Status of VORPAL Friction Force Simulations for the RHIC II Cooler

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1. Tech-X Corporation

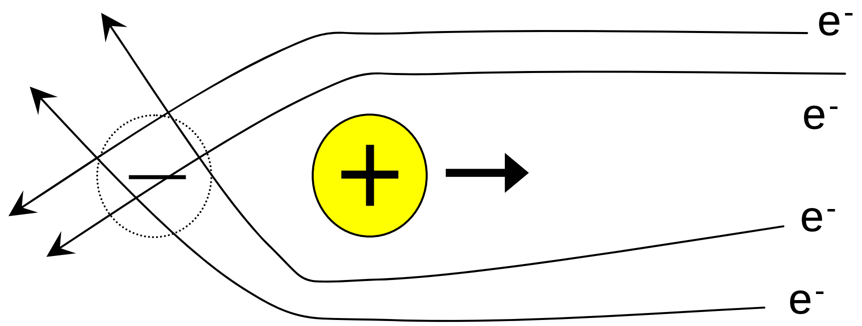
2. Brookhaven National Lab

Overview

- We are considering unmagnetized high-energy cooling
  - simulations of dynamical friction for RHIC II parameters
- Question arose at BNL cooling workshop (May, 2006)
  - Can VORPAL characterize the effect of magnetic field errors?
    - as a function of integrated transverse field amplitude?
    - especially, as a function of wavelength?
  - Physical intuition suggests the following (S. Nagaitsev)
    - small wavelengths should modify ρ_{min} (logarithmic effect)
    - large wavelengths should increase effective rms e- velocities
- Our simulation results were difficult to interpret
  - this requires full understanding of finite-time effects
  - simple Coulomb log must be reconsidered for finite time
- Error-field & field-free simulations are now understood
  - beginning to re-examine many error-field simulations

Dynamical friction has been long understood

- Case of isotropic plasma, with no external fields, was first explained 65 years ago
  - S. Chandrasekhar, *Principles of Stellar Dynamics* (U. Chicago Press, 1942).
  - B.A. Trubnikov, Rev. Plasma Physics **1** (1965), p. 105.
  - *NRL Plasma Formulary*, ed. J.D. Huba (2000).



- Physics can be understood in two different ways
  - Binary collisions (integrate over ensemble of e-/ion collisions)
  - Dielectric plasma response (ion scatters off of plasma waves)
- Finite-time effects provide a new twist on old physics

Idea for Electron Cooling is 40 Years Old

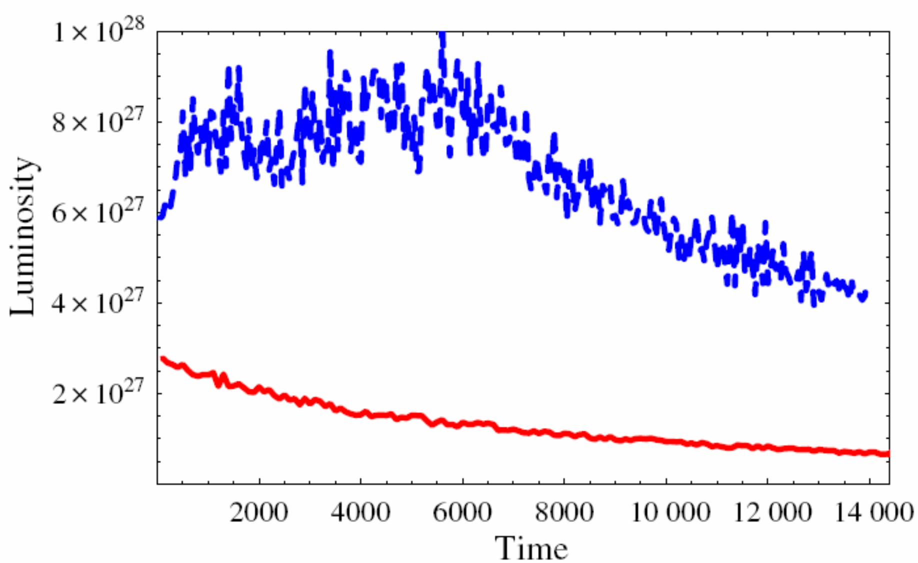
- Budker developed the concept in 1967
  - G.I. Budker, At. Energ. **22** (1967), p. 346.
- Many low-energy electron cooling systems:
  - continuous electron beam
  - electrons are nonrelativistic & cold
  - electrons are magnetized in a solenoid field
    - suppresses transverse temperature & increases friction
- Fermilab has shown cooling of relativistic p-bar's
  - S. Nagaitsev *et al*., PRL **96**, 044801 (2006).
  - 4.3 MeV e-'s ($\gamma \approx 8$) from a customized DC source
  - electrons are *unmagnetized* (solenoid for focusing)
- RHIC II, eRHIC need "high-energy" cooler
  - 100 GeV/n $\Rightarrow \gamma \approx 108 \Rightarrow$ 54 MeV bunched electrons
  - Cooling is less efficient; new parameter regime

