frequencies



- Optimal gain & max. cooling rate for ToF cooling
  - single particle cooling ($\propto G$)
    versus multi-particle diffusion ($\propto G^2$)

$$\lambda_{max} f_0 \approx \frac{W}{N} \xrightarrow[\text{rms size accounting}]{\text{Bunched beam cooling}} \boxed{\lambda_{max} f_0 \approx \frac{2\pi^{5/2}}{n_\sigma^{\ 2}} \frac{\sigma_s}{C} \frac{W}{N}} \quad n_\sigma = \frac{(\Delta p / p)_{max}}{\sigma_p}, \quad W = f_0 \left(n_{max} - n_{min} \right),$$

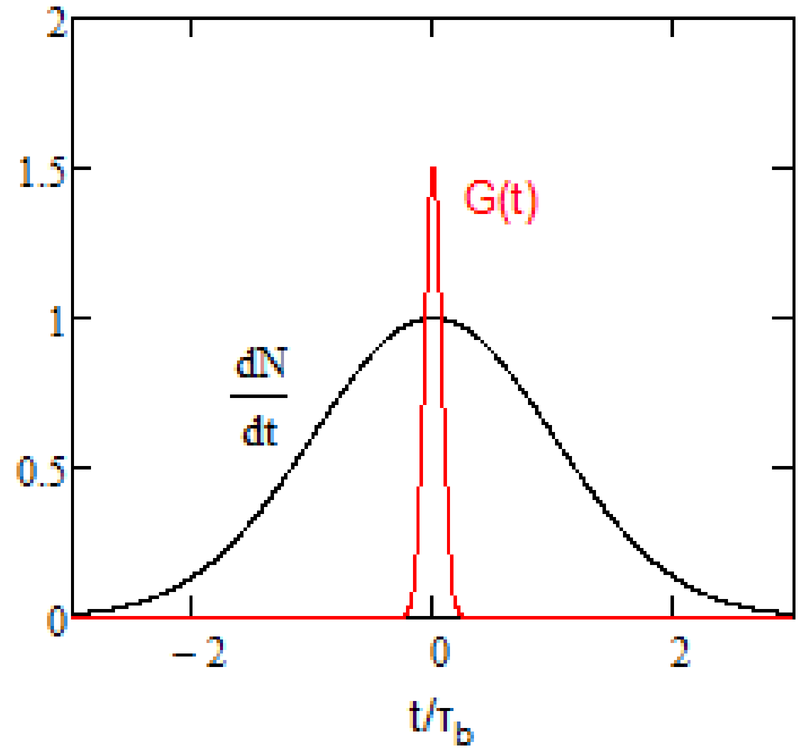
  - Bandwidth, for Gaussian band: $W = 2\sqrt{\pi}\sigma_f \quad \left(G(\omega) = G_0 \exp\left(-\omega^2 / 2\left(2\pi\sigma_f\right)^2 \right) \right)$

Gain Duration in OSC

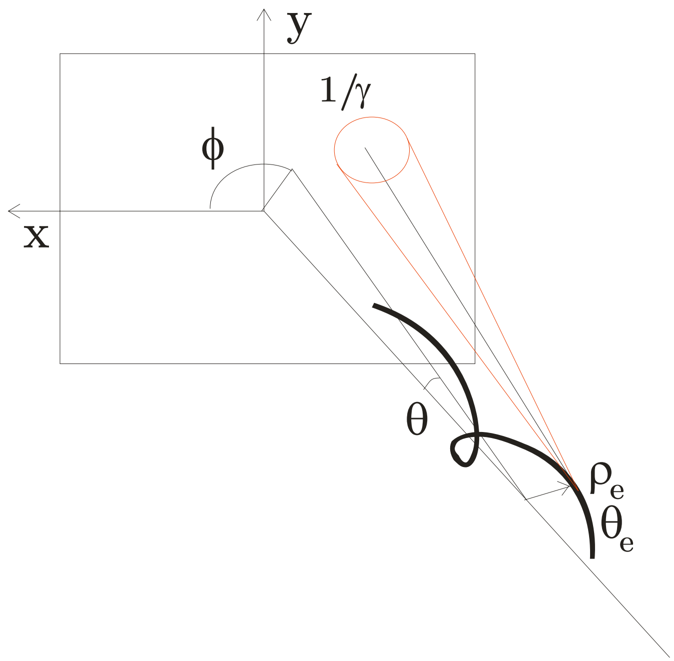
- Optimal gain does not depend on the active time of the amplifier
  - Reduction of average cooling for the gain length shorter than the bunch length

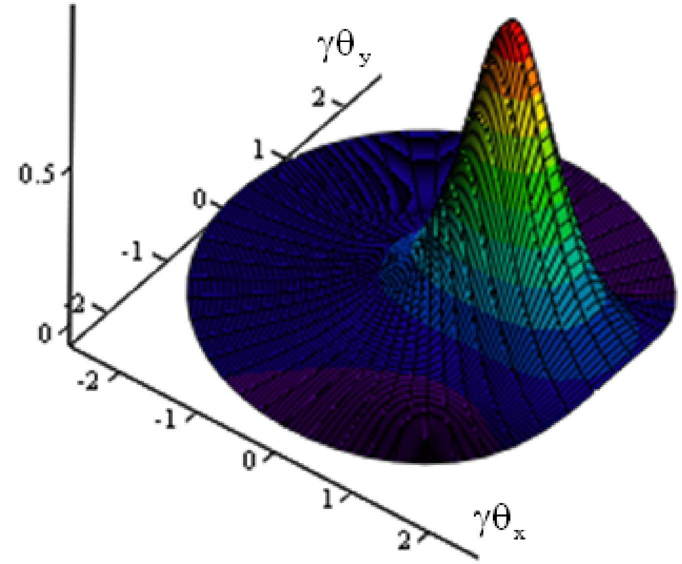
$$\lambda_{opt} = \frac{2\pi^{5/2}}{n_\sigma^{\ 2}} \frac{\sigma_s}{C} \frac{W}{N} \frac{\sigma_g}{2\sigma_s}, \quad \sigma_g \ll \sigma_s$$

- That basically excludes optical parametric amplifiers and FELs as candidates for OA of heavy particles



Basics of OSC – Radiation from Undulator

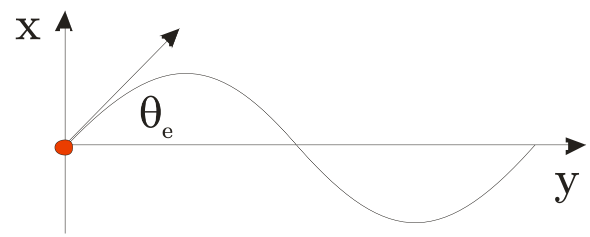




E_x for K=1

- Radiation of ultra-relativistic particle is concentrated in 1/γ angle
- Undulator parameter: $K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi}\frac{eB_0}{mc^2}$
- For K ≥ 1 the radiation is mainly radiated into higher harmonics
- Radiation wave length

$$\lambda = \frac{\lambda_{wgl}}{2\gamma^2}\begin{cases}\left(1+\gamma^2\left(\theta_e^2+\theta^2\right)\right) & -\ helical\ undulator\\ \left(1+\gamma^2\left(\frac{1}{2}\theta_e^2+\theta^2\right)\right) & -\ flat\ undulator\end{cases}$$



- Only 1st harmonic radiation interacts in the kicker undul. resonantly

Basics of OSC – Cooling Rates: Linear & Nonlinear

- Partial slip factor: describes a long. particle displacement on the way from pickup to kicker with $\Delta p/p \neq 0$ & no betatron motion

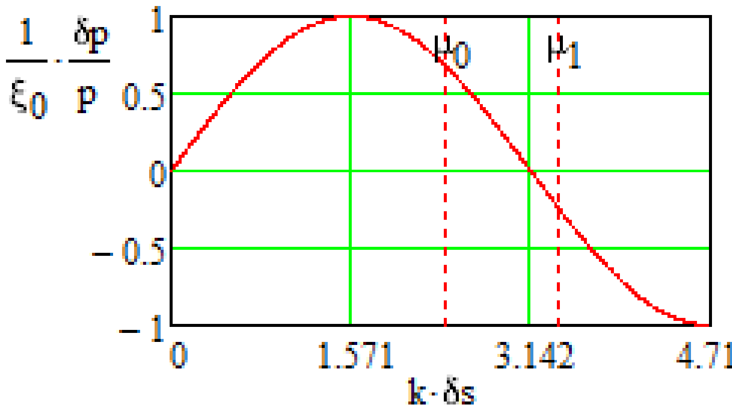
$$\Delta s = \tilde{M}_{56}(\Delta p / \mathrm{p}) \quad \Leftrightarrow \quad \tilde{M}_{56} = M_{51}D_1 + M_{52}D_1' + M_{56}$$

- Kick strength: $\delta p / p = -\xi_0 \sin(k\,\Delta s)$ => $\delta p / p = -\xi_0\, k\,\Delta s$
  - Cooling rates:

$$\lambda_x = \frac{k\xi_0}{2}\left(M_{56} - \tilde{M}_{56}\right)$$
$$\lambda_s = \frac{k\xi_0}{2}\tilde{M}_{56}$$

=>

$$\lambda_x + \lambda_s = \frac{k\xi_0}{2} M_{56}$$



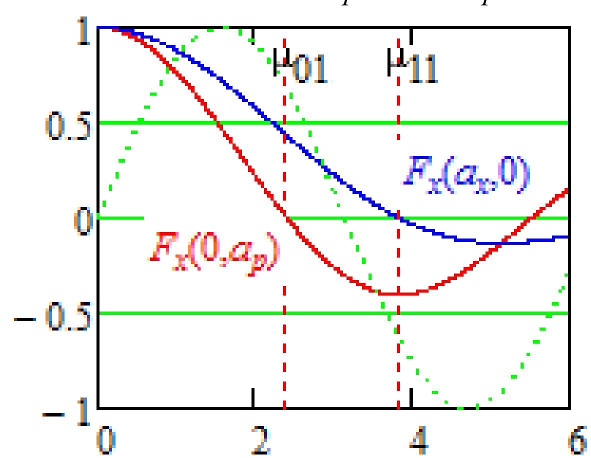
- But cooling force depends on Δs nonlinearly
  - Averaging over bet. & synchr. motions: $k\Delta s = a_x \sin(\psi_x - \psi_0) + a_p \sin(\psi_p)$

=>

$$\lambda_{s,x}(a_x, a_p) = F_{s,x}(a_x, a_p)\lambda_{s,x}$$
$$F_x(a_x, a_p) = \frac{2}{a_x} J_0(a_p) J_1(a_x)$$
$$F_p(a_x, a_p) = \frac{2}{a_p} J_0(a_x) J_1(a_p)$$

=>

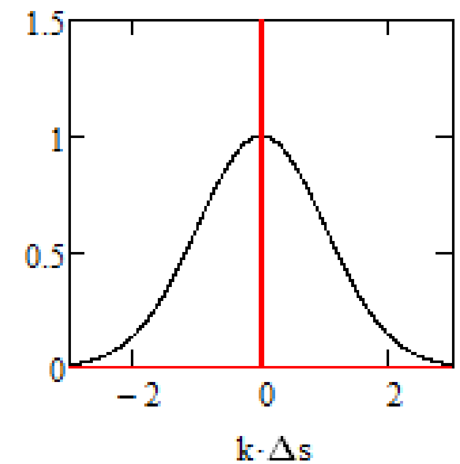
Cooling boundaries:
$a_x,\ a_p < \mu_{01}$,
$\mu_{01} \approx 2.405$



