

A large, semi-circular graphic on the right side of the slide. It contains a dense field of small, multi-colored dots (blue, cyan, green, yellow, and red) representing particles. A thick, curved line in red and orange arcs across the top and left of this field, likely representing a beam boundary or a specific particle trajectory.

Simulation of Laser Cooling of Heavy Ion Beams at High Intensities

L. Eidam, D. Winters, O. Boine-Frankenheim



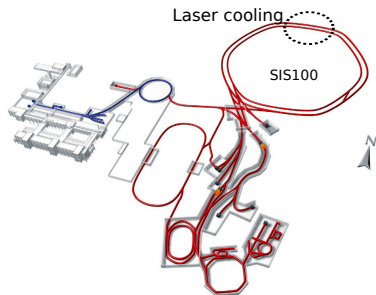
- ▶ Motivation
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Motivation

Laser Cooling at FAIR



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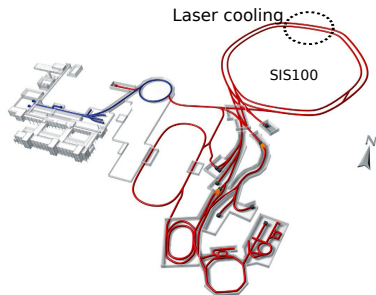
- ▶ first time of laser cooling at high beam energies
- ▶ high beam intensities available

Motivation

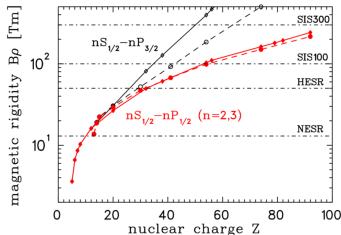
Laser Cooling at FAIR



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Doppler shift: $\lambda' = \frac{\lambda_L}{\gamma(1-\beta)}$
for Li-like ions: ($\lambda_L = 257 \text{ nm}$)



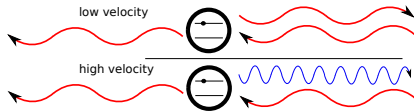
[U. Schramm, 2004]

- ▶ first time of laser cooling at high beam energies
- ▶ high beam intensities available
- ▶ high magnetic rigidity allows cooling of heavy ions

Cooling at High Energies



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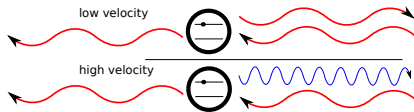
► $\langle \Delta p_{lab} \rangle = \frac{\hbar \omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta)$

► $\frac{\langle \Delta p_{lab} \rangle}{p}$ nearly constant

Cooling at High Energies



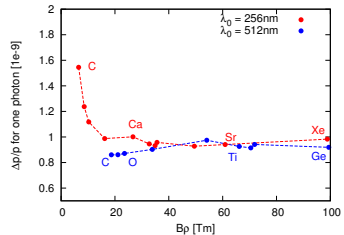
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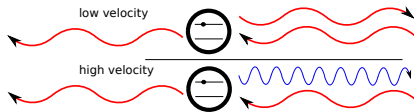
Li-like ($2p_{1/2} - 2s_{1/2}$):



Cooling at High Energies

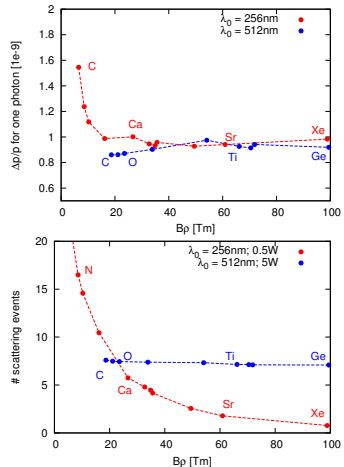


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- ▶ $\langle \Delta p_{lab} \rangle = \frac{\hbar \omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta)$
- ▶ $\frac{\langle \Delta p_{lab} \rangle}{p}$ nearly constant
- ▶ for low/high energies ions with similar cooling force exist
⇒ pick the right ion & transition
- ▶ adiabatic damping
⇒ no pre-cooling necessary

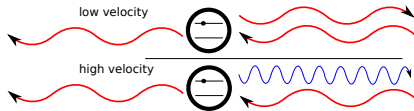
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Cooling at High Energies



