Crystalline Beam Study with Andy Sessler

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Michigan State U., Skyworks Inc., Hiroshima U.

COOL'15, JLab, October 1, 2015

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Outline

- Introduction – remembering Andy as our mentor, role model, colleague, and friend
- Crystalline beam in storage rings
- Condensed matter methods for the beam rest frame
- Towards ultra low temperature
- Summary

Andy Sessler in Muir Woods, California, 1993

What Are Crystalline Beams Good For?

- Low (i.e. → zero) emittance frontier of particle beams
- Highest possible beam density
- Extremely high luminosity colliders (e.g. for rare isotopes)
- Rich fundamental physics of a new state of matter

A Brief History

- 1979: Novosibirsk group saw anomaly in e-cooled proton beam
  - Dementiev, Dikansky, Medvedko, Parkhomchuk, D.V.Pestrikov, Skrinsky …
- 1984: Parkhomchuk conceived the concept of crystalline beams
- 1985: Schiffer, Rahman, Hasse et al started theoretical study with the molecular dynamics method and laser cooling experiments
  - Kienle, Habs, Hasse, Avilov, Hofmann, Hangst, Poulsen …
- 1987: Diedrich et al; Gilbert et al; Walther … experimental observation of Coulomb crystals in various kinds of traps
- 1992: Wei, Li, Okamoto, Sessler obtained conditions for crystallization, maintenance, and cooling in actual storage rings
- More current works:
  - TSR and ASTRID groups on experimental laser cooling
  - Okamoto et al on theoretical study of 3-D laser cooling
  - GSI group (Steck et al 1996) observed 1-D ordering in ESR beam
  - CRYRING (Danared et al 2002) 1-D ordering observed
  - Noda, Ikegami, et al on S-LSR: 1-D ordering and dispersion-free ring
  - Meshkov, Katayama, Moehl et al on colliding 1D strings for rare ions
  - Colliding crystals and storage ring lattice with high/imaginary transition energy
  - Further work with traps including at PALLAS and at Hiroshima University

NAP-M and Electron Cooling

G.I. Budker, Atomnaya Energiya 22 (1967) 346;
A.N. Skrinsky, V.V. Parkhomchuk, Sov. J. Part. Nucl. 12 (1981) 223

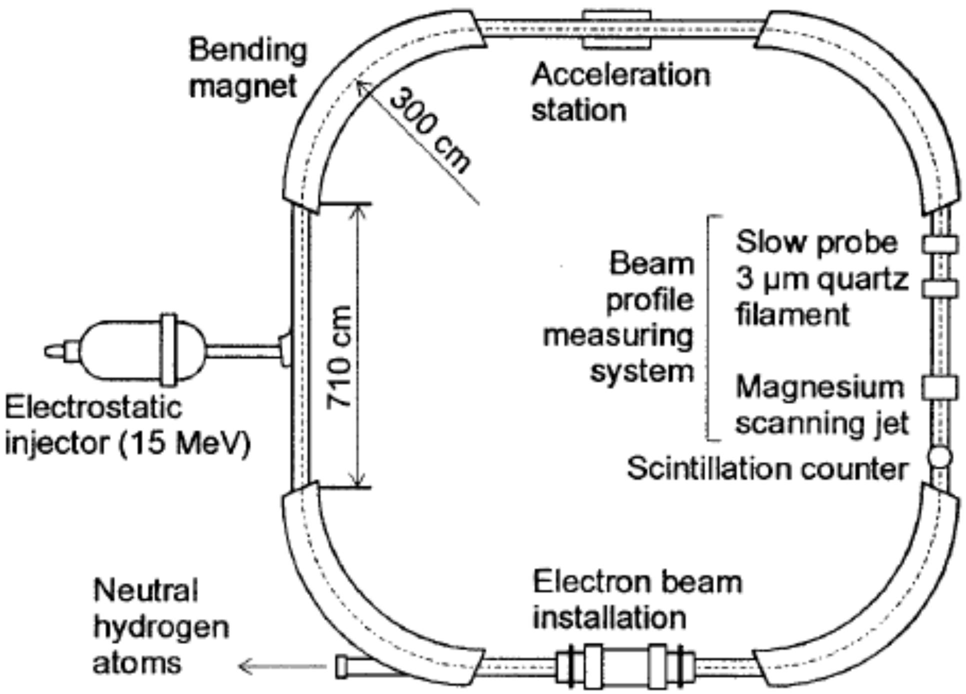


Figure 8. Layout of the proton cooler ring NAP–M

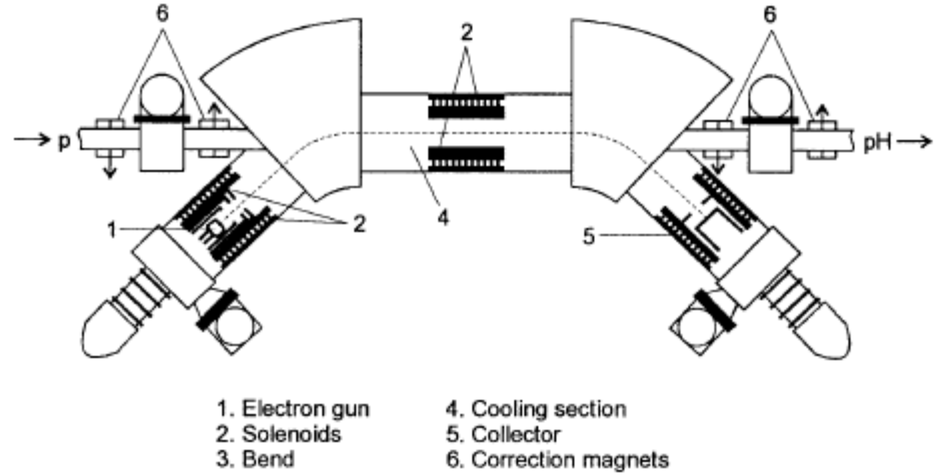
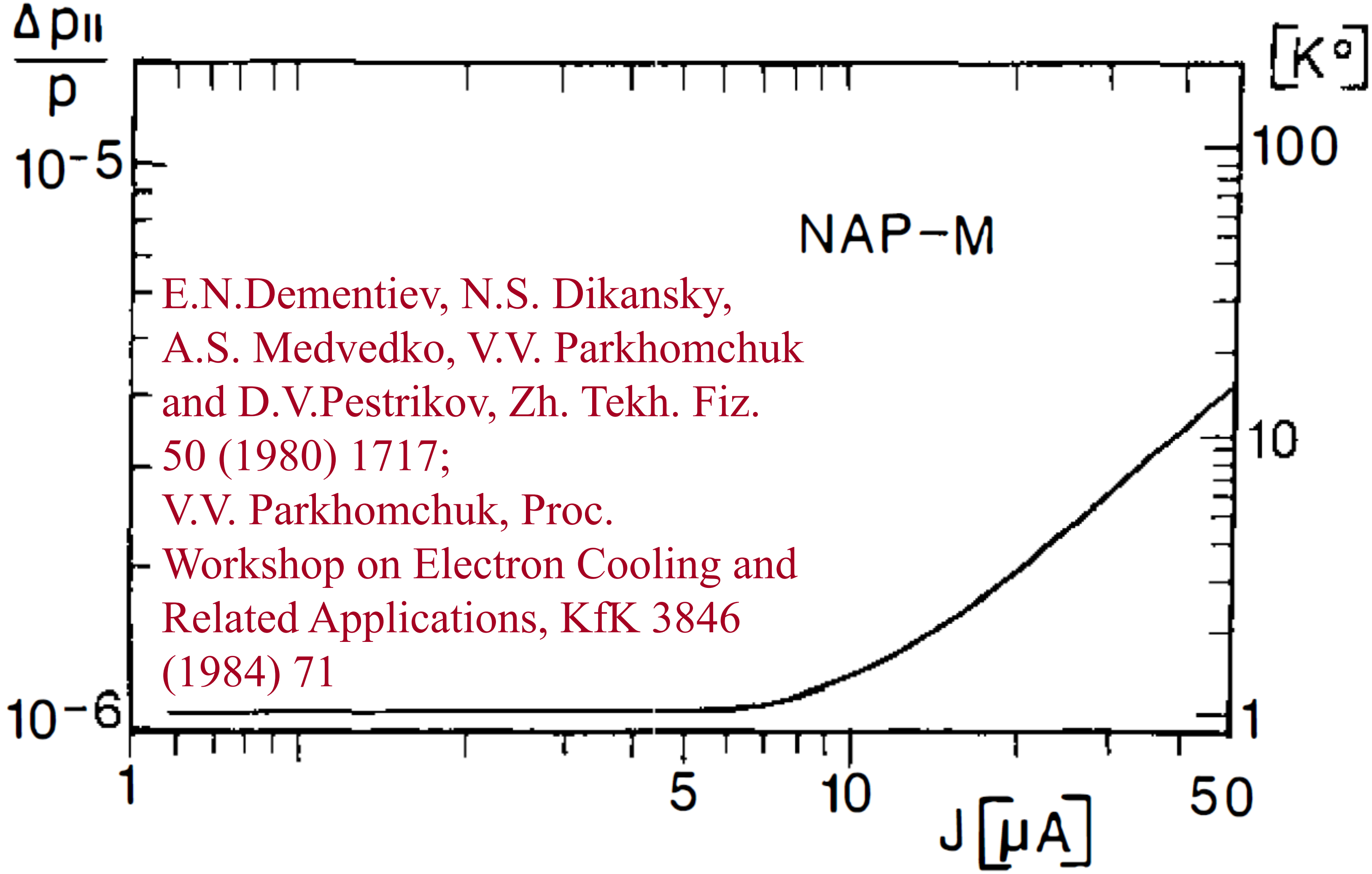


Figure 9. Layout of the electron-beam device for electron cooling.