Basics of OSC – Cooling Ranges

- Longitudinal displacement (sample lengthening) in pickup is

$$a_p = k\tilde{M}_{56}\left(\Delta p / p\right)$$

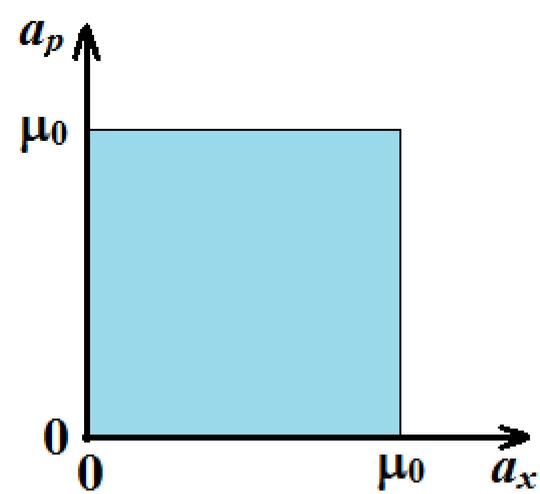


- In the linear approximation the cooling rates do not depend on beta-functions in OSC straight
- However for the horizontal betatron motion the sample lengthening on the way from pickup to kicker depends on β-function

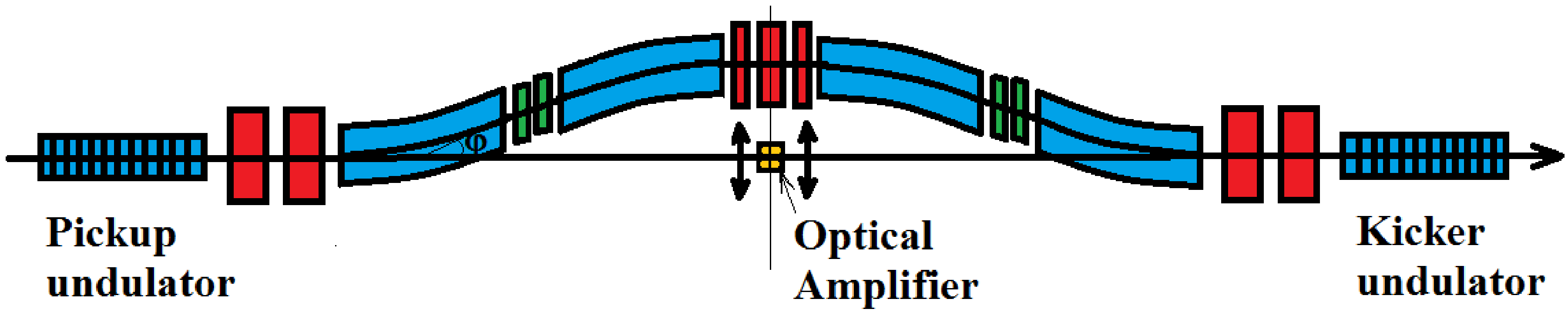
$$a_x = k\sqrt{\varepsilon\left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}{}^2\right)}, \quad \text{where } \varepsilon = \beta_p\theta^2 - 2\alpha_p x\theta + \gamma_p x^2$$

where β_p, α_p are the β- and α-functions in the pickup

- Cooling requires the both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_{01} \approx 2.405$
- Sample lengthening is described by the same above equations



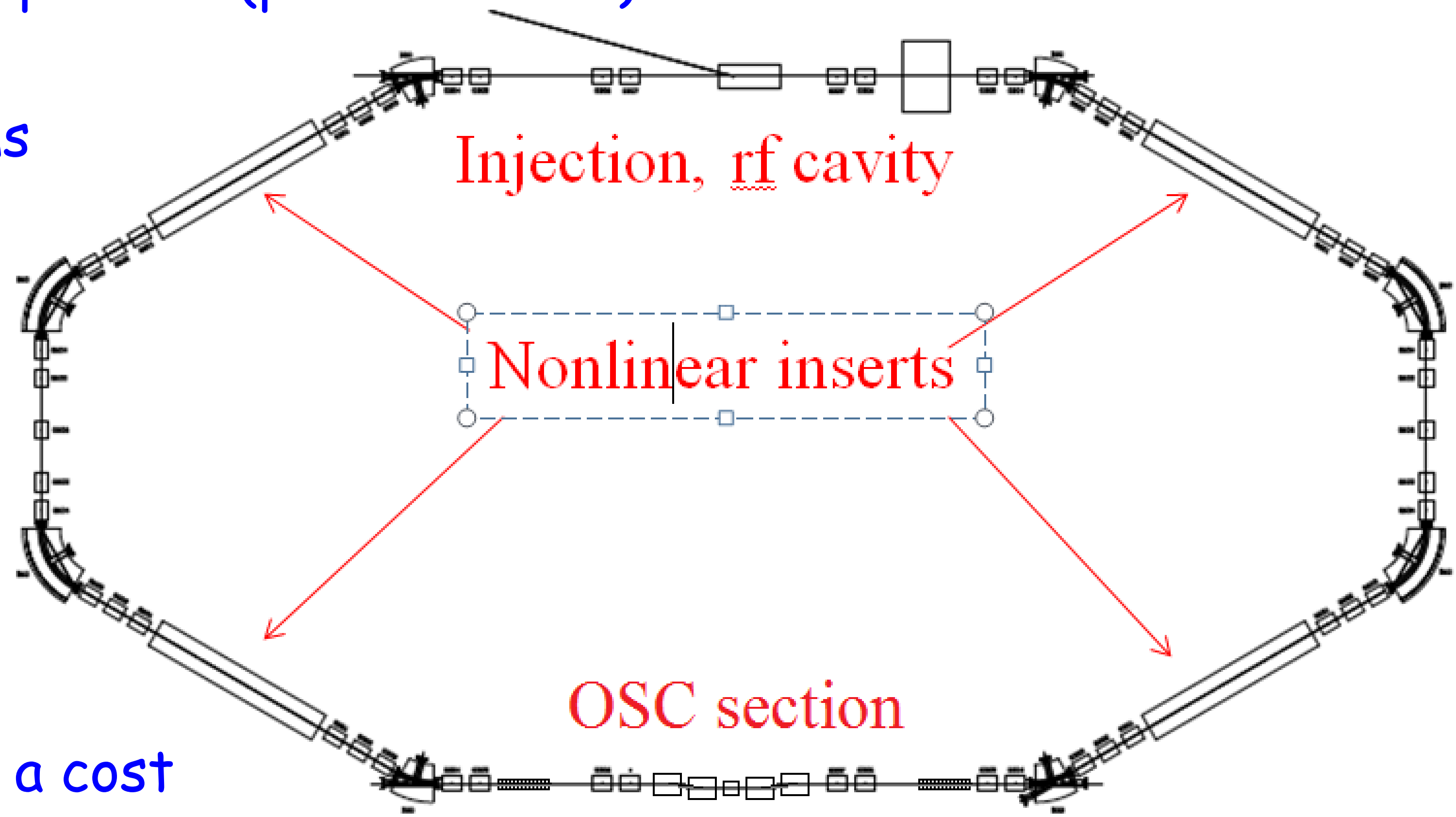
Cooling Chicane



- Cooling chicane delays the beam by the same amount as optical amplifier + optical system
- Four rectangular dipoles
  - if no horizontal focusing => $M_{56} = \tilde{M}_{56}$ => no horizontal cooling
- Quad in the center makes M_{56} and $\tilde{M}_{56}$ different
  - ⇨ horizontal cooling
  - Extra two quads make a triplet: more opportunities for future
- Two sextupoles at each side to correct non-linear sample lengthening
- Optical amplifier is actually located inside dipoles and quadrupole

Test of OSC in Fermilab

- Fermilab is constructing a dual purpose ring called IOTA to test:
  - Integrable optics
    - 150 MeV electrons
    - 2.5 MeV protons (pc≈70 MeV/c)
  - OSC
    - electrons
- Part of ASTA program
  - Top energy injection from SC linac
- Test in a small electron ring is a cost effective way to test the OSC



OSC Chicane Optics Optimization

Dispersion in the chicane center

- In the first approximation
  the orbit offset in the chicane (h),
  the path lengthening (Δs),
  the defocusing strength of Q_d (Φ)
  and dispersion in the chicane center (D^*)
  determine the entire cooling dynamics
- Δs is set by delay in the amplifier => M_{56}
  (Δs = 3 mm is chosen, includes delay in lenses)
- Choose $\left(dD/ds\right)^* = 0$ => $D\big|_{s=\pm L_t} \approx D^*$
- $\Phi D^* h$ determines the ratio of decrements
  - Choose: $\lambda_x \approx 2\lambda_s$ => $\Phi D^* h \approx 4\Delta s/3$
- For the wave length of λ=2.2 μm and momentum spread of $\sigma_p = 1.1\cdot 10^{-4}$
  ⇨ Cooling acceptance for longitudinal degree of freedom ($n_{\sigma p}$=3.7)
- Thus $\Phi D^* h$ determines the ratio of cooling rates and cooling acceptance in momentum

This is the first limitation which sets the wave length to be ≥ 2 μm

$$M_{56} \approx 2\Delta s \,,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h \,,$$

$$\frac{\lambda_x}{\lambda_s} = \frac{\tilde{M}_{56}}{M_{56} - \tilde{M}_{56}} \approx \frac{\Phi D^* h}{2\Delta s - \Phi D^* h} \,,$$

$$k\sigma_p \left(\frac{\Delta p}{p}\right)_{\max} \tilde{M}_{56} < \mu_{01}$$

$$\xrightarrow{n_{\sigma p}\sigma_p = \left(\frac{\Delta p}{p}\right)_{\max}}$$

$$n_{\sigma p} \approx \frac{\mu_{01}}{\left(2\Delta s - \Phi D^* h\right) k\sigma_p} \,,$$

OSC Chicane and Limitations on IOTA Optics (2)

Beta-function in the chicane center

- Behavior of the horizontal β-function determines the cooling range for horizontal degree of freedom
  - ♦ At optimum $\alpha^* = 0$
  - ⇨ Cooling acceptance:

$$\varepsilon_{\max} = \frac{\mu_{01}{}^2}{k^2\left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}{}^2\right)} \xrightarrow[\alpha_p \approx \frac{L_t}{\beta^*}]{\beta_p \approx \frac{L_t{}^2}{\beta^*}} \approx \frac{\mu_{01}{}^2}{k^2 \Phi^2 h^2 \beta^*}$$