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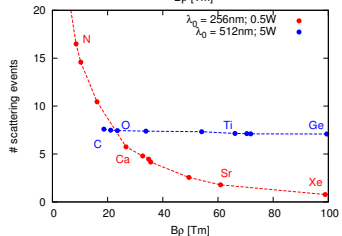
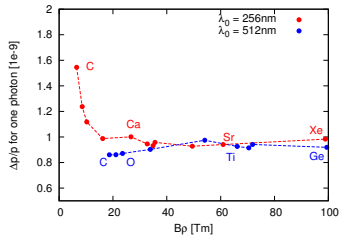


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⇒ pick the right ion & transition
- ▶ adiabatic damping
⇒ no pre-cooling necessary

- ▶ speed of synchrotron motion:

$$Q_s = -\frac{L\eta}{2\pi} \frac{\delta_0}{\tilde{z}_0} \quad \text{with} \quad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

Li-like (2p1/2 - 2s1/2):



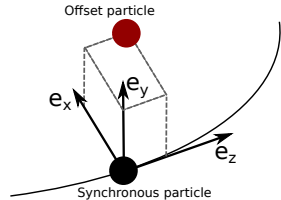


- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Solve Vlasov-Fokker-Planck Equation:

$$\frac{\partial f}{\partial t} - \eta \beta c \delta \frac{\partial f}{\partial z} + \frac{qV(z,t)}{p_0} \frac{\partial f}{\partial \delta} = \left(\frac{\partial f}{\partial \delta} \right)_c$$

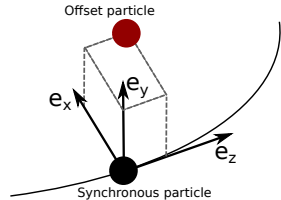
- ▶ use macro particles (PIC-code)
- ▶ macro particles : $N_{macro} \approx 10^5 - 10^7$
- ▶ discretization of time (T_{rev})



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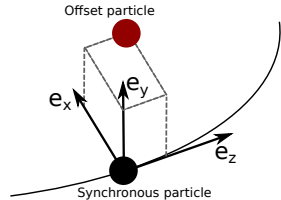
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- ▶ assume fast transverse motion is unaffected by cooling process
⇒ 1D longitudinal problem



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⇒ 1D longitudinal problem



No direct coulomb interactions
⇒ results not exact at very low temperatures
⇒ concentrate on cooling process

Simulation Tool

Modeling of Laser Force



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simple two level system in electromagnetic mode $\Rightarrow$ optical Bloch equations

$$\dot{\rho} = \frac{i}{\hbar} [\rho, H] - \frac{1}{\tau} \rho \quad \text{with} \quad H = \begin{pmatrix} \hbar\omega_e & (\hbar\frac{\Omega}{2})e^{-i\omega t} \\ (\hbar\frac{\Omega}{2})e^{i\omega t} & \hbar\omega_g \end{pmatrix}; \quad \rho = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$$

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- ▶ spontaneous emission rate:

$$k(\delta) = \frac{1}{\tau} \cdot \rho_{ee}(\delta)$$

- ▶ cooling Force:

$$F_{Laser}(\delta) = \Delta p_{lab} \cdot k(\delta)$$

Simulation Tool

Modeling of Laser Force



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- ▶ if $U_j \leq \frac{\Delta E}{\tau} \rho_{ee}(\delta_j)$: $\Delta\delta = \Delta p_{lab} / p_0$

U_j : uniform random number

- ▶ $\Delta p_{lab} = \frac{\hbar\omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta) \cdot 2U_{j2}$

Simulation Tool

Modeling of Laser Force



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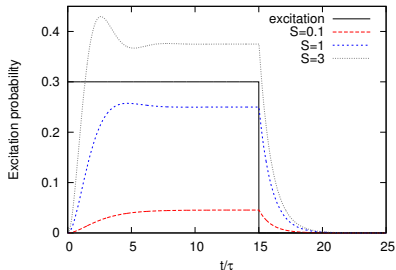
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$$\rho_{ee}^{Sat} = \frac{1}{2} \frac{S}{1 + S + (2\Delta\nu \cdot \tau)^2}$$