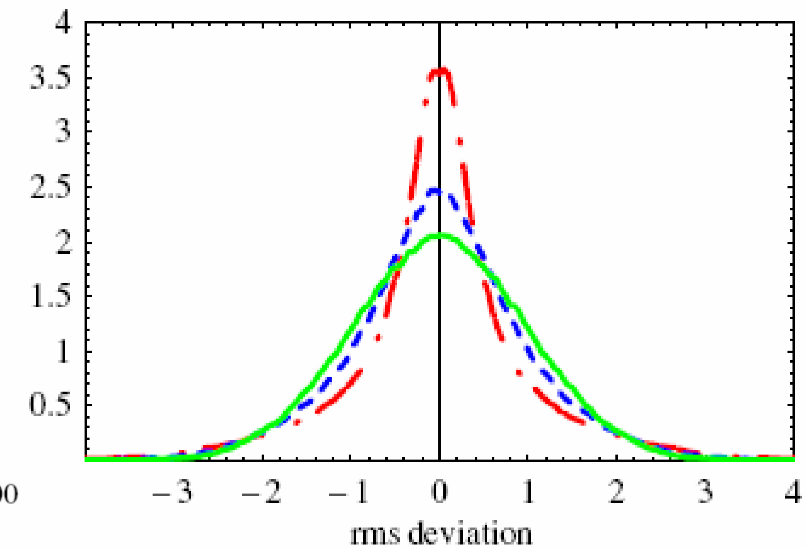
The Parallel ***VORPAL*** Framework is used to Simulate the Microphysics of Electron Cooling

- Electromagnetic PIC for laser-plasma
  - Nieter & Cary, *J. Comp. Phys.* (2004).
- Electrostatic PIC for beams & plasma
  - Messmer & Bruhwiler, *Comp. Phys. Comm.* (2004)
- Algorithms for simulating electron cooling physics
  - Fedotov, Bruhwiler, Sidorin, Abell, Ben-Zvi, Busby, Cary & Litvinenko, *Phys. Rev. ST/AB* (2006).
  - Fedotov, Ben-Zvi, Bruhwiler, Litvinenko & Sidorin, *New J. Phys.* (2006).
  - Bell, Bruhwiler, Fedotov, Sobol, Busby, Stoltz, Abell, Messmer, Ben-Zvi & Litvinenko, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam," *J. Comp. Phys.*, in preparation.
- SRF Cavities, Electron guns, Dielectric structures (PBG)…
  - Nieter *et al.*, *J. Comp. Phys.*, in preparation.
  - Dimitrov, Bruhwiler, Smithe, Messmer, Cary, Kayran & Ben-Zvi, *Proc. ICFA Beam Dynamics Workshop on Energy Recovery Linacs* (2007), in press.
  - Werner & Cary, *J. Comp. Phys.*, in preparation.
- Large software development team (Tech-X & CU)
  - C++/MPI, object-oriented, template techniques, multi-physics
  - parallel or serial; cross-platform (Linux, AIX, OS X, Windows)
- Actively used throughout the beam & plasma communities
  - BNL, JLab, Fermilab, LBL, ANL, some universities, also outside the USA
  - commercial customers
- Development and use has been supported by several agencies since 2000
  - US DOE Office of Science (HEP, NP, FES & ASCR)
  - NSF (original grant); DOD (AFOSR, OSD)

VORPAL supports use of BETACOOL

- BETACOOL code is used to model many turns
  - A.O. Sidorin *et al*., *Nucl. Instrum. Methods* A **558**, 325 (2006).
  - A.V. Fedotov, I Ben-Zvi, D.L. Bruhwiler, V.N. Litvinenko, A.O. Sidorin, New J. Physics **8**, 283 (2006).
  - a variety of electron cooling algorithms are available
    - in particular, models for the dynamical friction force
  - many mechanisms for emittance growth are included
- VORPAL is used to study microphysics of friction
  - to increase understanding & make BETACOOL more effective