- For known rms emittance, ε, we can rewrite it as following

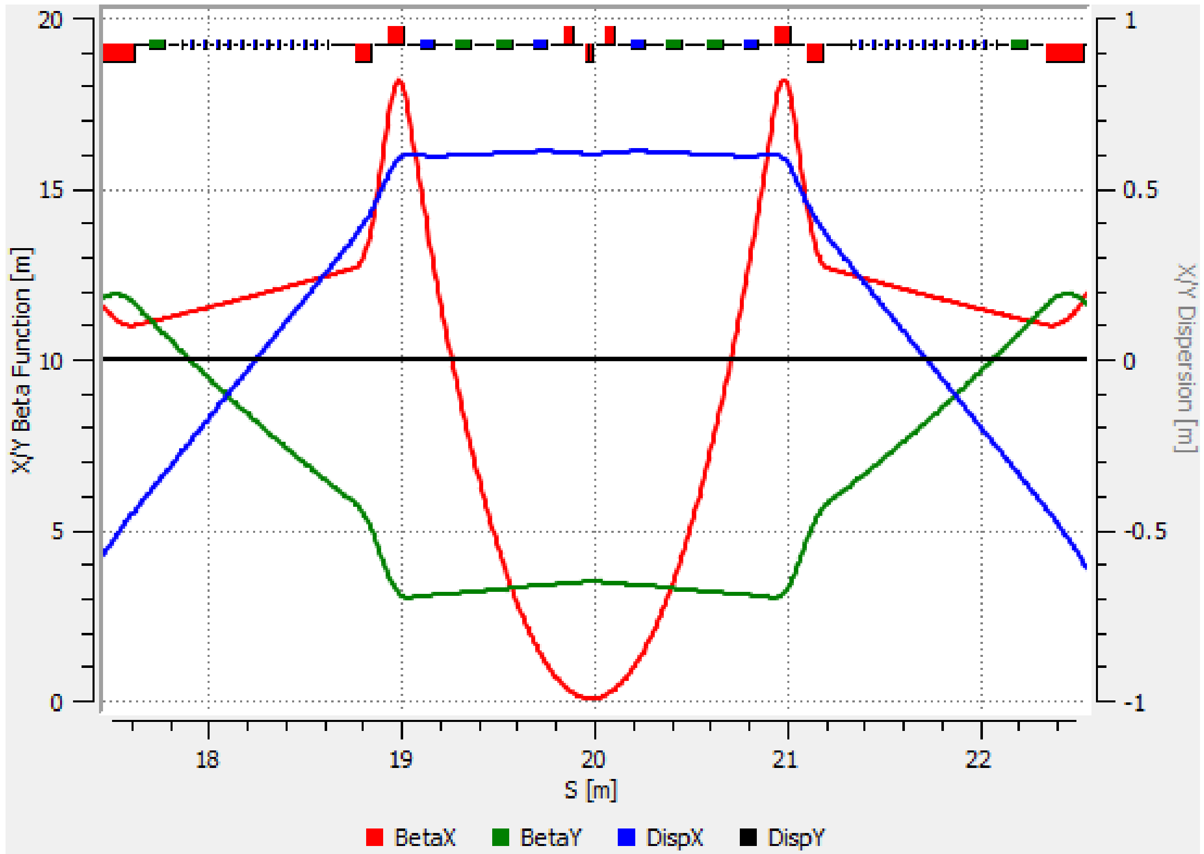
$$n_{\sigma x} \equiv \sqrt{\frac{\varepsilon_{\max}}{\varepsilon}} \approx \frac{\mu_{01}}{k\Phi h\sqrt{\varepsilon\beta^*}} \xrightarrow{\Phi D^* h = 2\Delta s \frac{\lambda_x}{\lambda_s + \lambda_x}} \boxed{n_{\sigma x} = \frac{\mu_{01}}{2k\Delta s}\left(1 + \frac{\lambda_s}{\lambda_x}\right)\sqrt{\frac{A_x{}^*}{\varepsilon}}} \qquad A_x{}^* = \frac{D^{*2}}{\beta^*}$$

- Thus the cooling range, $n_{\sigma x}$, determines the dispersion invariant $A_x{}^*$
- Average value of A_x in dipoles determines the equilibrium emittance.
  - ♦ $A_x{}^*$ is large and A_x needs to be reduced fast to get an acceptable value of the equilibrium emittance (ε)
- Getting sufficiently large cooling acceptance requires long wave length of the radiation: another reason for $\lambda \geq 2$ μm

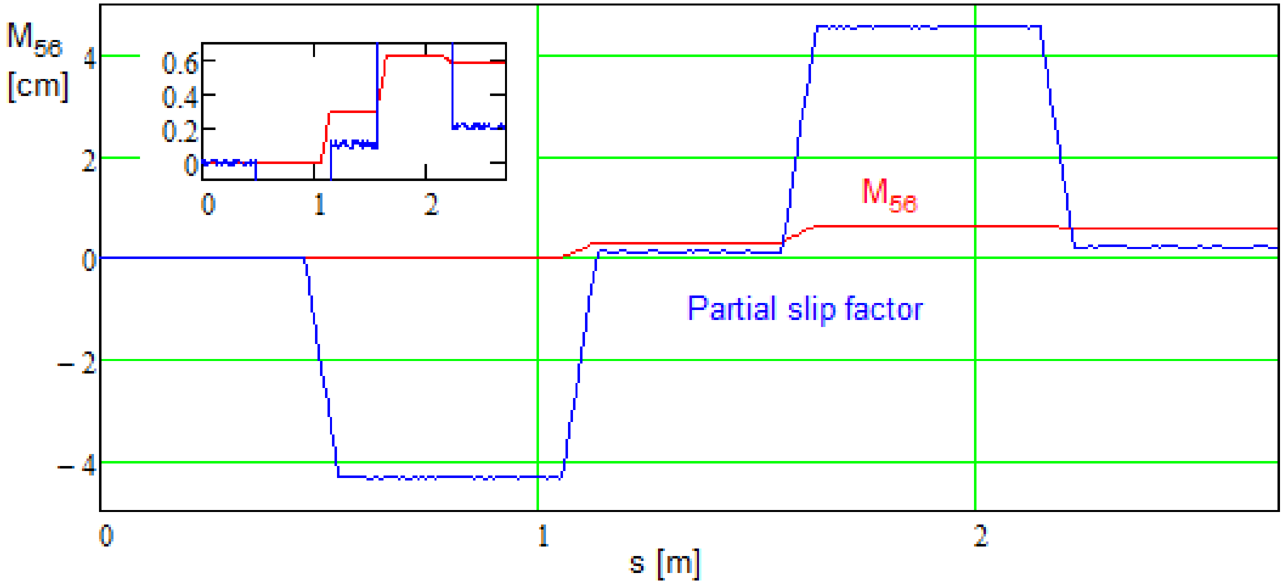
Linear Beam Optics for Cooling Chicane

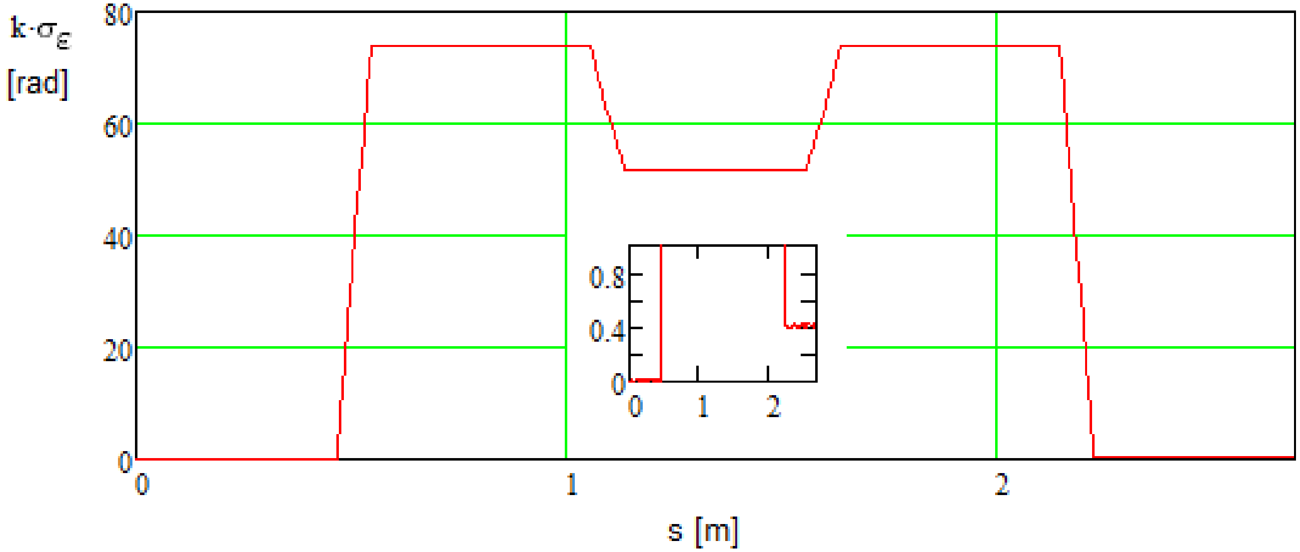
Major parameters of cooling chicane

Beam energy	100 MeV
Dipole type	Rbend
B of dipole	3.06 kG
L of dipole	8 cm
Orbit offset, h	43 mm
Delay, Δs	3 mm
GdL of Qd quad	830 Gs
β_x*	5.2 cm
D_x*	60 cm
Cooling rates ratio, λ_x/λ_s	1.7
Basic wave length, λ	2.2 μm
Cooling range in momentum, $(\Delta p/p)_{max}$	$\pm 1.2 \cdot 10^{-3}$ (3.7σ)
Cooling range in hor. plane, ε_{max} (Linear appr.)	0.31 μm (5.9σ)
Geometric acceptance	5 μm



Sample Lengthening on the Travel through Chicane





Sample lengthening due to momentum spread (top) and due to betatron motion (bottom)

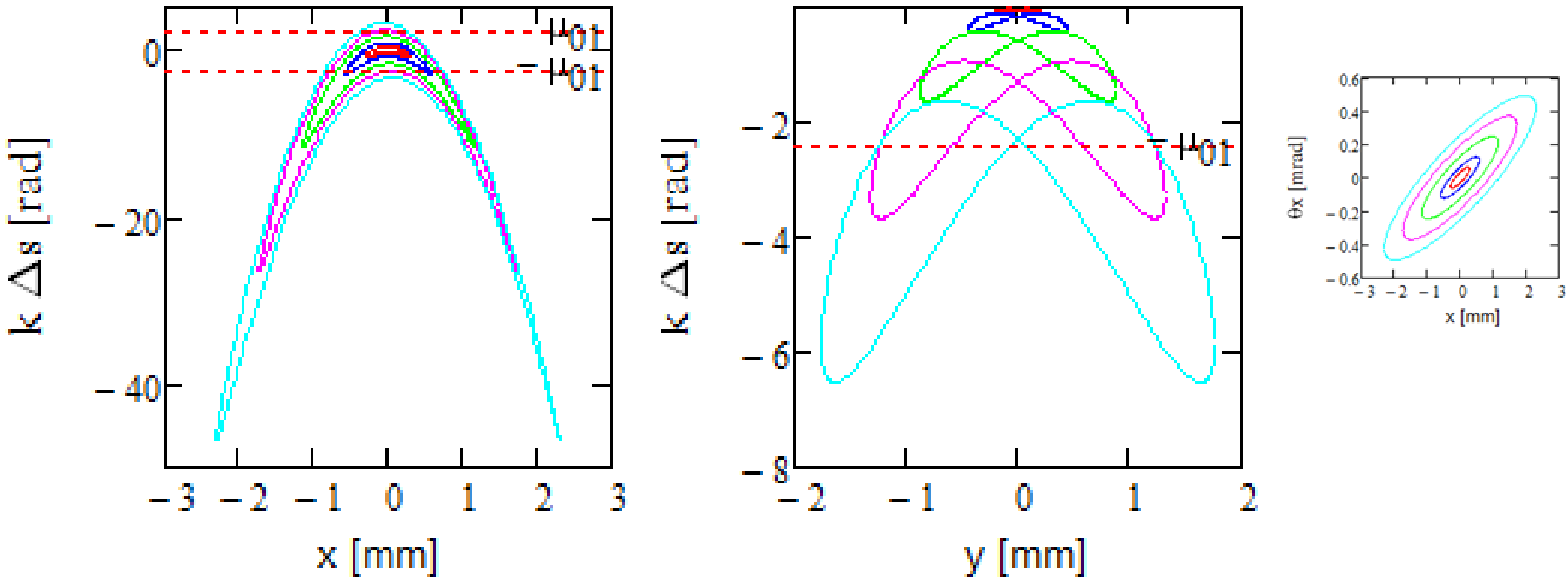
- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening, $\Delta(BL)/(BL)_{dipole}<10^{-3}$

Non-liner Sample Lengthening

- Major contribution to the 2nd order lengthening comes from particle angle:

$$\Delta s_2 = \int_{-L_q/2}^{L_q/2} \frac{\theta(s)^2}{2} ds \xrightarrow{\beta^* \ll L_t} \approx \frac{1}{2}\frac{\varepsilon}{\beta^*} L_q$$