NAP-M Observation, 1980

Anomaly in the Proton Storage Ring with Electron Cooling



E.N.Dementiev, N.S. Dikansky, A.S. Medvedko, V.V. Parkhomchuk and D.V.Pestrikov, Zh. Tekh. Fiz. 50 (1980) 1717; V.V. Parkhomchuk, Proc. Workshop on Electron Cooling and Related Applications, KfK 3846 (1984) 71

D'Artagnan et les Trois Mousquetaires in the NAP-M control room, Russia, 1975

V. Parkhomchuk, A. Skrinsky, I. Meshkov, and N. Dikansky

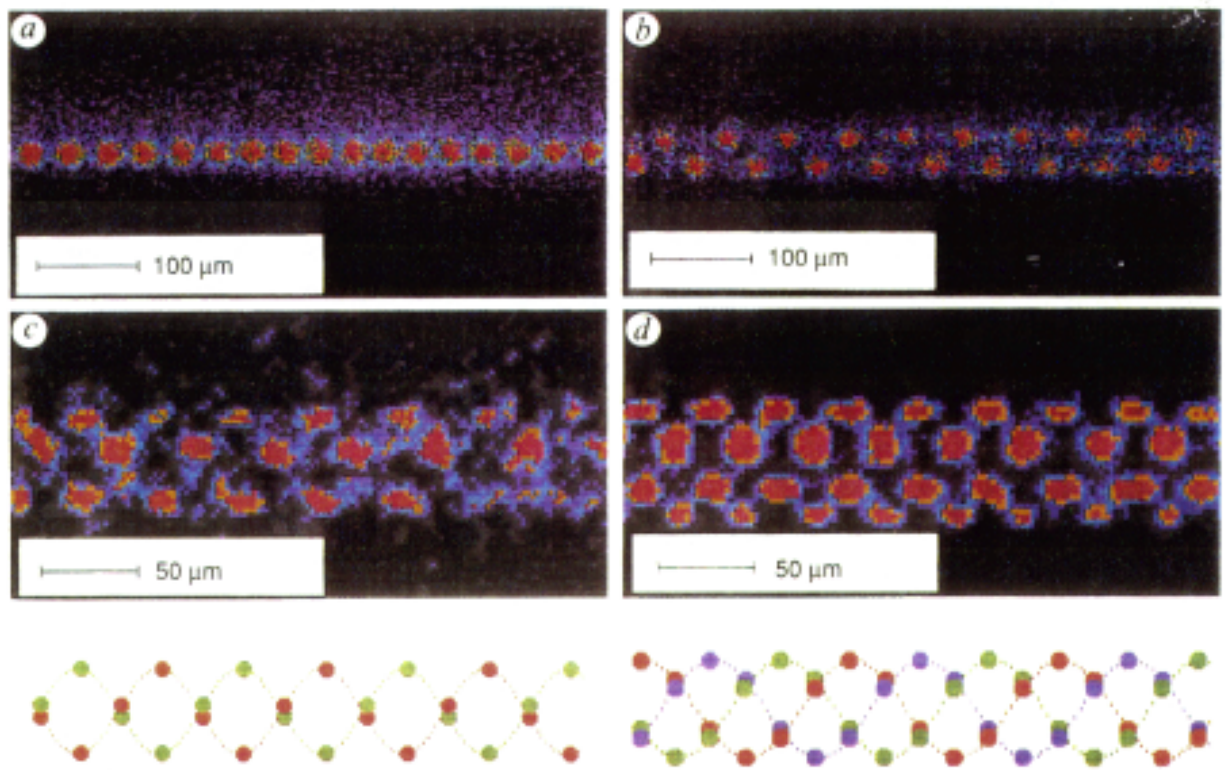
кф-мн, снс И.Н. Мешков, кф-мн, снс Н.С. Диканский в пультовой НАП-М. 1975 г.

Electron Cooling Team at BINP, 1976

I. Meshkov, B. Sukhina, D. Pestrikov, V. Ponomarenko,
V. Parkhomchuk, N. Dikansky

Coulomb Crystals Observed in Traps

FIG. 2 Colour-coded images of crystalline structures of laser-cooled $^{24}Mg^{+}$ ions. The intensity increases from violet to blue, yellow and red. Individual ions could be resolved in these images. The ions arrange themselves in minimum energy configurations. *a*, For low ion density ($\lambda = 0.29$) the ions form a string along the field axis; *b*, increasing the ion density changes the configuration to a zig-zag ($\lambda = 0.92$). At still higher ion densities the ions form ordered helical structures on the surface of a cylinder: *c*, two interwoven helices at $\lambda = 1.9$; *d*, three interwoven helices at $\lambda = 2.6$. Experimental images are displayed above, visualizations below.



- Gilbert, Bollinger, Wineland: Nature, Vol. 357, 28 May 1992

RFQ Ring Schematic Layout

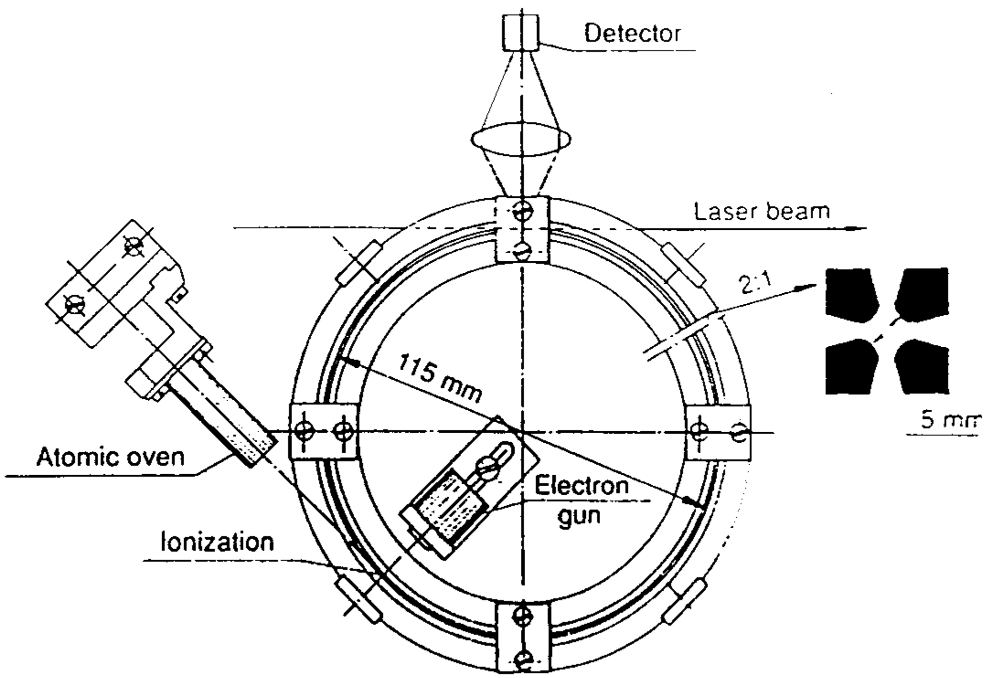


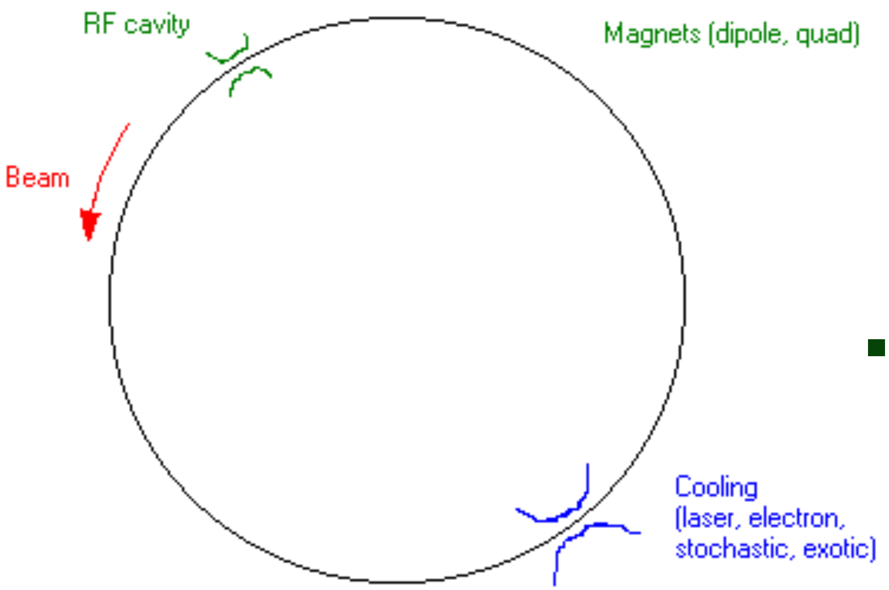
FIG. 1 Quadrupole storage ring, with the atomic beam oven and electron gun. The storage ring consists of four circular electrodes, and the diameter of the toroidal storage volume is $2R = 115$ mm. The insert shows an enlarged cross-section with opposite electrodes having a separation of $2r_0 = 5$ mm. The laser beam enters the storage volume tangentially. Resonance fluorescence is detected with a photomultiplier tube or an imaging photon detector system.