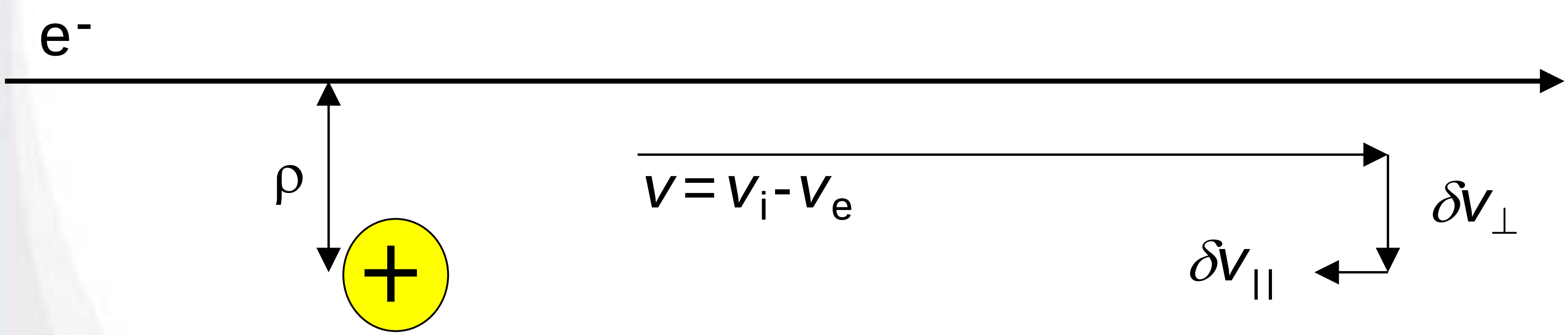
Physical Parameters from 2006 RHIC II Design

Parameter	Frame	Value
Electron Density, n_e	Beam	$9.50 \times 10^{13}\ e^-/m^3$
RMS e^- x, y-velocity, Δ_x, Δ_y	Beam	$2.8 \times 10^5\ m/s$
RMS e^- z-velocity, Δ_z	Beam	$9.0 \times 10^4\ m/s$
Interaction time, τ	Beam	2.47×10^{-9} sec
90° collision impact parameter, ρ_{min}	Beam	2.2×10^{-7} m
Mean Coulomb Log value, Λ	Beam	8.2
Relativistic γ	Lab	108
Relativistic $\beta = v/c$	Lab	0.99957
Undulator length	Lab	80 m
Undulator strength	Lab	10 Gauss or 1.0×10^{-3} Tesla
Undulator wavelength, λ	Lab	8 cm

Dynamical Friction, Coulomb log & Finite-time Effects

Perturbative calculations lead to Coulomb log

- Assume each e- trajectory is infinite and straight
  - integrate Coulomb force along trajectory to obtain $\delta v_\perp \sim \rho^{-1}$
    - integrating over all angles leads to zero
  - by symmetry, $\delta v_{||} = 0$ for each trajectory
    - however, energy conservation requires $\delta v_{||} \approx - \delta v_\perp^2 / 2v \sim \rho^{-2}$
- Approximation is very good for large ρ
  - assumption of infinite trajectories not valid for finite τ
  - choose physically reasonable cutoff ρ_{max}
- Approximation breaks down for small ρ
  - choose cutoff ρ_{min}, for which $\delta v_\perp = v_{rel} \rightarrow$ 90 deg scattering



Dynamical friction & finite-time effects on ρ_{max}

$$\mathbf{F} = -\frac{4\pi n_e k^2}{m_e} \int \log\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{\mathbf{v}}{|\mathbf{v}|^3} f(\mathbf{v}_e) d^3 v_e$$

$$k = Ze^2/(4\pi\epsilon_0) \qquad \mathbf{v} = \mathbf{v}_i - \mathbf{v}_e \qquad \rho_{min} = \frac{k}{m_e |\mathbf{v}|^2}$$