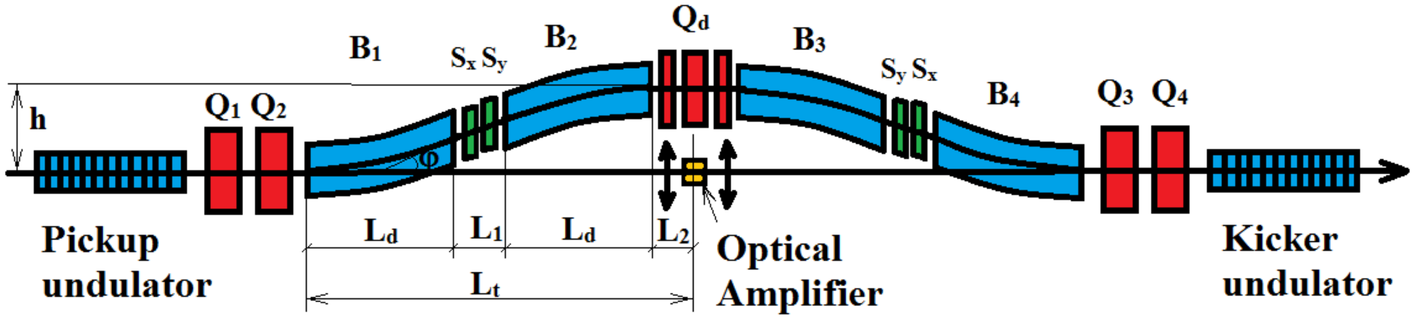
  - $\beta_x^* \ll \beta_y^*$ => hor. betatron oscillations make much larger contribution

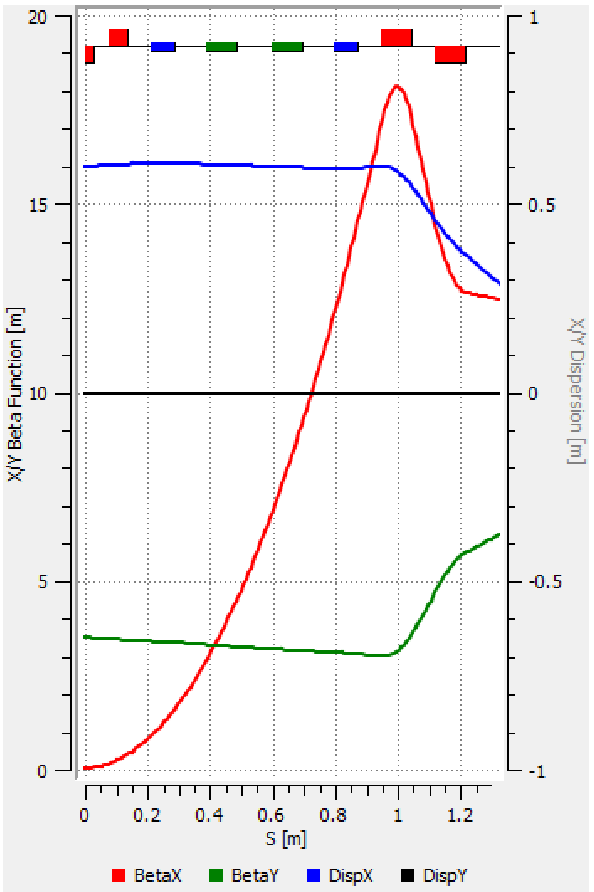


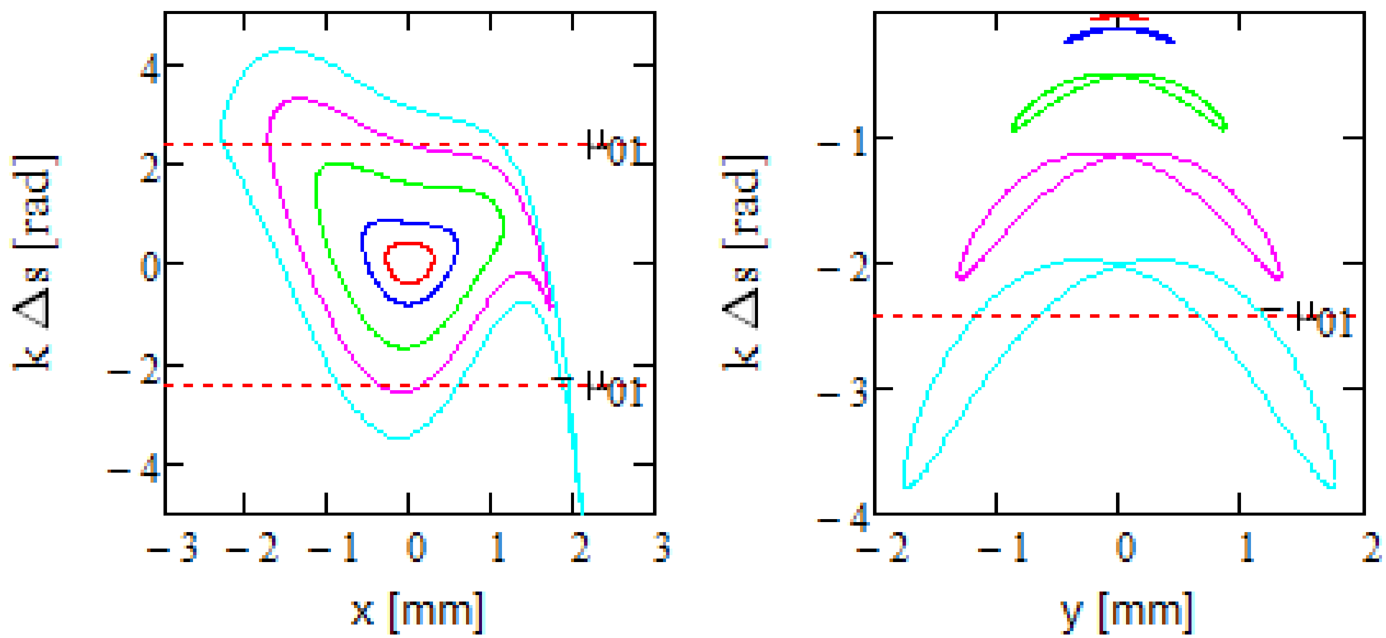
Dependence of normalized long. particle displacement in the kicker, kΔs, on ⊥ particle position in the pickup for particles located at ellipses of 1σ, 2σ, 4 σ, 6σ and 8σ (referenced to equilibrium H. emittance in the absence of x-y coupling). Left and right – H. and V. betatron motions, respectively. Horizontal lines mark cooling boundaries.

Correction of Non-liner Sample Lengthening

- Two sextupoles to correct V & H lengthening
  - Large β_x difference in F and D sextupoles

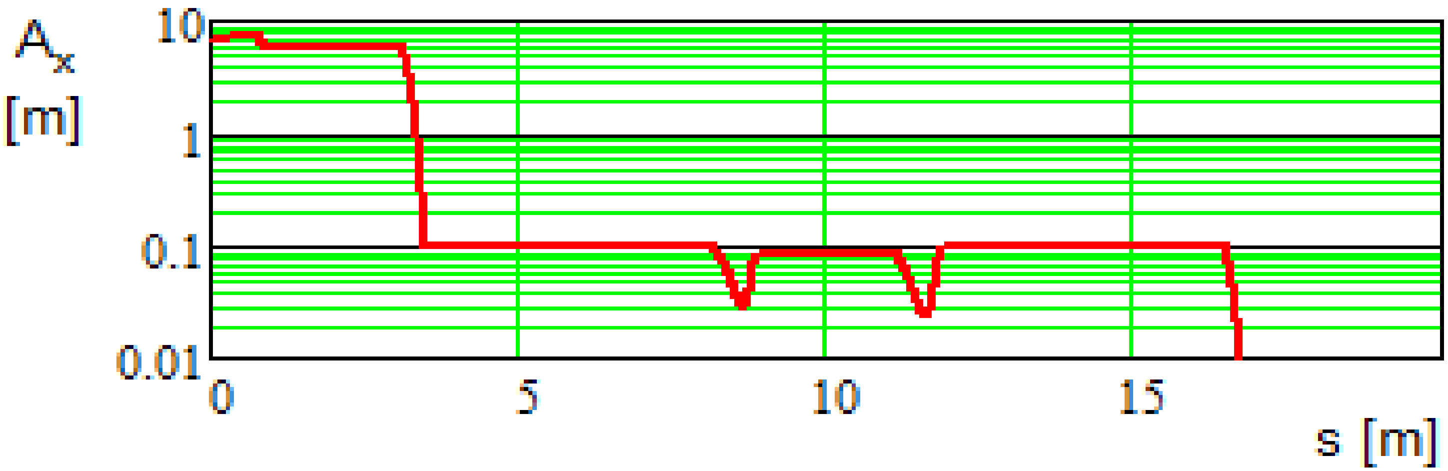


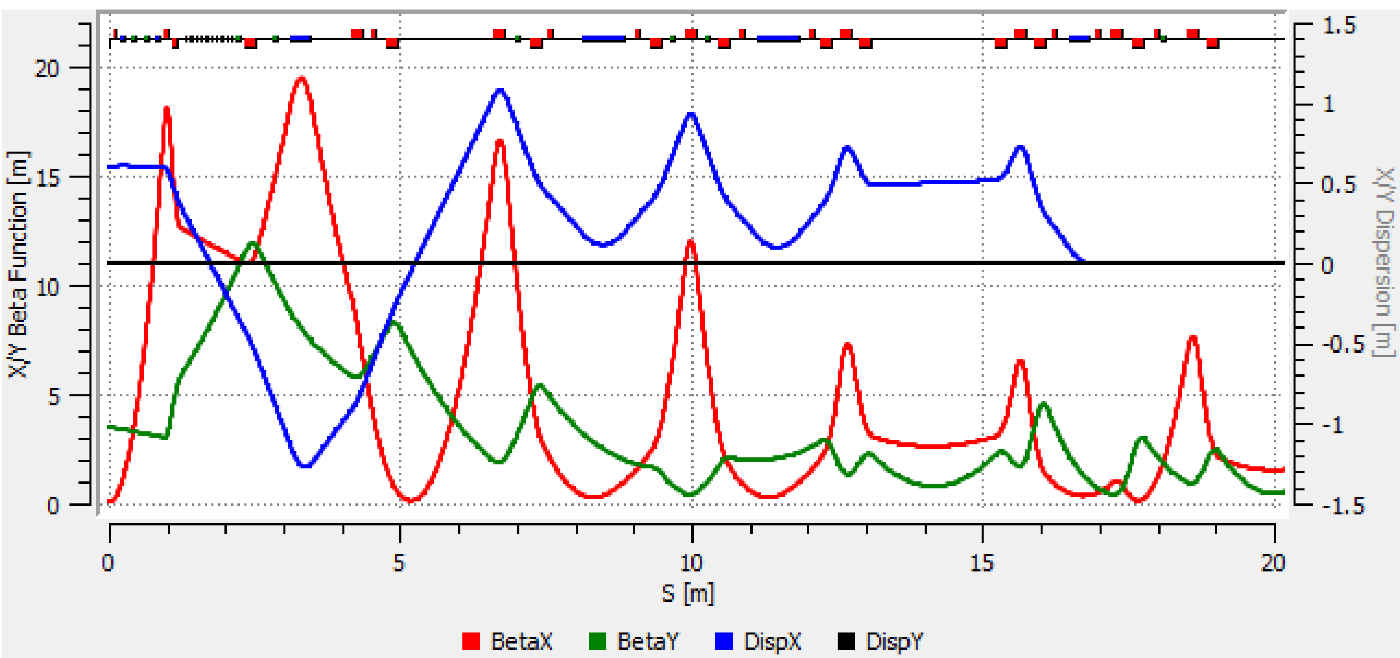




- Model implies perfect rectangular dipoles with rigid edge
  - Vertical edge focusing due to finite gap is accounted

IOTA Optics for OSC





Optics functions and dispersion invariant for IOTA half ring

- Focusing at the edges of OSC insert is adjusted to reduce A_x in the ring
  ⇨ Small horizontal emittance