- Stationary $^{24}Mg^{+}$ ions
- Table top trap
- Multi-layer structure observed
- Structure in agreement with classical prediction

31^{st} Workshop of the INFN Eloisatron Project on Crystalline Beams and Related Issues, Erice, Italy, 1995

J. Wei, A.G. Ruggiero, A.M. Sessler, J. Hangst, and J. Schiffer

General Relativity Derivation of EOM
To Rigorously Adopt Condensed Matter Methods



- Equations of motion in tensor form

$$\frac{DP^i}{d\tau} = g^{ik} F_k, \qquad F_i U^i \equiv 0,$$

  - Covariant differentiation

$$\frac{DP^i}{d\tau} \equiv \frac{dP^i}{d\tau} + \Gamma^l_{ik} U^k P^l, \ \Gamma^l_{ik} = \frac{g^{lm}}{2}\left(\frac{\partial g_{mi}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^m}\right)$$

- EOM in the laboratory frame

$$m_0 \frac{d(\Gamma \mathbf{u})}{dt} = e\mathbf{E} + \frac{e}{c}\mathbf{u}\times\mathbf{B}, \ \ m_0 c^2 \frac{d\Gamma}{dt} = e\mathbf{u}\cdot\mathbf{E}$$

  - Time track of a circulating reference particle

$$f^i(\tau) = (R\cos\theta, 0, R\sin\theta, \gamma\tau) \qquad \theta = \omega\gamma\tau$$

  - Transformation of variables

$$X^i = f^i(\tau) + \breve{\alpha}_{i\kappa} x^\kappa, \ \ \text{and} \ \ x^4 = \tau$$

- EOM in the rotating beam rest frame

$$\begin{cases} \frac{m_0}{\Gamma}\left(\ddot{x} - 2\gamma^2\omega\dot{z} - \gamma^4\omega^2 x\right) - m_0\Gamma\gamma^2\omega^2 R(1-\chi) = \\ \quad = e(1-\chi)E'_x + \frac{e}{\Gamma c}\left[B'_z\dot{y} - B'_y(\dot{z} + \gamma^2\omega x)\right] - \frac{\partial V}{\partial x}, \\ \frac{m_0}{\Gamma}\ddot{y} = e(1-\chi)E'_y + \\ \quad + \frac{e}{\Gamma c}\left[B'_x(\dot{z} + \gamma^2\omega x - B'_z(\dot{x} - \gamma^2\omega z)\right] - \frac{\partial V}{\partial y}, \\ \frac{m_0}{\Gamma}\left(\ddot{z} + 2\gamma^2\omega\dot{x} - \gamma^4\omega^2 z\right) = \\ \quad = e(1-\chi)E'_z + \frac{e}{\Gamma c}\left[B'_y(\dot{x} - \gamma^2\omega z) - B'_x\dot{y}\right] - \frac{\partial V}{\partial z}, \end{cases}$$

Beam Rest-frame Hamiltonian

$$H_{bend} = \sum_i \frac{1}{2}\left[P_{ix}^2 + P_{iy}^2 + P_{iz}^2\right] - \gamma x_i P_{iz} + \frac{1}{2}\left[(1-n)x_i^2 + n y_i^2\right] + V_{ci}$$

$$H_{non-bend} = \sum_i \frac{1}{2}\left[P_{ix}^2 + P_{iy}^2 + P_{iz}^2\right] + \frac{1}{2}\left[-n\, x_i^2 + n\, y_i^2\right] + V_{ci} + U_s$$

$$V_{ci} = \sum_{j\neq i} \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}} \qquad \mathrm{n} = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial x}, \quad \frac{\partial U_s}{\partial z} = -\frac{Z_0 e \xi E_s}{m_0 c^2}\left(\frac{\rho}{\xi\beta\gamma}\right)^2$$

Condition 1: Ground State Existence

- The storage ring is alternating-gradient (AG) focusing operating below the transition energy

$$\gamma < \gamma_T$$

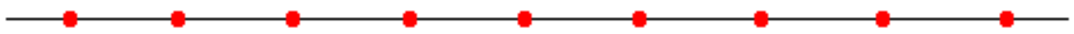
  - In the negative-mass regime there exists no ground state; the Hamiltonian is not bounded
  - Criterion of stable kinematic motion under Coulomb interaction when particles are subject to bending in a storage ring

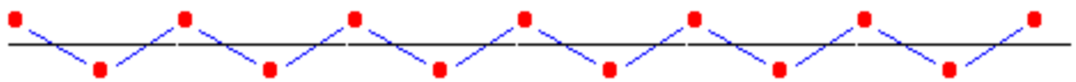
$$\bar{H} = \nu_x J_x + \nu_y J_y + \frac{1-\gamma^2 F_z}{2}\bar{P}_z^2 + \bar{V}_C$$

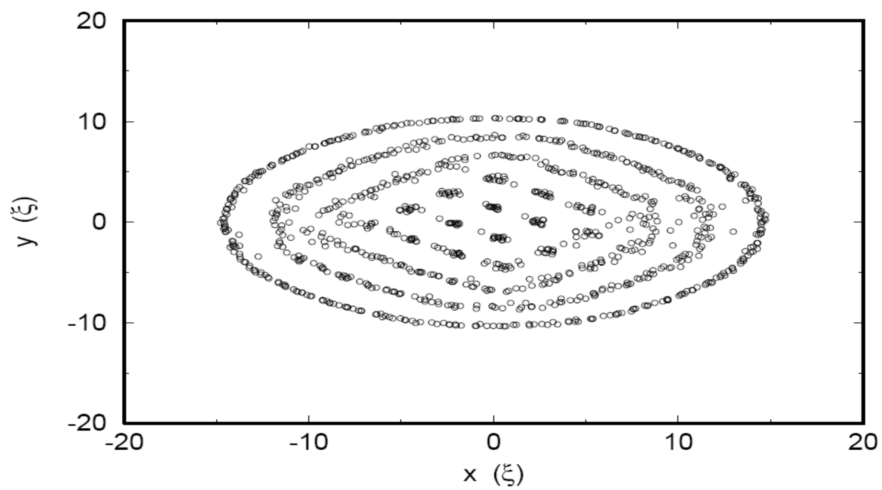
$$\langle F_z \rangle = \frac{\rho}{2\pi R}\oint F_z dt = \frac{\rho}{2\pi R}\oint_{\text{bend}} D dt \equiv \frac{1}{\gamma_T^2}$$

Crystalline Beam Structures

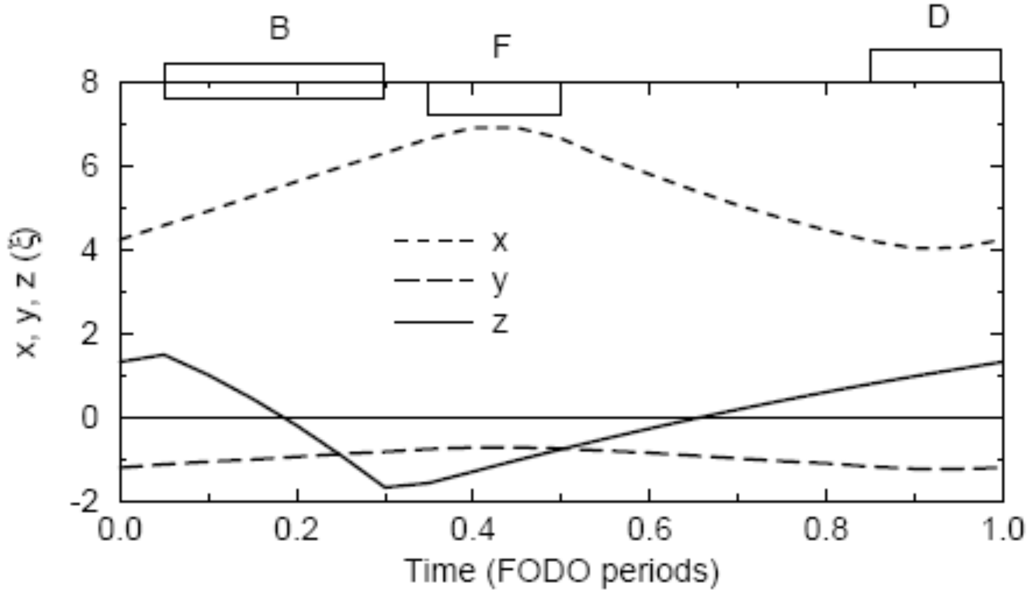
- From 1-D string, to 2-D zig-zag, to 3-D multi-shell helices