Simulation Tool

Modeling of Laser Force



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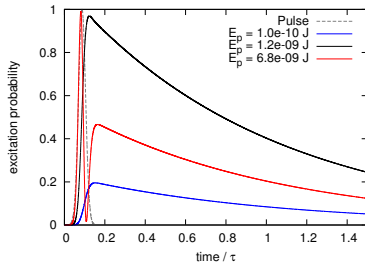
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pulsed laser:





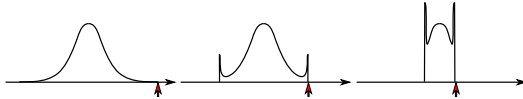
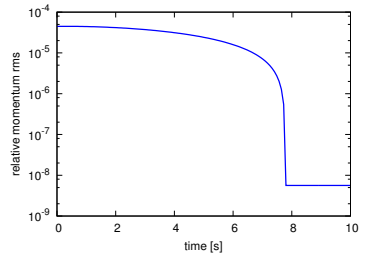
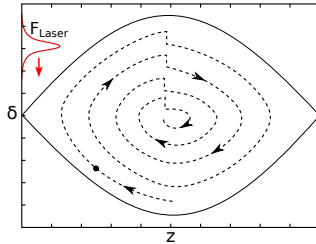
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Cooling at Low Intensities

Cooling Process



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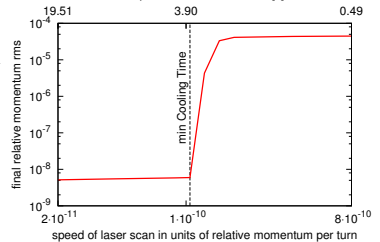
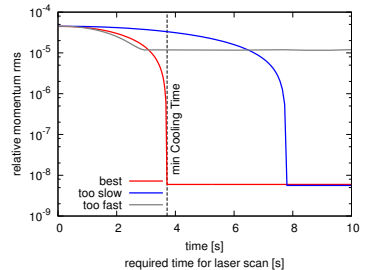
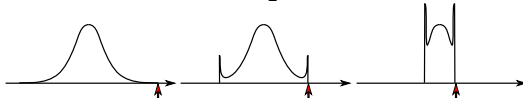
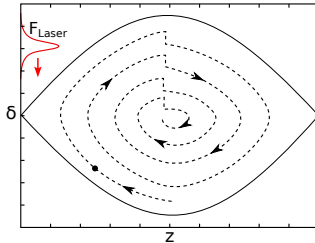


Cooling at Low Intensities

Cooling Process



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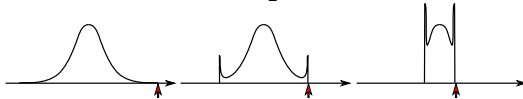
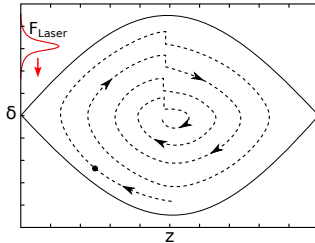


Cooling at Low Intensities

Cooling Process



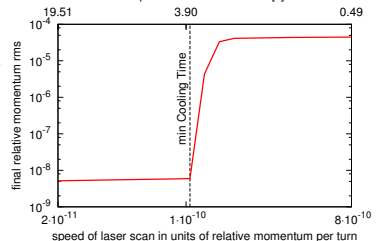
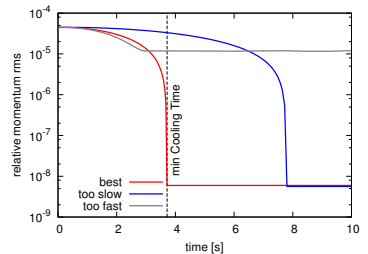
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- influence of synchrotron frequency?