IOTA Optics

Main Parameters of IOTA storage ring for OSC

Circumference	40 m
Nominal beam energy	100 MeV
Bending field of main dipoles	4.8 kG
Tunes, Q_x / Q_y	5.45/3.45
Natural chromaticities, ξ_x/ ξ_y	-18 / -7.4
Chromaticities with OSC sextupoles	253 / -67
Geometric acceptance	5 μm
Dynamic acceptance♥	0.15 μm (4σ)
RMS hor. emittance, SR, no coupling	9.1 nm
Rms momentum spread, σ_p	$1.07 \cdot 10^{-4}$
SR cooling times (ampl.), (τ_x/ τ_y/ τ_s)	1.7/2/1.1 s
Cooling ranges* (before OSC), $n_{\sigma x}$/ $n_{\sigma s}$	5.9 / 3.7

♥ Chromaticities are compensated to about zero with ring sextupoles

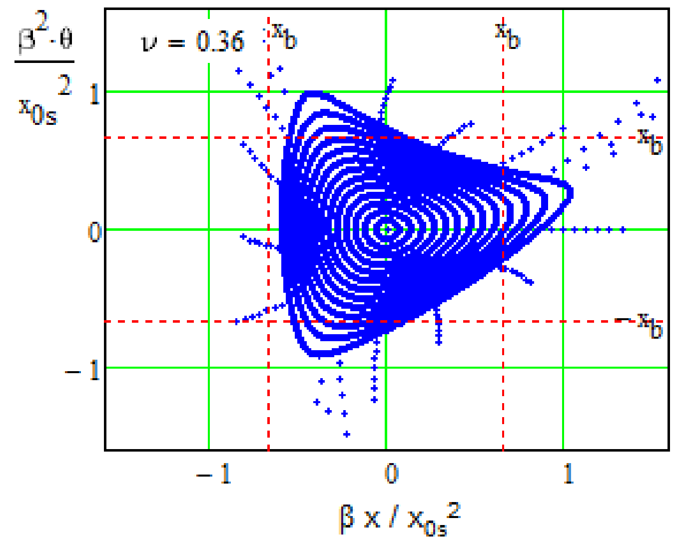
- Energy is reduced 150→100 MeV to reduce ε, σ_p Operation on coupling resonance reduces horizontal emittance and introduces vertical OSC damping
- Tunes are chosen to maximize the dynamic aperture limited by OSC and ring sextupoles

Dynamic Aperture Limitation by Sextupoles of OSC Insert

- In vicinity of 3rd order resonance:

$$\tilde{x}_b \equiv \frac{x_{\max}\beta_x}{x_{0S}^2} \approx 25\left|[\nu]-\frac{1}{3}\right| \quad \text{<=>} \quad \varepsilon_b \approx \frac{625 x_{0S}^4}{\beta_x^3}\left([\nu]-\frac{1}{3}\right)^2$$

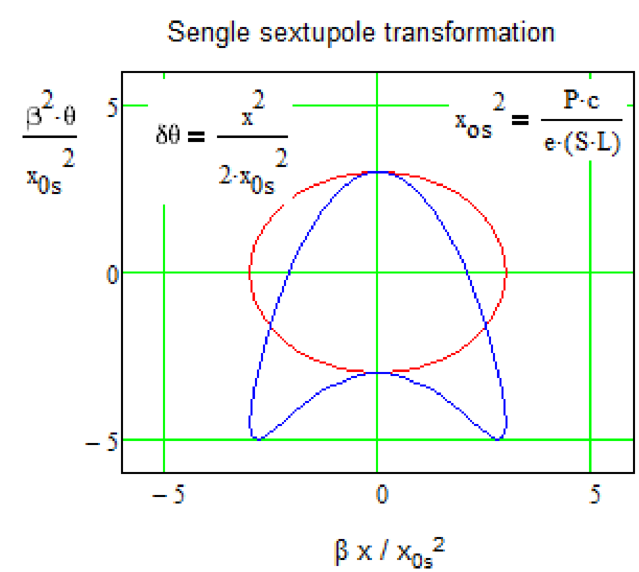
where: $$x_{0S}^2 = \frac{pc}{e(SL)}$$



- Far from the resonance the stability boundary can be estimated from the phase space distortion

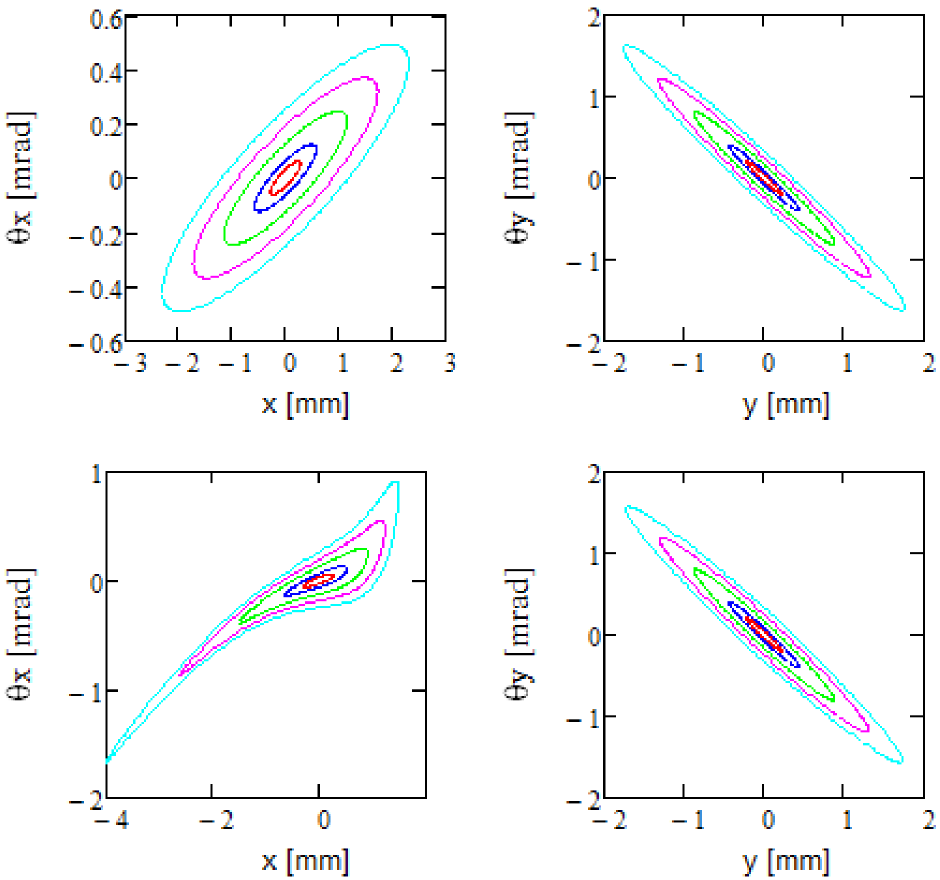
$$\tilde{x}_b \approx 3 \quad \text{<=>} \quad \varepsilon_b \approx \frac{9 x_{0S}^4}{\beta_x^3}$$

  - Transition happens at detuning $\Delta\nu \approx 0.1$

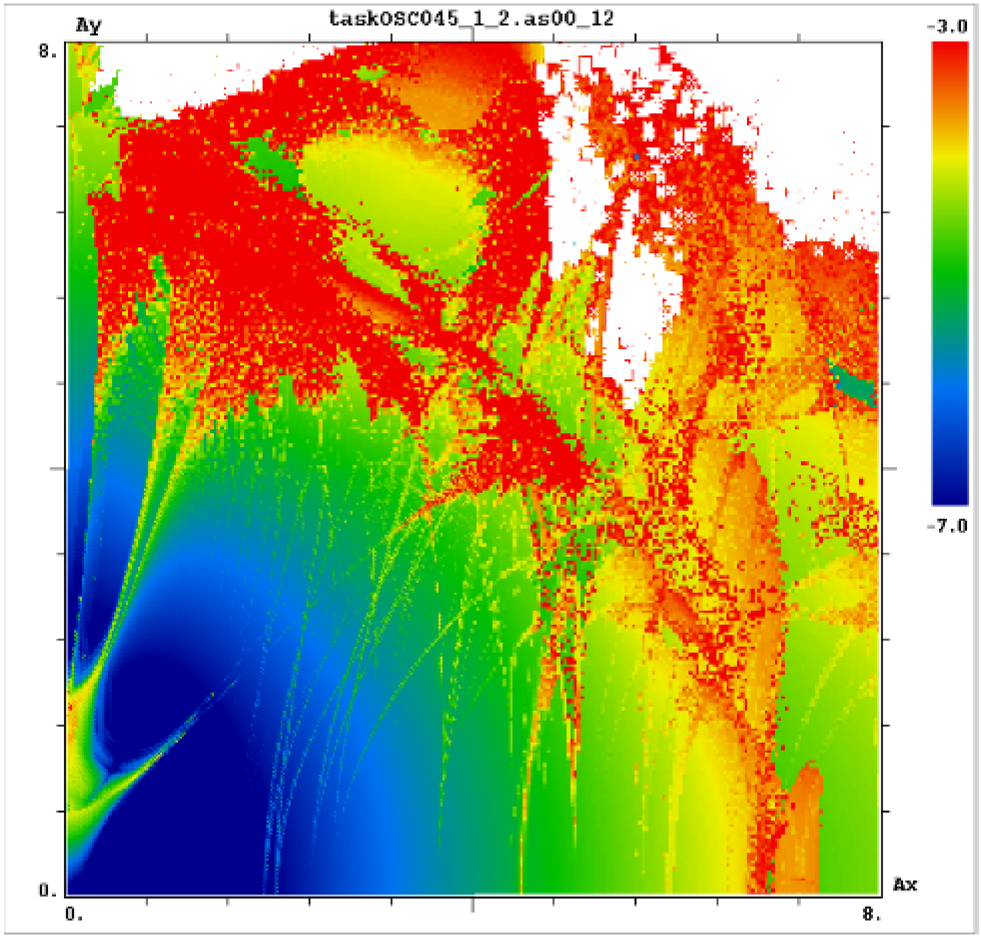


- Two sextupoles have $\Delta Qx \approx 180$ deg.
  - Good compensation of non-linearities in vicinity of resonance (weak sext.)
  - Almost no compensation far from resonance
- First estimate of dynamic aperture used
  - phase space distortion after a pass through cooling chicane
- Then: Tunes, sextupole families, FMA and tracking

Dynamic Aperture Limitation by Sextupoles of OSC Insert(2)



Hor.(left) and vert.(right) phase spaces after 1 pass through the chicane with (bottom) and without (top) sextupole correction. Initially particles were located at ellipses of 1σ, 2σ, 4σ, 6σ *and* 8σ.