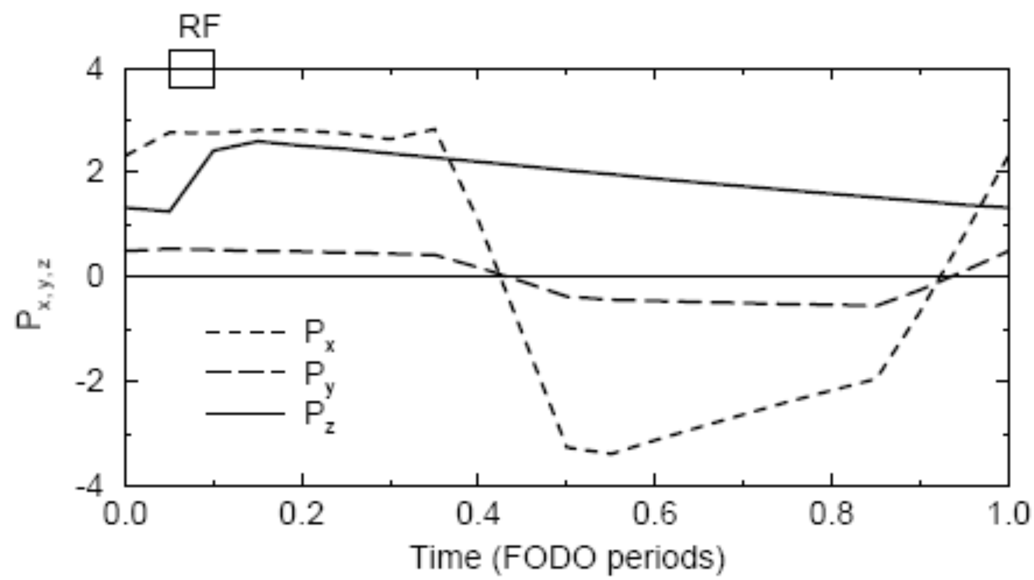




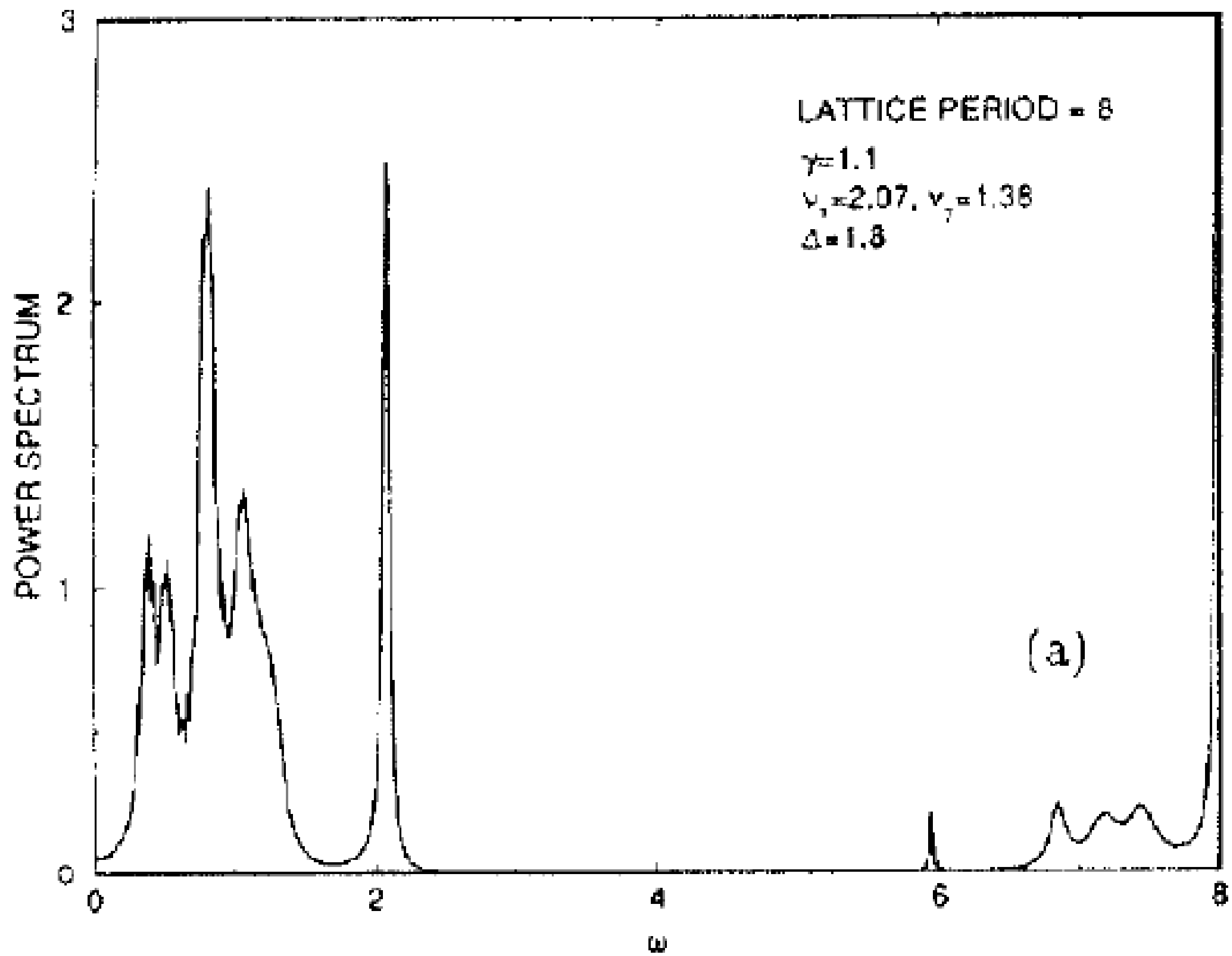


Trajectory of A Crystalline Beam Particle
In a Storage Ring of 10 FODO Cells





Vibrational Phonon Spectrum



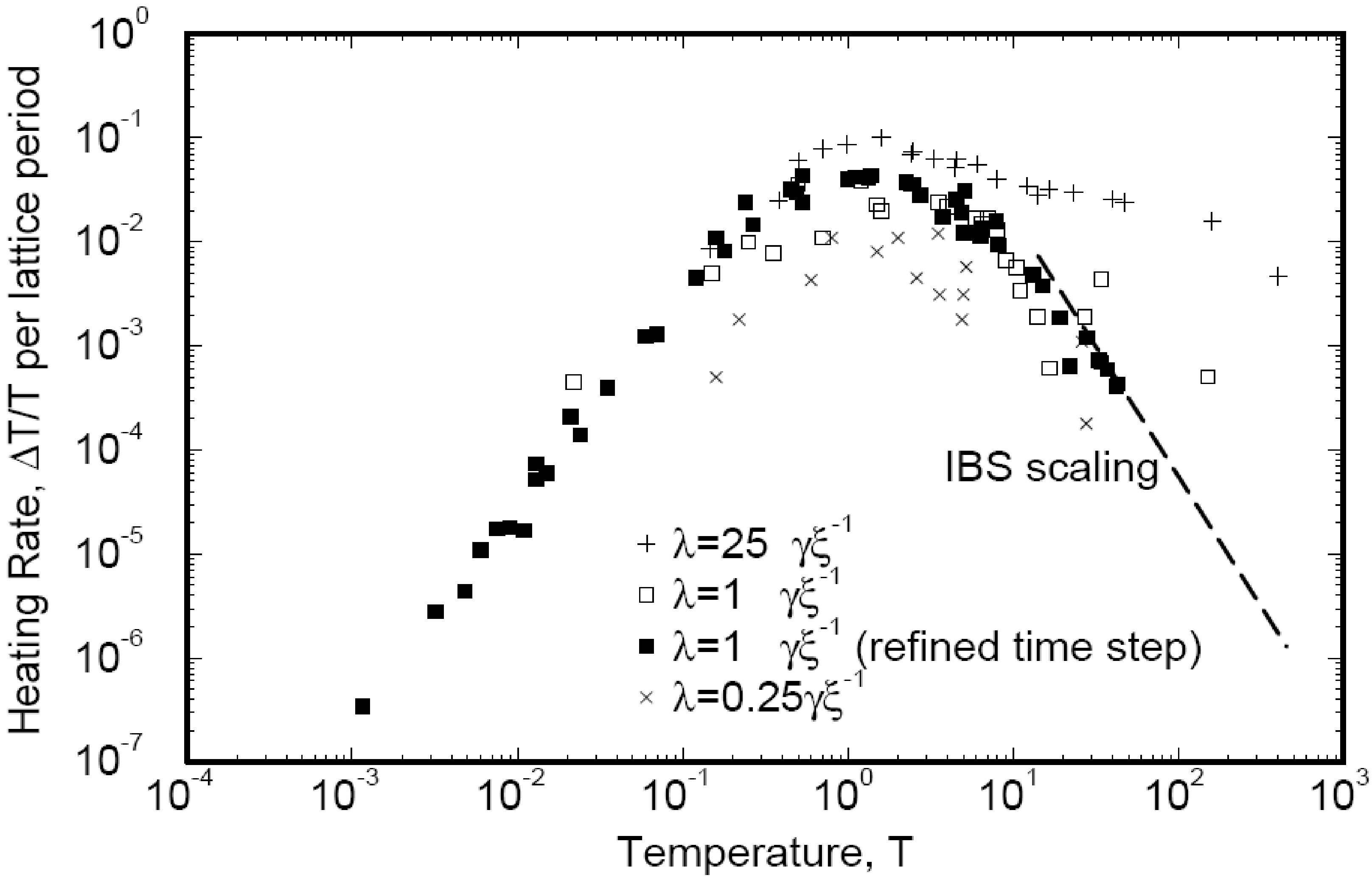
Condition 2: Linear Resonance Avoidance

- The bare transverse phase advances per lattice period need to be less than 90°

$$\nu_{x,y} < \frac{N_{\rm sp}}{4}$$

- There is no linear resonance between the phonon modes of the crystalline structure and the machine lattice periodicity (127°condition)
- Linear resonance stopbands are not crossed during the entire cooling process as the 3-D beam density is increased (90°condition)

Beam Heating Due To Intrabeam Scattering



Condition 3: Tapered Cooling Force

- 3-D cooling to overcome the intra-beam scattering heating
- Horizontally "tapered" cooling force

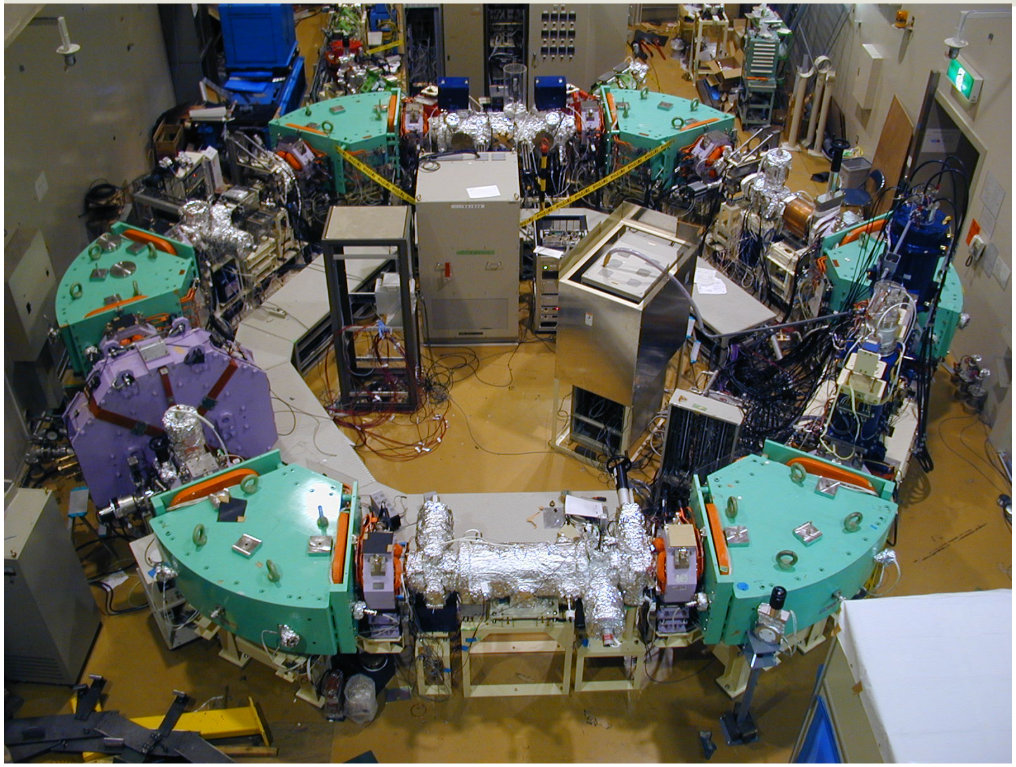
$$\Delta p_z = -f_z(p_z - C_{xz}x)$$

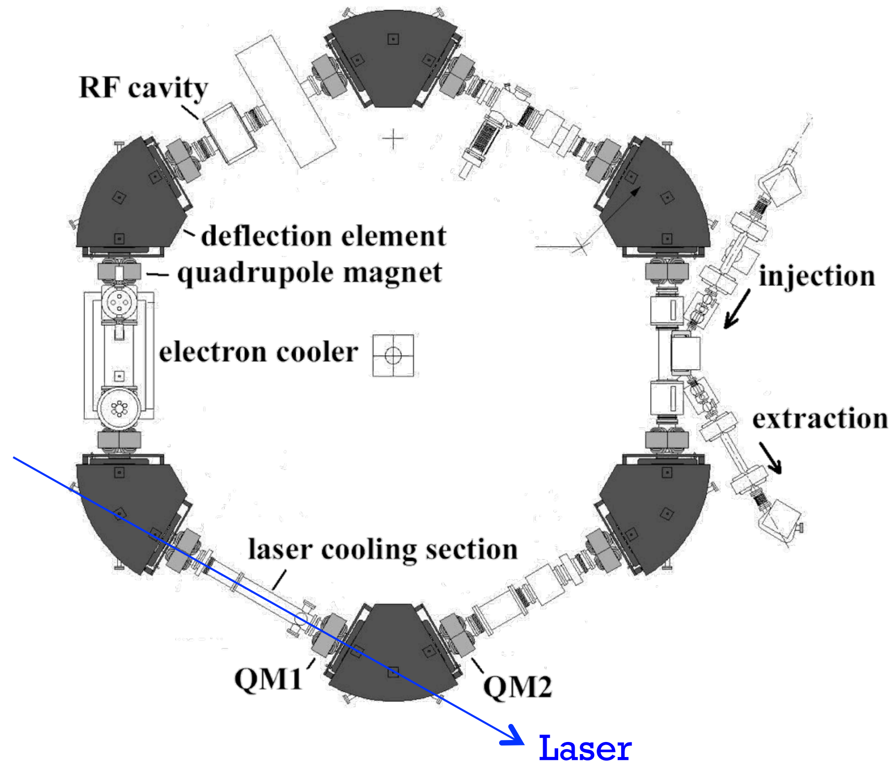
  - Cooling force needs to conform to the dispersive nature of a crystalline structure
  - Particles of different momentum move with the same angular velocity, not the same linear velocity