Synchrotron tune:

$$Q_s = \frac{-L\eta \cdot \delta}{2\pi \hat{z}} \quad \text{with} \quad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$



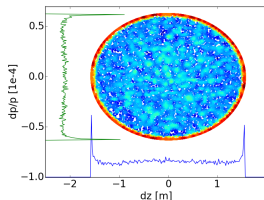
Cooling at Low Intensities

Dependence of Synchrotron Tune



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high $Q_s (\approx 10^{-3})$



- ▶ verified in storage rings
- ▶ symmetric reduction of momentum spread and bunch length

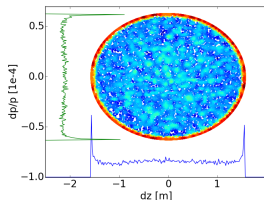
Cooling at Low Intensities

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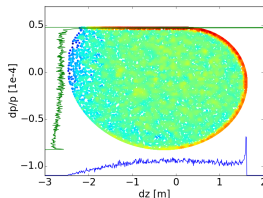
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high Q_s ($\approx 10^{-3}$)



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medium Q_s ($\approx 10^{-4}$)



- ▶ particles are captured by laser force
- ▶ non-symmetric

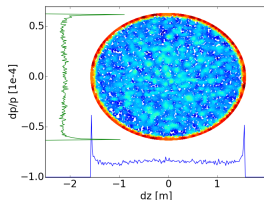
Cooling at Low Intensities

Dependence of Synchrotron Tune



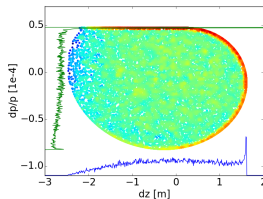
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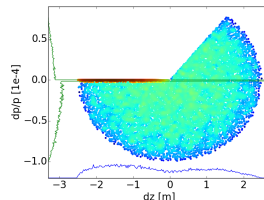
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medium Q_s ($\approx 10^{-4}$)



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- ▶ non-symmetric

low Q_s ($\approx 10^{-5}$)



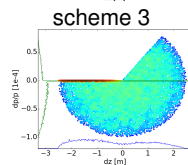
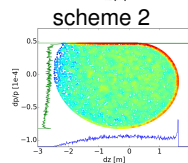
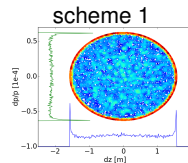
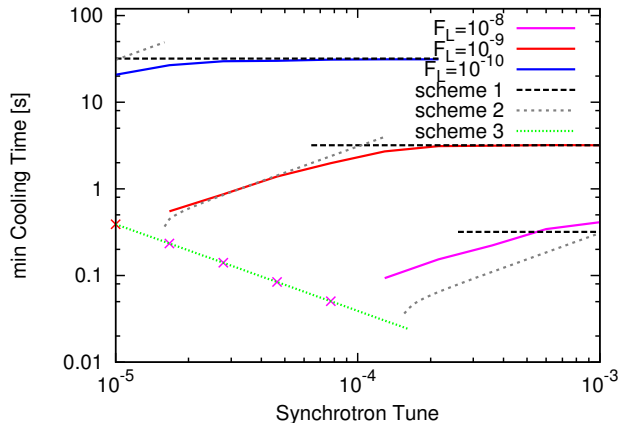
- ▶ laser position is fixed
- ▶ laser counteracts rf kick
- ▶ particles are rotated into laser force

Cooling at Low Intensities

Required Cooling Time



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$$\delta_{Laser} = 10^{-9}$$

$$T_{excitation} = 10\tau$$

$$\Delta_{FWHM} = 10^{-7}$$

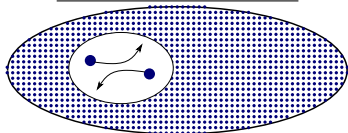
$$\hat{\delta} = 10^{-4}$$

$$\hat{z} = 2.5 m$$



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Intra Beam Scattering

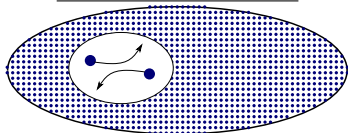


Space Charge

- ▶ transverse oscillations heat longitudinal motion



Intra Beam Scattering

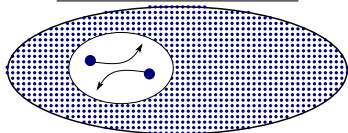


Space Charge

- ▶ transverse oscillations heat longitudinal motion
- ▶ described by diffusion in FPE

$$\delta_{IBS} = Q_j \cdot \sqrt{2D_{zz} \frac{L_{acc}}{\beta \alpha_0}}$$

Intra Beam Scattering



Space Charge

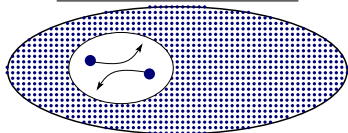
- ▶ transverse oscillations heat longitudinal motion
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$$\delta_{IBS} = Q_j \cdot \sqrt{2D_{zz} \frac{L_{acc}}{\beta \alpha_0}}$$