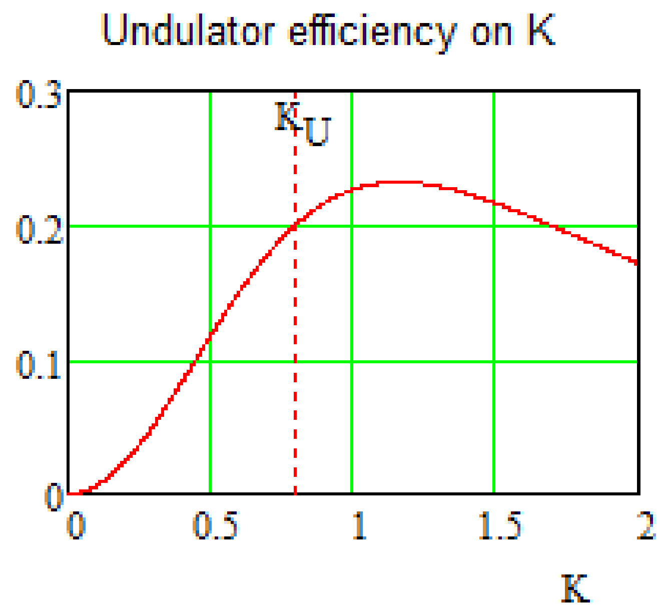


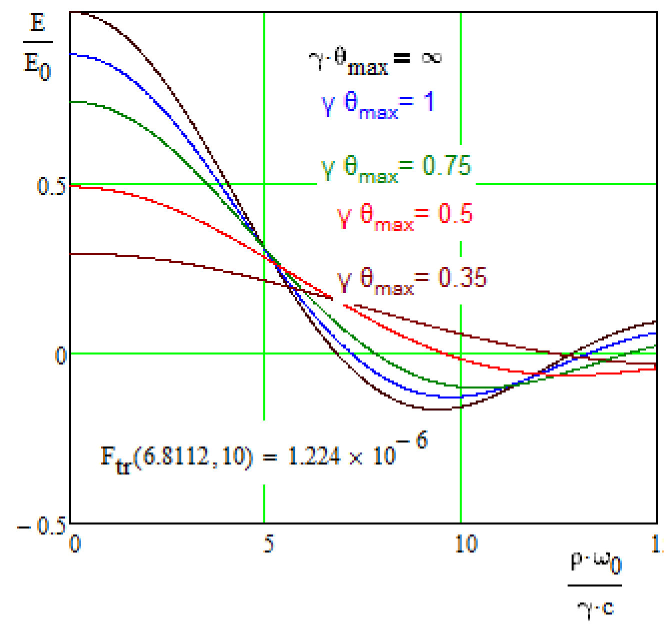
FMA (frequency map analysis) for dimensionless betatron amplitudes

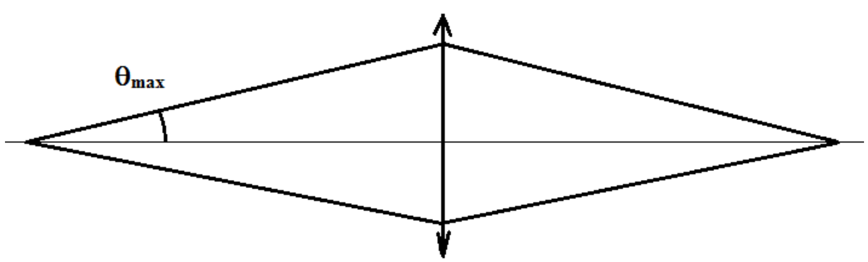
- 4σ dynamic aperture is obtained
  - further improvements are expected
- But, first, the light optics has to be better understood

Undulators

Radiation wavelength at zero angle	2.2 μm
Undulator parameter, K_U	0.8
Undulator period	12.9 cm
Number of periods, *m*	6
Total undulator length, L_w	0.77 m
Peak magnetic field	664 G
Distance between centers of undulators	3.3 m
Energy loss per undulator per pass	22 meV
Average power per undulator for N_e=10^6	26 nW
Optical system aperture (2a)	13 mm
Radiation spot size in the kicker, HWHM	0.35 mm
$\gamma\theta_{max}$	0.63



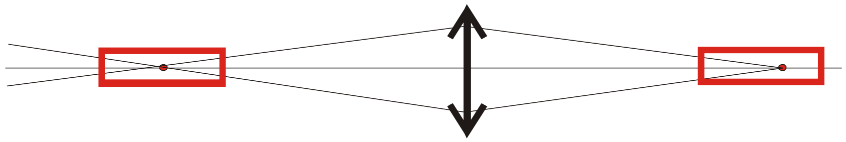




- Larger θ_{max} => larger bandwidth (more problems with optics) => faster cooling

Passive and Active OSC

- For $K << 1$ focused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

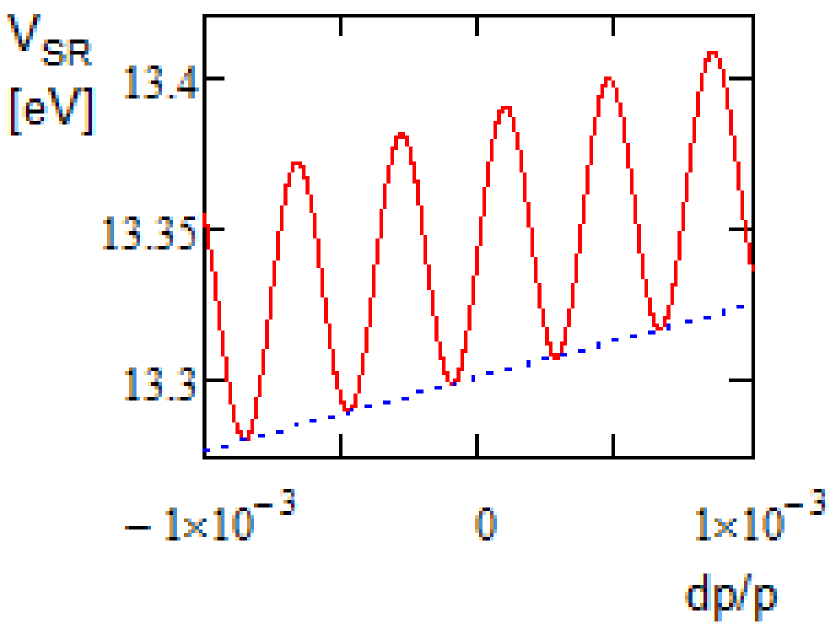


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1 = \mathbf{E}_2} 2\cos(\phi/2)\mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

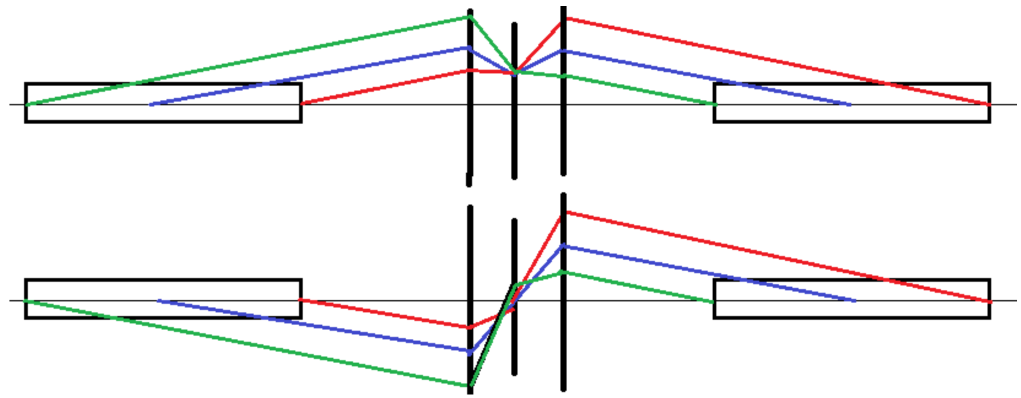
$$\Delta U \propto \left|E^2\right| = 4\cos(\phi/2)^2\left|\mathbf{E}_1{}^2\right| = 2(1+\cos\phi)\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left|\mathbf{E}_1{}^2\right|$$

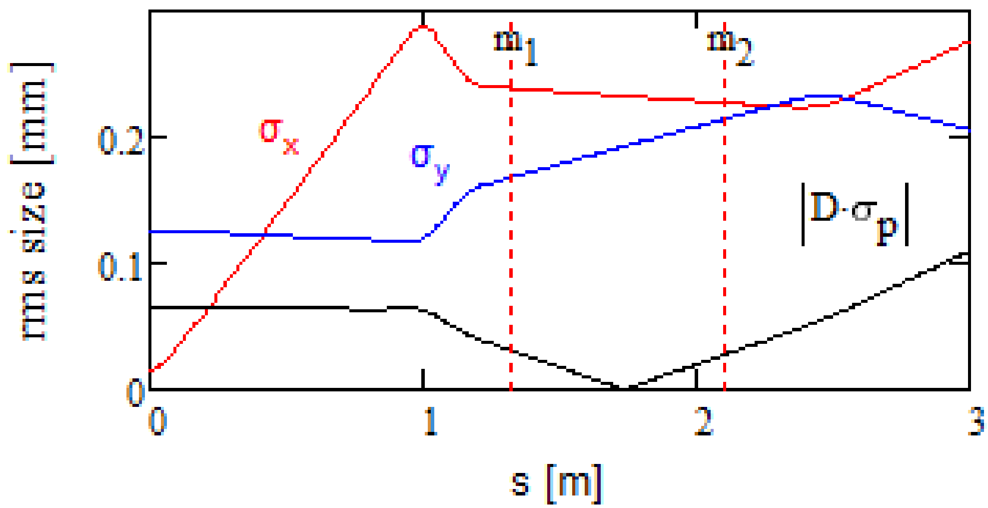
- OSC can be achieved even in the absence of optical amplifier
- Passive OSC increases SR damping rates by ~ an order of magnitude
- It should be easier to get larger bandwidth in a passive OSC
  - Bandwidth is limited by dispersion in lens material (~6 mm glass)
- Gain in active OSC – 10 dB (next talk)
  - $\sqrt{10} \approx 3$ times faster cooling
  - Bandwidth loss has to be less than 3 times



Optical system for Passive OSC

- 3 lens system
  - Transfer matrix = ±**I**
    => no depth of field problem
- Two solutions
  I: **M**= -**I** (D center lens)
  II: **M** = +**I** (F center lens)
- I is preferred
  - Weaker focusing for all lenses
  - Smaller focusing chromaticity
  - Suppression of divergence of radiation and its particle in the kicker undulator
    - Beam (center to center):
      $M_{11}=M_{22}=-1.07$
      $M_{33}=M_{44}=-2.07$

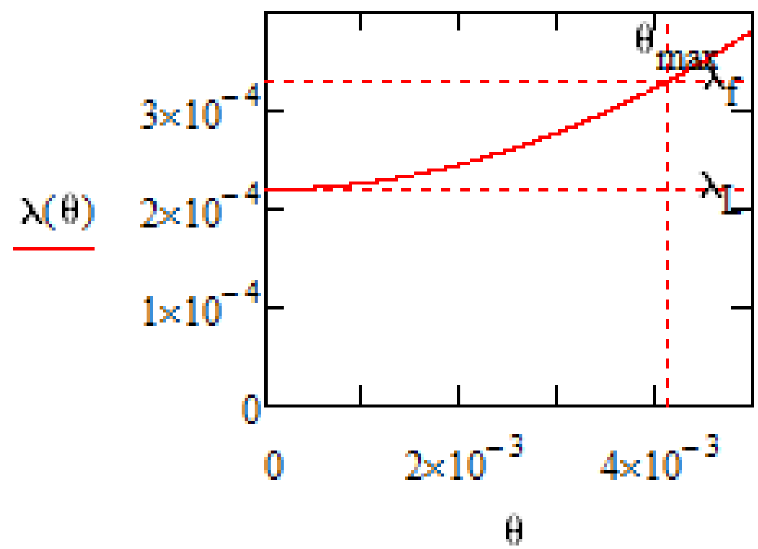




Rms beam sizes in absence of OSC,
σ_x=0.25 mm - in undulators
Radiation HWHM – 0.35 mm

Cooling Rates and Other Beam Parameters

Band	2.2 – 3.3 μm
Damp. rates (*x*=*y*/*s*)	6.3/5.2 s^{-1}
Geometric acceptance	5 μm
Dynamic acceptance	0.15 μm
Average vacuum (H_2 equiv.)	2 10^{-10} Torr
Vacuum lifetime	50 min.
SR loss per turn	13.3 eV
RF voltage	30 V
Harmonic number	-0.178
RF bucket height, $(\Delta p/p)_{max}$	10^{-3}
RMS bunch length (no OSC)	22 cm
Number of particles per bunch	10^6
Touschek lifetime	1.3 hour
IBS H.emit. growth rate	0.1 s^{-1}
IBS L.emit. growth rate	0.44 s^{-1}