Beam Ordering Studies in Europe & Japan

	NAP-M	TSR	ASTRID	ESR	CRYRING	S-LSR	
E_u [MeV/u]	65.7	1.9	0.00417	360	7.4	7	
Circumference [m]	47.25	55.4	40	108.36	51.63	22.55	
γ	1.07	1.002	1.00000444	1.384	1.00789	1.00746	
γ_T	1.18	2.96	4.34	2.67	2.25	1.23	
N_{SP}	4	2	4	6	6	6	
$\frac{\nu_x}{N_{SP}} / \frac{\nu_y}{N_{SP}}$	0.338 / 0.315	1.285 / 1.105	0.345 / 0.33	0.383 / 0.383	0.383 / 0.383	0.27 / 0.20	0.35/0.19
Species	p	$^{7}Li^{+}$	$^{24}Mg^{+}$	$^{197}Au^{79+}$	$^{129}Xe^{36+}$	p	$^{24}Mg^{+}$
Cooling Method	EC	LC	LC	EC	EC	EC	LC
ξ [μm]	4.6	4.15	21.8	12.7	11.2	4.8	3.3
$T_{Bx,y}$ / T_{Bz} [K]	50 / 1	-- / 3	>0.1 / 0.001	13580.6 / <10	27.2 / 18.1	9.05 / 1.54	20 / 0.4
$T_{x,y}$ / T_z	13.9 / 0.28	-- / 0.75	>0.132 / 0.00132	1.68 / <0.001	0.014 / 0.009	2.64 / 0.45	186 / 3.7
N_0 (anomaly)	2×10^{7}	--	5.5×10^{8}	4000	1000 -- 10000	2000	--
N_0 (1-D to 2-D)	6.0×10^{6}	1.4×10^{7}	1.1×10^{6}	7.9×10^{6}	4.7×10^{6}	2.9×10^{6}	4.6×10^{6}
Observations	Schottky anomaly	Indirect transverse cooling	Schottky anomaly	1-D ordering	1-D ordering	1-D ordering	Indirect transverse laser cooling