- ▶ non-Gaussian distribution $\Rightarrow$ local diffusion model (BETACOOOL)

$$D_{zz} = \frac{e^4 Z^4 \Lambda \sqrt{2}}{128 \pi m_i^2 \epsilon_0^2 c_0^3 \gamma^3 \beta^3 \langle \beta^{1/2} \rangle \epsilon_{\perp}^{3/2}} \cdot n$$

Intra Beam Scattering



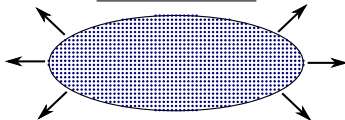
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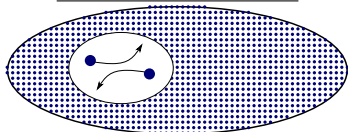
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Space Charge



- ▶ describes interaction of particle with electric field of the bunch in perfectly conducting pipe

Intra Beam Scattering

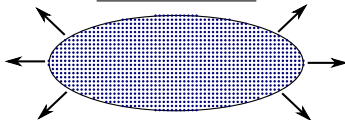


- ▶ transverse oscillations heat longitudinal motion
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$$\delta_{IBS} = Q_j \cdot \sqrt{2D_{ZZ} \frac{L_{acc}}{\beta \alpha_0}}$$

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Space Charge



- ▶ describes interaction of particle with electric field of the bunch in perfectly conducting pipe
- ▶ space charge impedance:

$$Z_{sc} = -i \cdot f \cdot \frac{gL_{acc}}{2\epsilon_0 \beta^2 c^2 \gamma^2} \text{ with } g = 1 + 2 \log(b/a)$$
- ▶ potential:

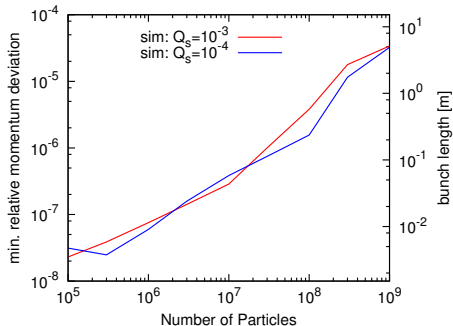
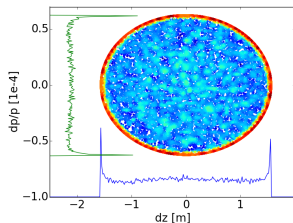
$$U_{sc}(z) = Z_{ion} \cdot q \cdot IFFT\{Z_{sc}(\omega) \cdot FFT\{\lambda(t)\}\}$$

Intensity Effects Intra Beam Scattering (IBS)



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high synchrotron tune $Q_s \approx 10^{-3}$



Intensity Effects

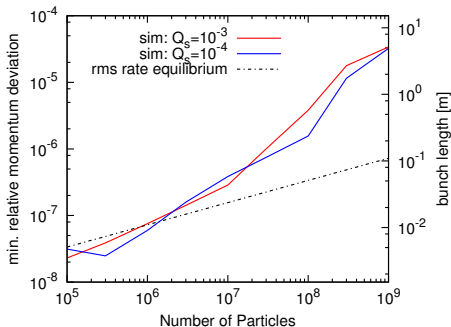
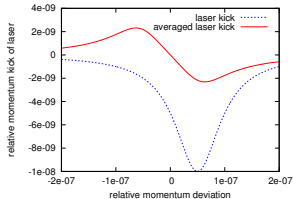
Intra Beam Scattering (IBS)



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rms rate equilibrium:



Intensity Effects

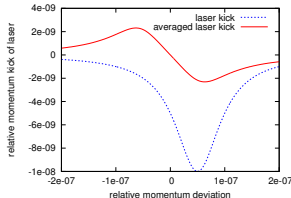
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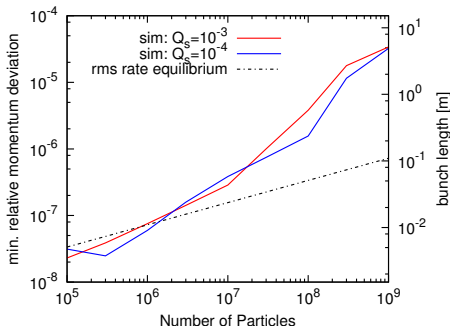
rms rate equilibrium:



$$\tau_{IBS}^{-1} + \tau_{Laser}^{-1} = 0$$

$$\tau_{Laser}^{-1} = \frac{1}{T_{rev}} \cdot \left. \frac{\partial \langle \Delta \delta \rangle_{syn}}{\partial \delta_{pos}} \right|_0$$

$$\langle \Delta \delta \rangle_{syn} = \frac{1}{\pi} \int_0^\pi \cos(\phi) \cdot \Delta \delta(\delta_{pos} \cdot \cos(\phi)) d\phi$$



Intensity Effects

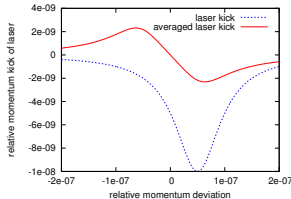
Intra Beam Scattering (IBS)



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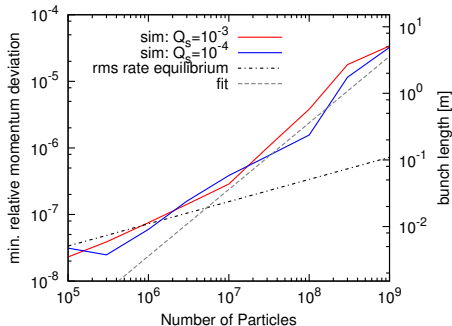
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$$\text{fit: } \langle \delta_{IBS} \rangle = f(\delta_{laser}) = \text{const.}$$

$$D_{||\max}^{IBS} \propto \frac{N}{L_{bunch}} \propto \frac{N}{\delta_{rms}}$$

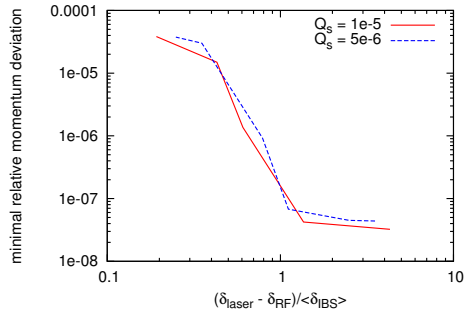
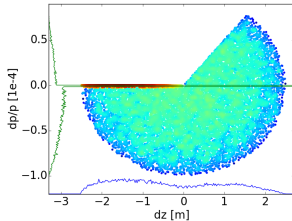
Intensity Effects

Intra Beam Scattering (IBS)



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low synchrotron tune $Q_s \approx 10^{-5}$



► limit: $\frac{\hat{\delta}_{laser} - \hat{\delta}_{rf}}{\langle \delta_{IBS} \rangle} > 1$

⇒ Barrier of laser has to be higher than IBS kicks

► longitudinal density increases only by factor 2

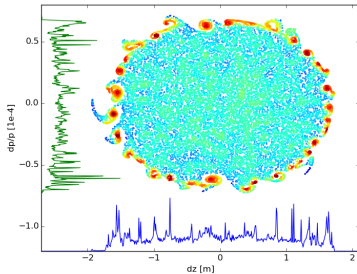
(changed $N_p \rightarrow \langle \delta_{IBS} \rangle$)

Intensity Effects Space Charge



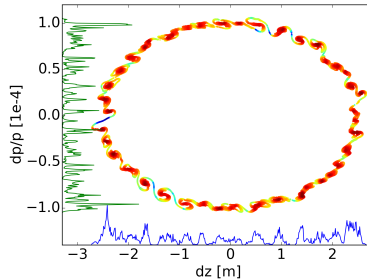
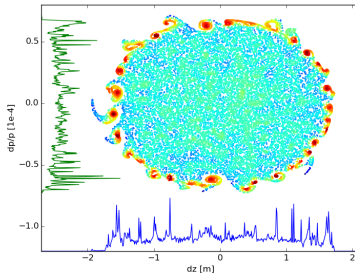
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high synchrotron tune $Q_s \approx 10^{-3}$