- Number of particles is limited by IBS and Touschek effect
- OSC cooling test will operate well below the optimal gain
    - No interaction through cooling system

Other Limitations

- Quantum effects play little role in the OSC cooling
  - However interesting studies of quantum behavior can be done
    - In particular, single electron cooling

Quantum Mechanical Treatment of Transit-Time Optical Stochastic Cooling of Muons

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(Dated: April 9, 2009)

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December 2000

Quantum theory of Optical Stochastic Cooling *

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Conclusions

- Optical stochastic cooling looks as a promising technique for future hadron colliders (not extremely high energy of course)
- Experimental study of OSC in Fermilab is in its initial phase
  - It is aimed to validate cooling principles and to demonstrate cooling with and without optical amplifier
- The beam intensity ranges from a single electron to the bunch population limited by operation at the optimum gain (10^8-10^9)
  - Single electron cooling - localization of electron wave function and essence of quantum mechanics
    - Quantum noise for passive cooling and cooling with OA

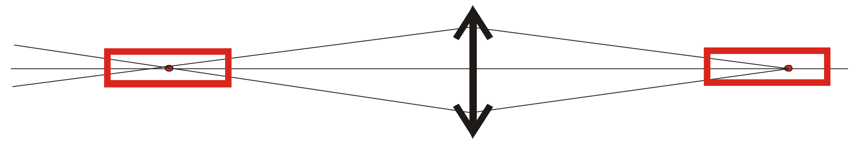
IOTA Timeline

FY15	**20 MeV e- commissioned** **HE beam line 40%** **IOTA parts 60%**
FY16	50 MeV e- commissioned 150 MeV CM2 to dump IOTA installed 60%
FY17	IOTA installed IOTA *e-* commissioned *p+* RFQ re-commiss'd 50% IOTA research starts with *e-*
FY18	Proton RFQ moved 100% *p+* RFQ commissioned, move to IOTA
FY19	IOTA research starts with *p+*
FY20	(IOTA research continues)

Backup Slides

Basics of OSC – Radiation Focusing to Kicker Undulator

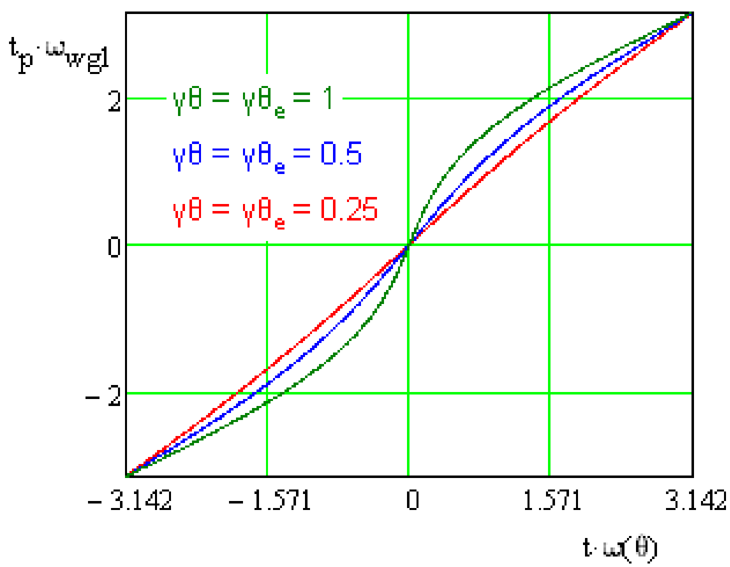
- Modified Kirchhoff formula

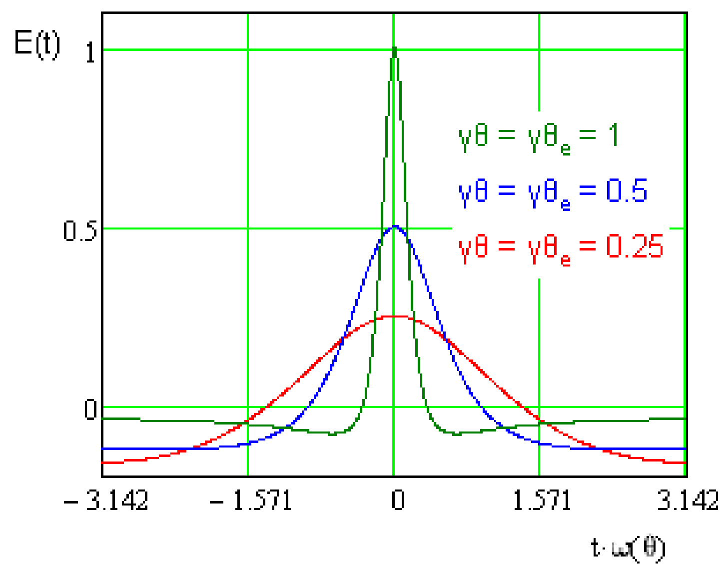


$$E(r) = \frac{\omega}{2\pi i c} \int_S \frac{E(r')}{|r - r'|} e^{i\omega|r-r'|} ds'$$

=> $$E(r) = \frac{1}{2\pi i c} \int_S \frac{\omega(r') E(r')}{|r - r'|} e^{i\omega|r-r'|} ds'$$

- Effect of higher harmonics
  - Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material



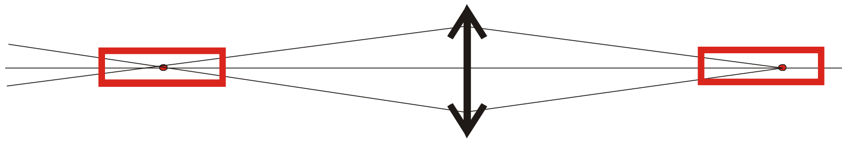


Dependences of retarded time (t_p) and E_x on time for helical undulator

- Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for K<<1

- For $K << 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

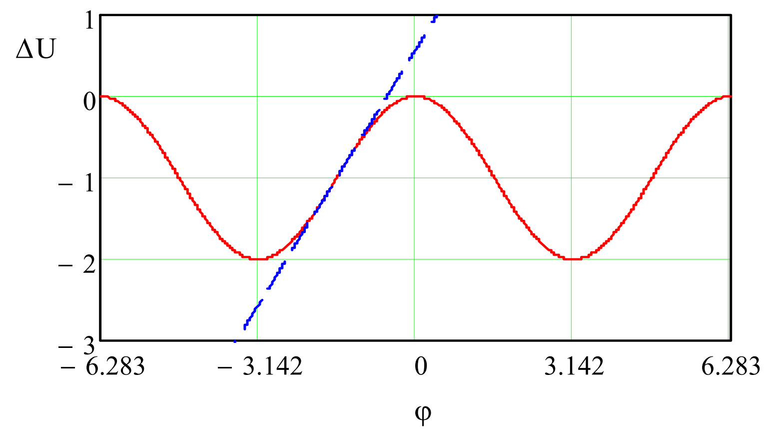


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1 = \mathbf{E}_2} 2\cos(\phi/2)\mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

$$\Delta U \propto \left|E^2\right| = 4\cos(\phi/2)^2 \left|\mathbf{E}_1{}^2\right| = 2(1+\cos\phi)\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left|\mathbf{E}_1{}^2\right|$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



  - SR damping: $\lambda_{\|\_SR} \approx \frac{2\Delta U_{SR}}{pc} f_0$
  - OSC: $\lambda_{\|\_OSC} \approx f_0 \frac{2\Delta U_{wgl}}{pc}(GkM_{56}) \xrightarrow{kM_{56}(\Delta p/p)_{\max}=\pi} f_0 \frac{2\Delta U_{wgl}}{pc}\left(\frac{G}{(\Delta p/p)_{\max}}\right)$

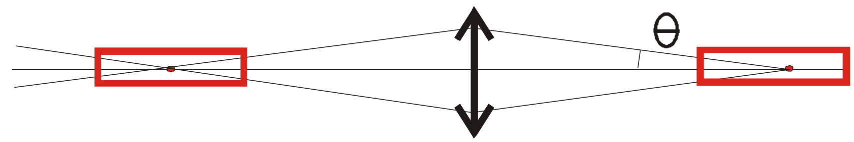
    where G - optical amplifier gain, $(\Delta p/p)_{max}$ - cooling system acceptance

⇨ $\lambda_{\|\_OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~1

Basics of OSC – Longitudinal Kick for K<<1(continue)

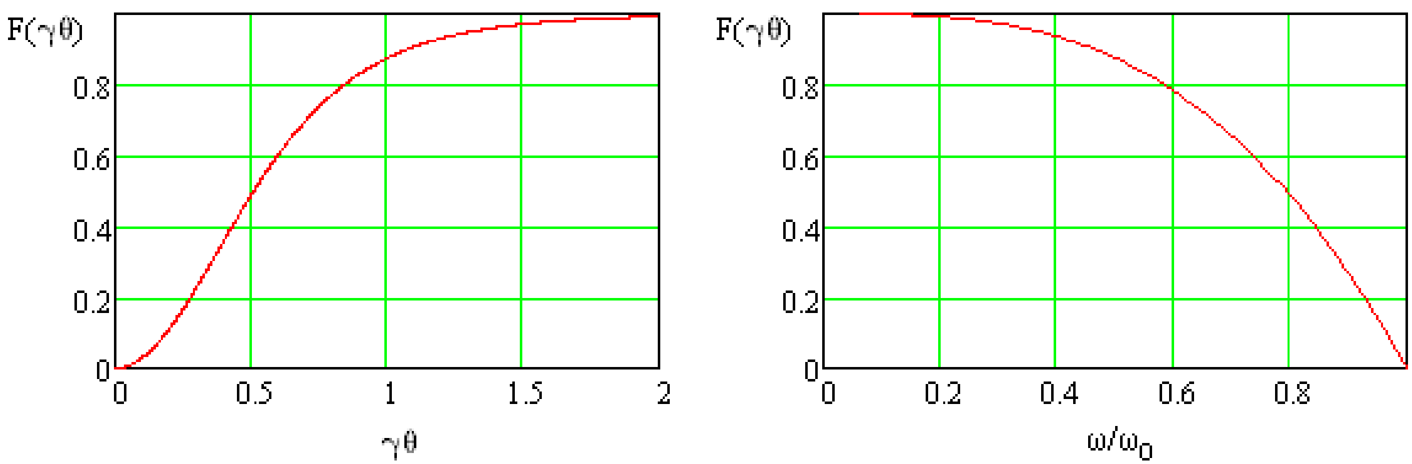
- Radiation wavelength depends on θ as

$$\lambda = \frac{\lambda}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{\|\_SR} = \lambda_{\|\_SR0} F\left(\gamma\theta_m\right), \qquad F\left(x\right) = 1 - \frac{1}{\left(1+x^2\right)^3}$$



- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0}\left(\frac{3\Delta\omega}{\omega}\right), \quad \frac{3\Delta\omega}{\omega} << 1$

where $\Delta U_{wgl0} = \frac{e^4 B^2 \gamma^2 L}{3m^2c^4}\begin{cases} 1, & \text{Flat wiggler} \\ 2, & \text{Helical wiggler}\end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