S-LSR, Kyoto University





Circumference	22.557 m
Superperiodicity	6
Ion Species	$^{24}Mg^{+}$, p
Kinetic Energy	~ 40 keV (Mg), 7 MeV (p)
Transition Gamma	1.67
Bending Curvature	1.05 m

Beam 1-D Ordering at S-LSR

T. Shirai et al, Phys. Rev. Lett. 98, 204801 (2007)

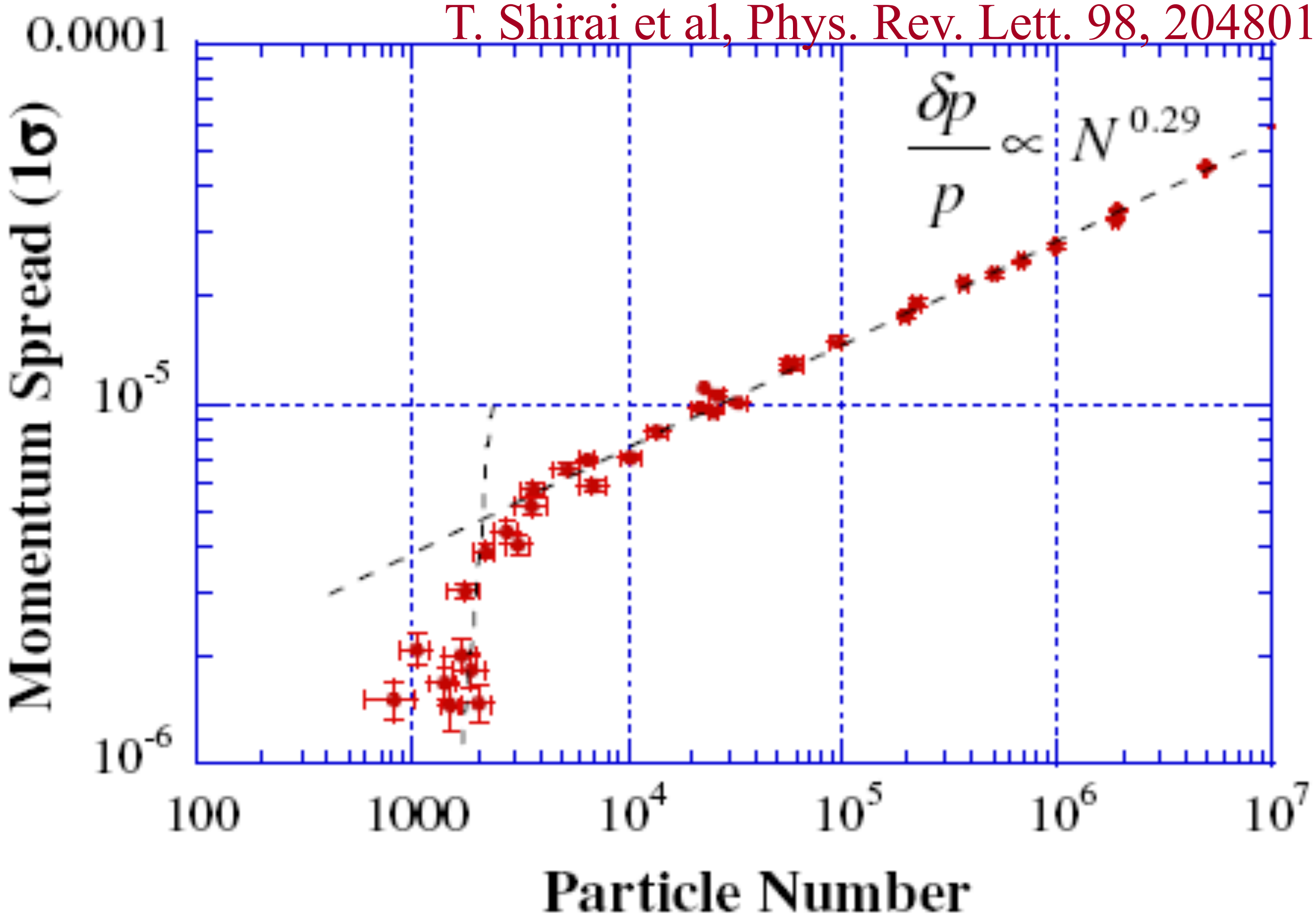
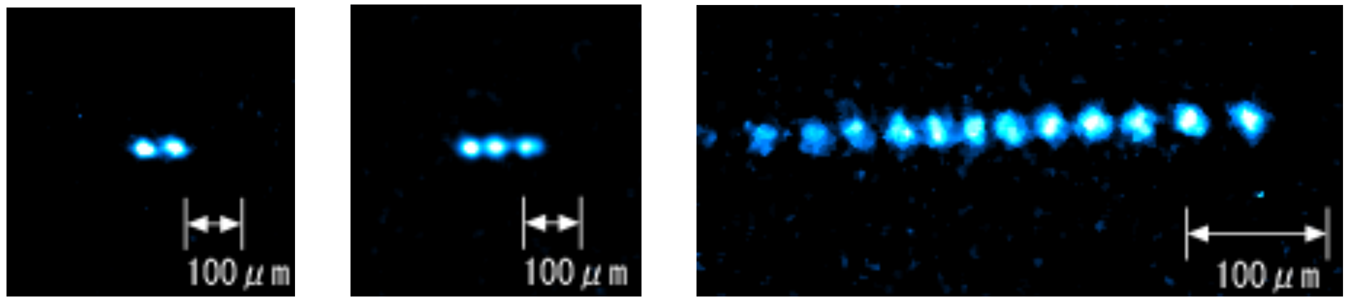


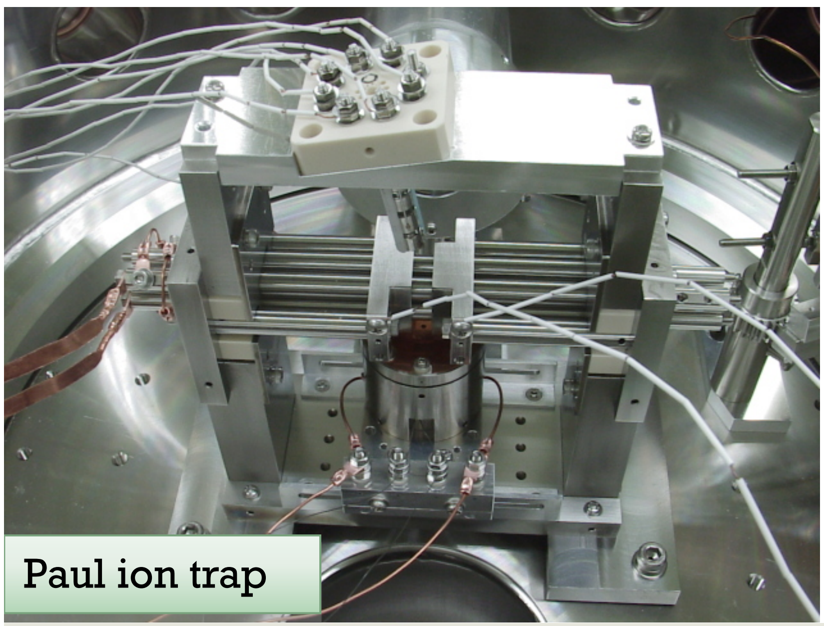
FIG. 1 (color online). Momentum spread as a function of the particle numbers in the ring with an electron current of 25 mA. The momentum spread drops at a particle number of 2000.