Intensity Effects Space Charge

high synchrotron tune $Q_s \approx 10^{-3}$



kind of two stream instability:

- ▶ perturbation in one stream produces bunching of second
⇒ bunching increases amplitude of perturbation
- ▶ in progress ⇒ comments are welcome

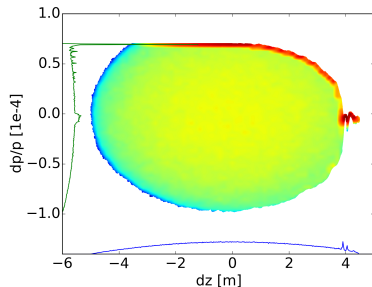
Intensity Effects

Space Charge



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medium synchrotron tune $Q_s \approx 10^{-4}$



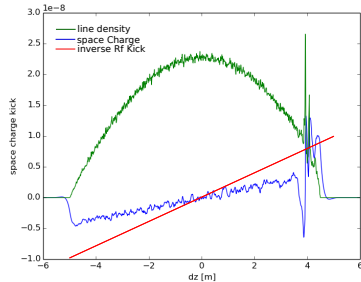
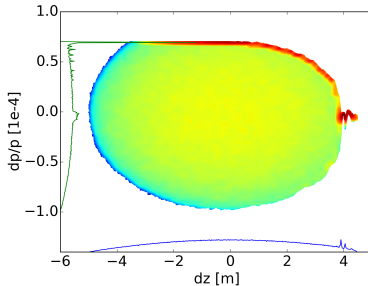
Intensity Effects

Space Charge



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medium synchrotron tune $Q_s \approx 10^{-4}$



- ▶ space charge compensates Rf-kick
⇒ synchrotron oscillation is stopped

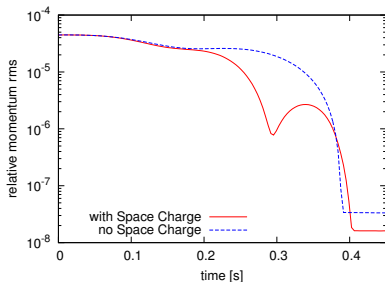
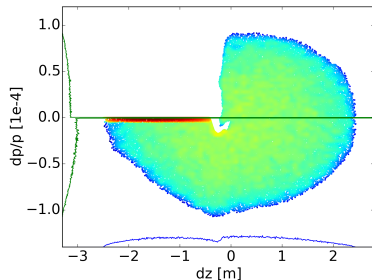
Intensity Effects

Space Charge



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low synchrotron tune $Q_s \approx 10^{-5}$



► space charge deforms bunch

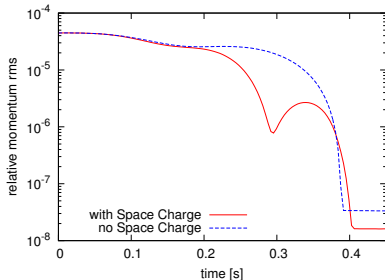
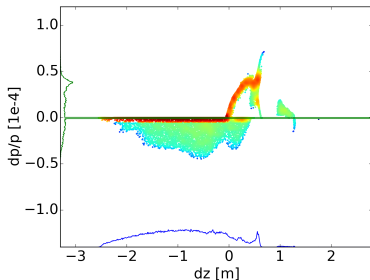
Intensity Effects

Space Charge



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low synchrotron tune $Q_s \approx 10^{-5}$



- ▶ space charge deforms bunch
- ▶ if deformation too strong $\Rightarrow$ space charge exceeds laser force (not stable)



- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Conclusion and Outlook



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- ▶ laser force implemented successfully in tracking code
- ▶ laser cooling looks promising at high beam energies
- ▶ cooling time and efficiency depend strongly on the way of laser scan
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Conclusion and Outlook



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- ▶ influence on transverse motion
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- ▶ combine everything for realistic conditions



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Thank you for your attention!