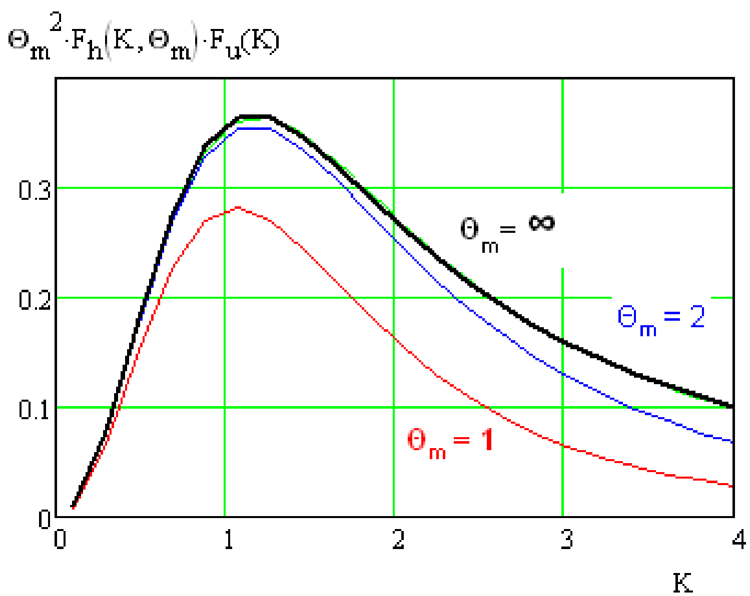
- For arbitrary undulator parameter we have

$$\Delta U_{OSC\_F} = \frac{1}{2}\frac{4e^4 B_0^{\,2}\gamma^2 L}{3m^2c^4} G F_f\left(K,\gamma\theta_{\max}\right) F_u\left(\kappa_u\right)$$

$$F_u\left(\kappa_u\right) = \mathrm{J}_0\left(\kappa_u\right) - \mathrm{J}_1\left(\kappa_u\right), \quad \kappa_u = K^2/\left(4\left(1+K^2/2\right)\right)$$

Fitting results of numerical integration yields:

$$F_h\left(K,\infty\right) \approx \frac{1}{1+1.07K^2+0.11K^3+0.36K^4}, \quad K \equiv \gamma\theta_e \le 4$$



- Dependence of wave length on θ:

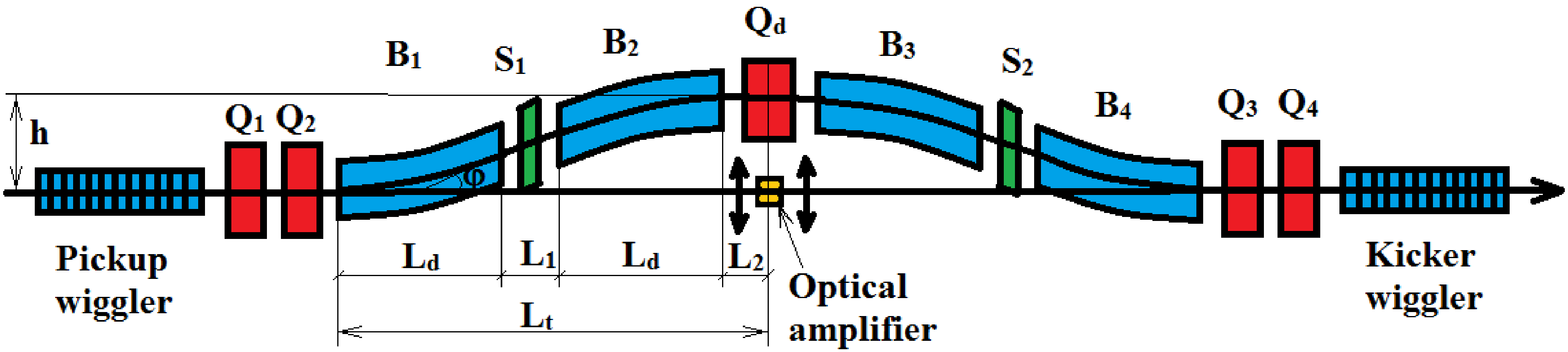
$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1+\gamma^2\left(\theta^2+\frac{\theta_e^{\,2}}{2}\right)\right)$$

$$K \equiv \gamma\theta_e$$

- Flat undulator is "more effective" than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed *B* a decrease of λ_{wgl} and, consequently, λ yields kick increase
  - but wavelength is limited by both beam optics and light focusing

Transfer Matrix for OSC Chicane



Chicane displaces the beam closer to its center

$$M_{ta}=\begin{pmatrix}1 & L_d & 0 & \frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & \varphi\\ -\varphi & -\frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & -\frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & -\varphi\\ \varphi & \frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 0 & 0\\ \Phi & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & -\frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & -\varphi\\ \varphi & \frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & \frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & \varphi\\ -\varphi & -\frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Leaving only major terms we obtain

$$M_{ta}=\begin{bmatrix}L_t\cdot\Phi+1 & L_t\cdot(L_t\cdot\Phi+2) & 0 & \Phi\cdot h\cdot L_t\\ \Phi & L_t\cdot\Phi+1 & 0 & \Phi\cdot h\\ -\Phi\cdot h & -\Phi\cdot h\cdot L_t & 1 & 2\cdot\Delta s\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\Delta s=\varphi^2\cdot\left(L_1+\frac{2}{3}\cdot L_d\right)$$

$$h=\varphi\cdot(L_1+L_d)$$

Matrix comparison:

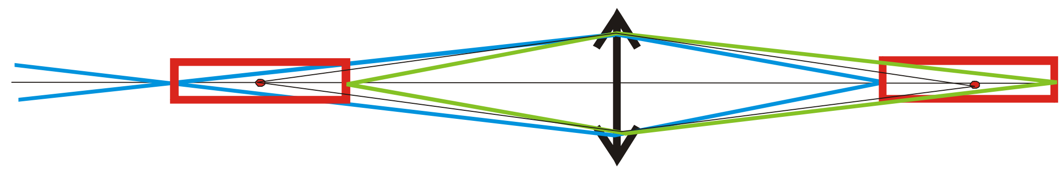
Exact (M_t) versus

approximate (M_{ta})

$$M_t=\begin{pmatrix}1.093 & 90.635 & 0 & 0.268\\ 2.152\times10^{-3} & 1.093 & 0 & 6.18\times10^{-3}\\ -6.18\times10^{-3} & -0.268 & 1 & 0.591\\ 0 & 0 & 0 & 1\end{pmatrix}\qquad M_{ta}=\begin{pmatrix}1.092 & 89.971 & 0 & 0.262\\ 2.148\times10^{-3} & 1.092 & 0 & 6.093\times10^{-3}\\ -6.093\times10^{-3} & -0.262 & 1 & 0.601\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Basics of OSC – Correction of the Depth of Field

- It was implied above that the radiation coming out of the pickup undulator is focused on the particle during its trip through the kicker undulator



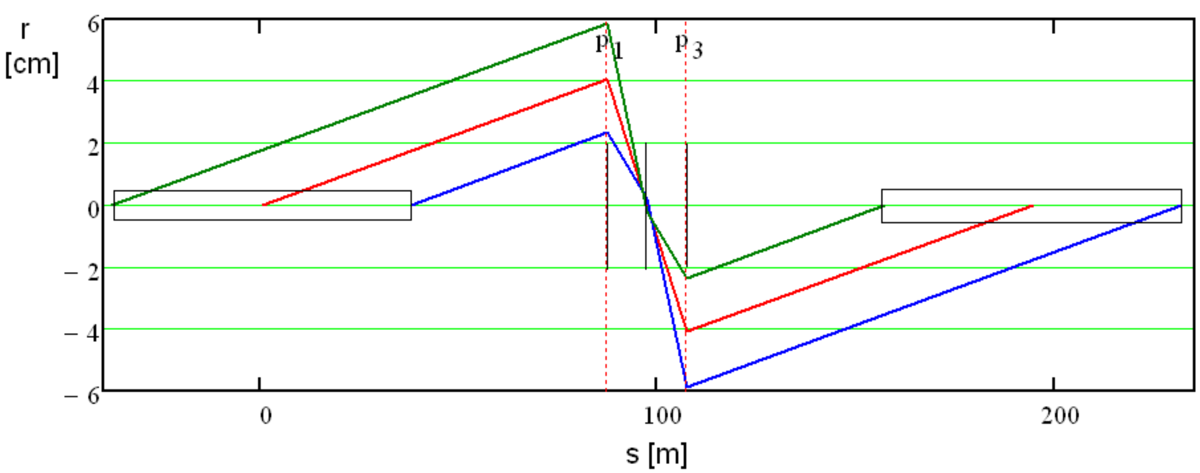
  - It can be achieved with lens located at infinity

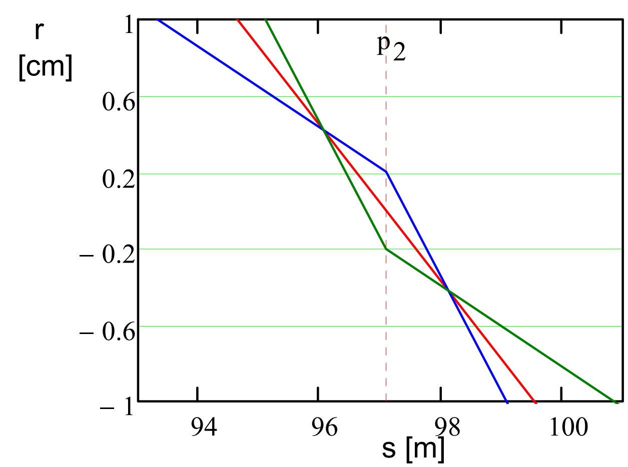
$$\frac{1}{2F+\Delta s}+\frac{1}{2F-\Delta s}=\frac{1}{F} \rightarrow \frac{1}{F-\Delta s^2/4F}=\frac{1}{F} \xrightarrow{F\to\infty} \frac{1}{F}=\frac{1}{F}$$

  - but this arrangement cannot be used in practice

- A 3-lens telescope can address the problem within limited space

$$\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_2^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$





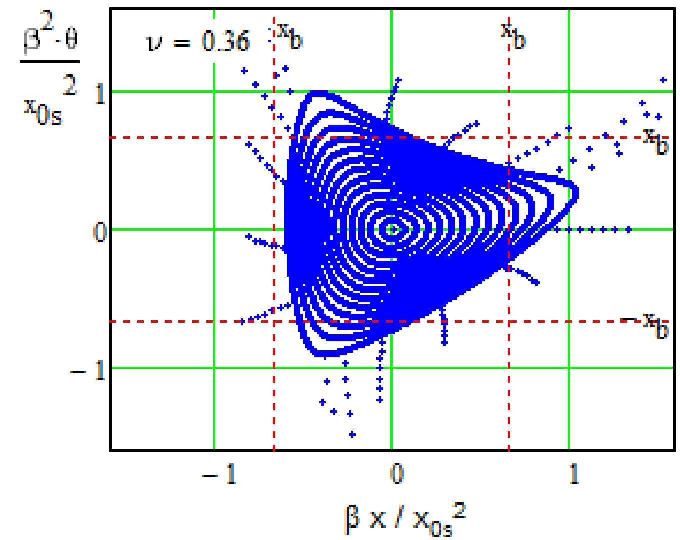
Dynamic Aperture Limitation by Sextupoles of OSC Insert

- Introduce dimensionless variables

$$\tilde{\theta} = \beta^2 \frac{\theta + \alpha x / \beta}{x_{0S}{}^2}, \quad \tilde{x} = \frac{\beta x}{x_{0S}{}^2} \quad \text{where} \quad x_{0S}{}^2 = \frac{pc}{e(SL)}$$

- Then the following transforms drive particle motion

$$\begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix}' = \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix}, \quad \tilde{\theta}' = \tilde{\theta} + \frac{\tilde{x}^2}{x_{0S}{}^2}$$



- In vicinity of 3rd order resonance:

$$\tilde{x}_b \approx 25\left|[\nu] - \frac{1}{3}\right| \;\Rightarrow\; \varepsilon_b \approx \frac{625 x_{0S}{}^4}{\beta^3}\left([\nu] - \frac{1}{3}\right)^2$$

- Far from the resonance the stability boundary can be estimated from the phase space distortion =>

$$\tilde{x}_b \approx 3 \;\Rightarrow\; \varepsilon_b \approx \frac{9 x_{0S}{}^4}{\beta^3}$$

- Transition happens at detuning Δν≈0.1

Sengle sextupole transformation