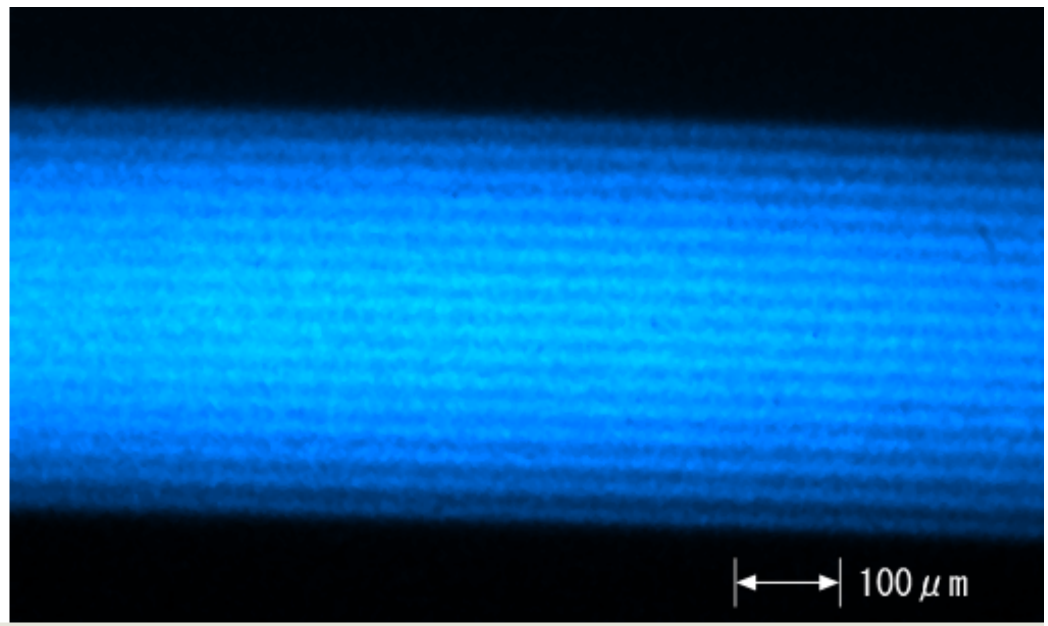
Coulomb Crystals in an Ion Trap, Hiroshima

String crystals ($^{40}Ca^{+}$)



Multi-shell crystal

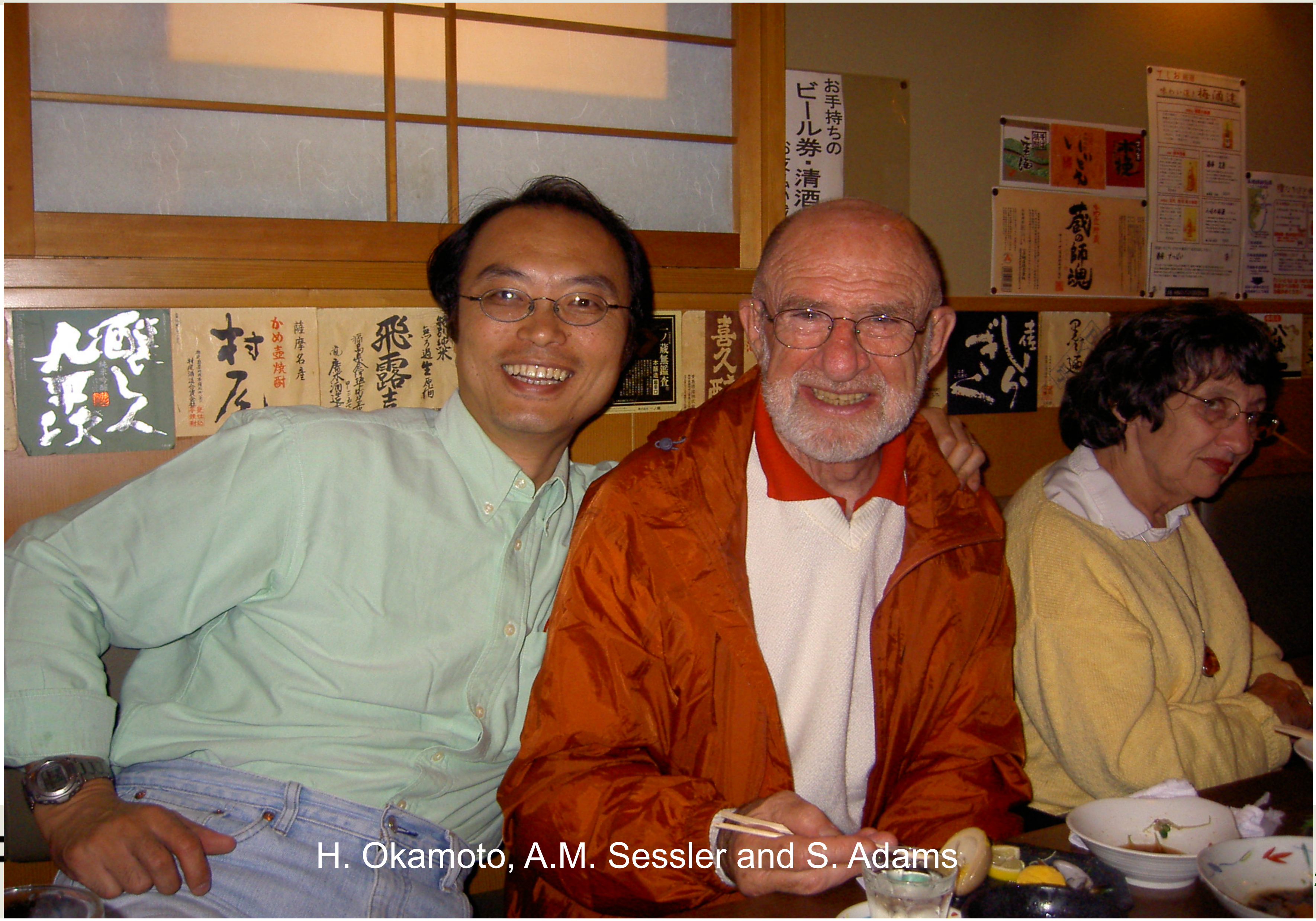




Andy Visiting BINP, 2002

G. Kulipanov, A.M. Sessler, V. Sidorov, B. Chirikov, N. Dikansky, I. Meshkov

Andy Visiting Hiroshima University, 2006



H. Okamoto, A.M. Sessler and S. Adams

Andy and Hiromi Visiting CERN, 2009

A. Hofmann, A.M. Sessler, E. Wilson, E. Keil, B. Zotter and D. Möhl

Andy Sessler Symposium, Berkeley, 2003

J. Wei, X.-P. Li, K. Takayama and S. Yu

Andy at PAC'05, Knoxville, 2005

Visiting Andy in October 2013

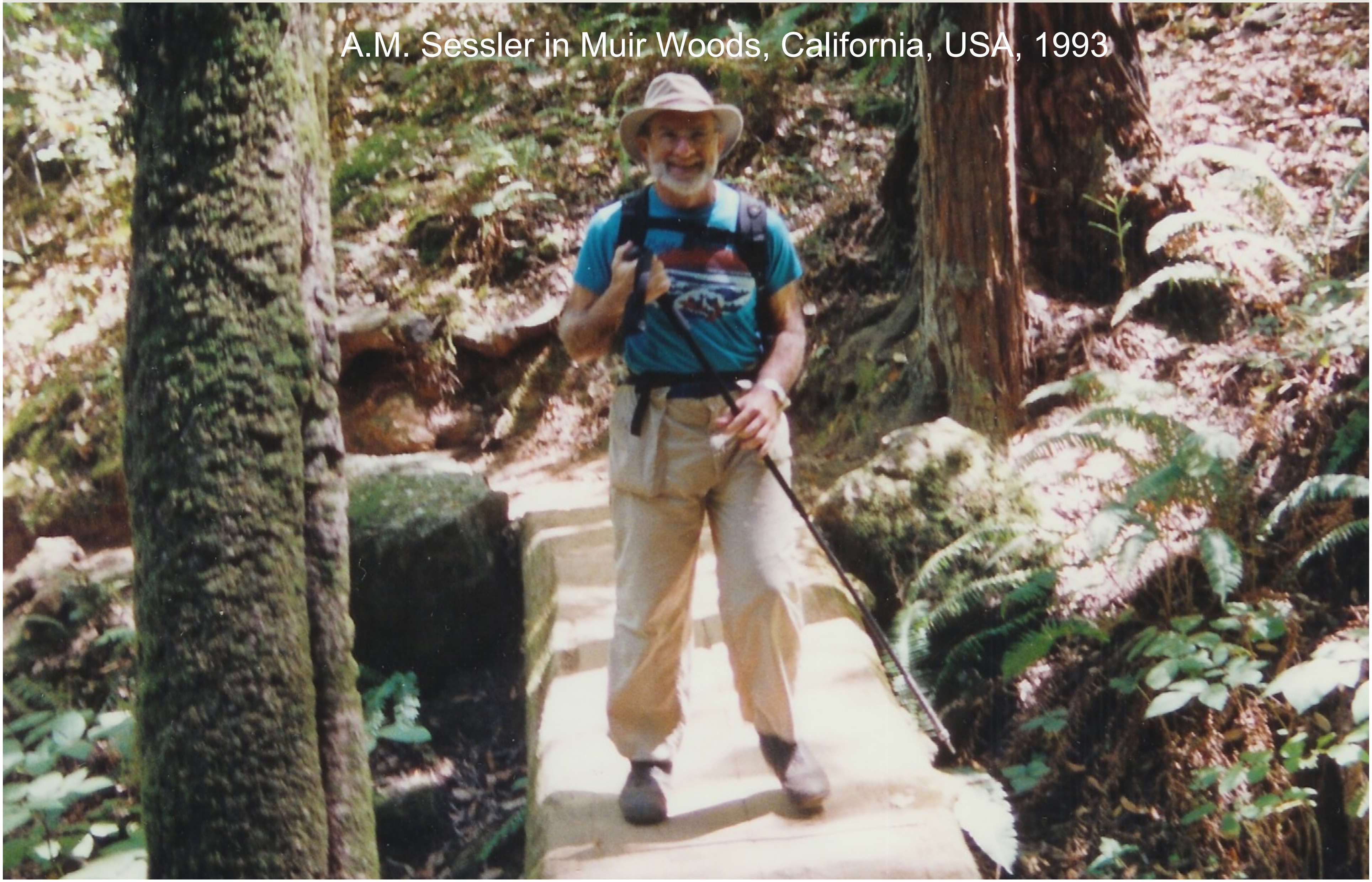
Andy Awarded 2013 Enrico Fermi Award

Summary

- The crystalline beam corresponds to the ultimate state of zero temperature and zero emittance of charged particle beams.
- Andy was fascinated by the rich and challenging physics and worked with us as his hobby for more than twenty years.
- The contributions range from the fundamental analytical formulation leading to guiding conditions of crystalline beam formation, to numerical methods and confirmation, and then to advising experimentalists in practical realization.

- We will always remember Andy as our mentor, role model, colleague, and very dear friend

Andy Sessler – Mentor, Role Model Colleague, and Friend


A.M. Sessler in Muir Woods, California, USA, 1993

References [1]

[1] *Handbook of Accelerator Physics and Engineering*, edited by A. Chao, K.H. Mess, M. Tigner, F. Zimmermann, World Scientific, 2013, p. 212.

[2] E.N.Dementiev, N.S. Dikansky, A.S. Medvedko, V.V. Parkhomchuk and D.V.Pestrikov, Zh. Tekh. Fiz. **50** (1980) 1717.

[3] V.V. Parkhomchuk, Proc. Workshop on Electron Cooling and Related Applications, KfK 3846 (1984) 71; V.V. Parkhomchuk, A. H. Skrinsky, Reports on Progress in Physics, **54** (1991) 919.

[4] J. P. Schiffer and P. Kienle, Z. Phys. A **321**, (1985) 181; A. Rahman and J. P. Schiffer, Phys. Rev. Lett. **57**, 1133 (1986); J. P. Schiffer and A. Rahman, Z. Phys. A **331**, 71 (1988).

[5] S. Schröder et al. Phys. Rev. Lett. **64**, 2901 (1990); H.-J. Miesner et al, Phys. Rev. Lett. **77**, 623 (1996).

[6] J. S. Hangst et al, Phys. Rev. Lett. **67** (1991) 1238, Phys. Rev. Lett. **76** (1991) 1238; N. Madsen, et al, Phys. Rev. Lett. **83** (1999) 4301; J. S. Hangst, et al, Phys. Rev. Lett. **74,** (1995) 86.

[7] M. Steck et al, Phys. Rev. Lett. **77** (1996) 3803.

[8] H. Danared et al, Phys. Rev. Lett. **88** (2002) 174801.

[9] T. Shirai et al, Phys. Rev. Lett. **98** (2007) 204801.

[10] J. Wei, X.-P. Li, A.M. Sessler, BNL Report 52381 (Brookhaven National Laboratory, 1993); J. Wei, PAC'01 (2001) 1678.

[11] J. Wei, X-P Li, A.M. Sessler, Phys. Rev. Lett. **73** (1994) 3089.

[12] H. Okamoto, A. M. Sessler, and D. Möhl, Phys. Rev. Lett., **72** (1994) 3977; H. Okamoto, Phys. Rev. E, **50**, (1994) 4982.

[13] Private communications with J. Wei (Oct. 2013).

[14] Web site: https://www.whitehouse.gov/blog/2014/02/03/ president-obama-welcomes-2013-fermi-award-winners-white-house

[15] J. Wei, X.-P. Li, A.M. Sessler, PAC'95 (1995) 2948.

[16] J. Wei, H. Okamoto, and A. M. Sessler, Phys. Rev. Lett., **80** (1998) 2606.

[17] C. Møller, *The Theory of Relativity*, Oxford, 1952.

[18] X. Nielsen, A.M. Sessler, Proc. Int. Conf. High Energy Acc. (1959) 239.

[19] J. Wei et al, Proc. 31st INFN Eloisatron Workshop on Crystalline Beams and Related Topics (Erice, Italy, 1995) 229; J. Wei et al, EPAC'06, 2841.

[20] I. Hofmann, J. Struckmeier, Proc. Workshop on Crystalline Ion Beams (1988) 140.

[21] R.W. Hasse, J.P. Schiffer, Ann. Phys. **203** (1990) 419.

[22] V.V. Avilov, Solid State Commun. **44**, (1982) 555.

[23] R.W. Hasse, Phys. Rev. Lett. **67** (1991) 600.

[24] X-P Li, A.M. Sessler, J. Wei, EPAC'94 (1994) 1379.

[25] X-P Li et al, PAC'05 (2005) 4111.

[26] L. von Hove, Phisica **16** (1950) 137.

[27] L.D. Landau and E.M. Lifshitz, Statistical Physics I, Pergamon Press, New York (1980).

References [2]

[28] G.I. Budker, Atomnaya Energiya **22** (1967) 346; A.N. Skrinsky, V.V. Parkhomchuk, Sov. J. Part. Nucl. **12** (1981) 223; Fiz. Elem. Chast. Atom. Yadra **12** (1981) 557.

[29] S. van der Meer, CERN Internal Report CERN/ISR-PO/72-31 (1972).

[30] D.J. Wineland, H. Dehmelt, Bull. Am. Phys. Soc., **20** (1975) 637.

[31] T. Hänsch, A. Shawlow, Opt. Commun., **13** (1975) 68.

[32] M. Nakao et al., Phys. Rev. ST AB **15**, (2012) 110102.

[33] H. Souda et al., Jpn. J. Appl. Phys. **52** (2013) 030202.

[34] H. Okamoto, J. Wei, Phys. Rev. E **58** (1998) 3817.

[35] Y. Yuri, H. Okamoto, Phys. Rev. ST AB **8** (2005) 114201.

[36] R. E. Pollock, Z. Phys. A **341** (1991) 95.

[37] M. Ikegami et al., Phys. Rev. ST AB **7** (2004) 120101; M. Ikegami, H. Okamoto, Y. Yuri, Phys. Rev. ST AB **9** (2006) 124201.

[38] K. Okabe and H. Okamoto, Jpn. J. Appl. Phys. **42** (2003) 4584.

[39] X.-P. Li *et al*., Phys. Rev. ST AB **9** (2006) 034201.

[40] H. Okamoto, Proc. COOL'13 (2013) 52.

[41] K. Osaki, H. Okamoto, Prog. Theor. Exp. Phys. **2014** (2014) 053G01.

[42] J. Wei, A.M. Sessler, EPAC'98 (1998) 862; J. Wei et al, COOL'07, (2007) 91.

[43] I. Meshkov et al, RIKEN Report: RIKENAF-AC-34 (2002).

[44] T. Katayama, D. Möhl, RIKEN Report RIKEN-AF-AC-39 (2002).

[45] H. Hasse, Phys. Rev. Lett. **83** (1999) 3430.

[46] H. Okamoto et al, Phys. Rev. E **69** (2004) 066504; I. Meshkov, A. Sidorin, Nucl. Instrum. Meth. A **532** (2004) 474.

[47] H. Okamoto, H. Tanaka, Nucl. Instrum. Meth. **437** (1999) 178; H. Okamoto et al., Nucl. Instrum. Meth. **733** (2014) 119; E. Gilson et al., Phys. Rev. Lett. **92** (2004) 155002.

[48] F. Diedrich et al, Phys. Rev. Lett. **59** (1987) 2931; D.J. Wineland et al, Phys. Rev. Lett. **59** (1987) 2935; S.L. Gilbert et al, Phys. Rev. Lett. **60** (1988) 2022; I. Waki et al., Phys. Rev. Lett. **68** (1992) 2007; N. Kjægaard and M. Drewsen, Phys. Rev. Lett. **91** (2003) 095002; N. Kjægaard, K. Mølhave, and M. Drewsen; M. Drewsen et al., Phys. Rev. Lett. **81** (1998) 2878.

[49] T. Schätz, U. Schramm, D. Habs, Nature (London) **412** (2001) 717; U. Schramm, M. Bussmann, D. Habs, Nucl. Instrum. Meth. A **532** (2004) 348; U. Schramm, T. Schätz, D. Habs, Phys. Rev. E **66** (2002) 036501.