- For $\omega_{pe}\tau >> 2\pi$ and $v_i << \Delta_e$
  - electron cloud screens ion charge; $\rho_{max} = \lambda_D = \Delta_e/\omega_{pe}$
- In opposite limit, $\omega_{pe}\tau < 2\pi$ (RHIC II param's)
  - no screening of ion charge; choose $\rho_{max} \sim \max(v_i,\Delta_e)\,\tau$
  - or calculate Coulomb log with finite-length trajectories
    - completely removes logarithmic singularity at large ρ

$$\Lambda = \ln\left(\frac{\rho_{max}}{\rho_{min}} \sqrt{\frac{\rho_{min}^2 + d^2}{\rho_{max}^2 + d^2}}\right) \qquad \text{for} \quad \tau \to 0,\ \Lambda \to \ln(1) = 0$$

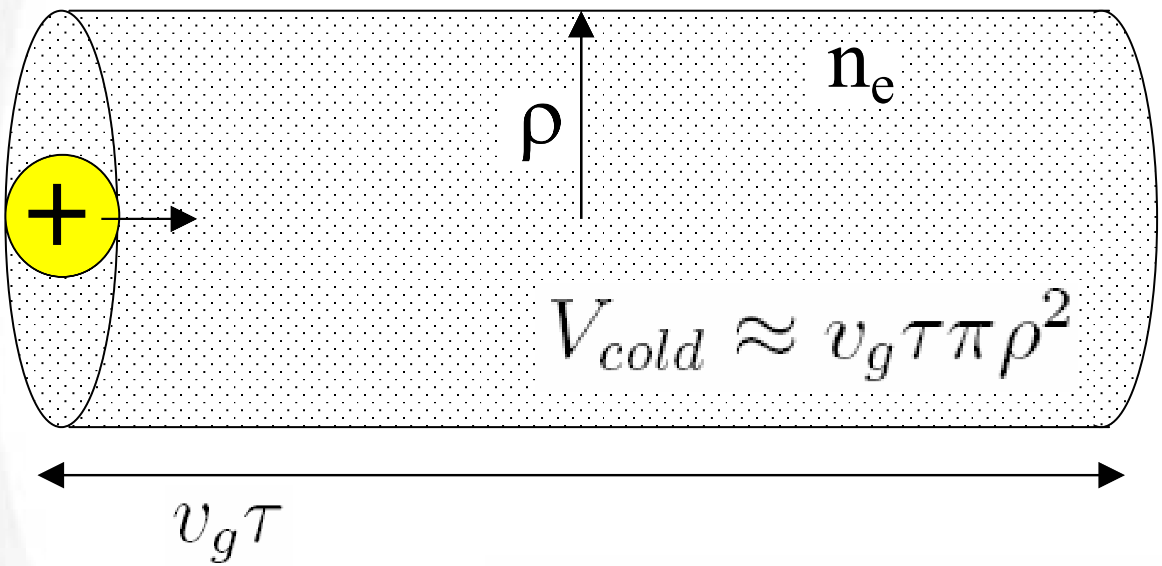
$$d = |\mathbf{v}_{rel}|\tau/2 \qquad \text{for} \quad \rho_{min} \ll d \ll \rho_{max} \quad \Lambda \to \ln(d/\rho_{min})$$

Finite-time effects limit # of collisions for small ρ

- Poisson statistics predict likelihood of N_c collisions
  - for all impact parameters less than ρ

$$P_k = \frac{\lambda^k}{k!} e^{-\lambda} \qquad P_{\geq 1} = 1 - P_0 = 1 - e^{-\lambda}$$

$$\lambda = n_e \langle v_{rel} \rangle \tau \pi \rho^2 = \mathrm{N_c}$$



$$v_g \rightarrow \langle v_{rel} \rangle (\Delta_x, \Delta_y, \Delta_z) = \int_{-\infty}^{\infty} d^3\mathbf{v}_e f(\mathbf{v}_e) |\mathbf{v}_{rel}|$$

Finite-time effects on ρ_{min} lead to concept of ρ_{res}

- Friction force integrals assume, for all ρ
  - there are plenty of trajectories to sample 4π *sr*
    - so perpendicular kicks average out
    - so longitudinal kicks accumulate correctly
  - and sufficient trajectories to sample e- velocities
- How many collisions are needed?
  - good agreement with simulations for $N_c = 120$

$$\rho_{res} = \sqrt{\frac{N_c}{\langle v_{rel} \rangle \tau \pi n_e}}$$

  - for RHIC II parameters shown above
    - $\rho_{min} \approx 2.2$ e-7 m $\rho_{res} \approx 1.8$ e-5 m
    - for $\rho_{max} \sim v_i\tau \approx 7.5$ e-4 m $\rightarrow$ >20% smaller Coulomb log

Review of Numerical Approach & Problem of Diffusion

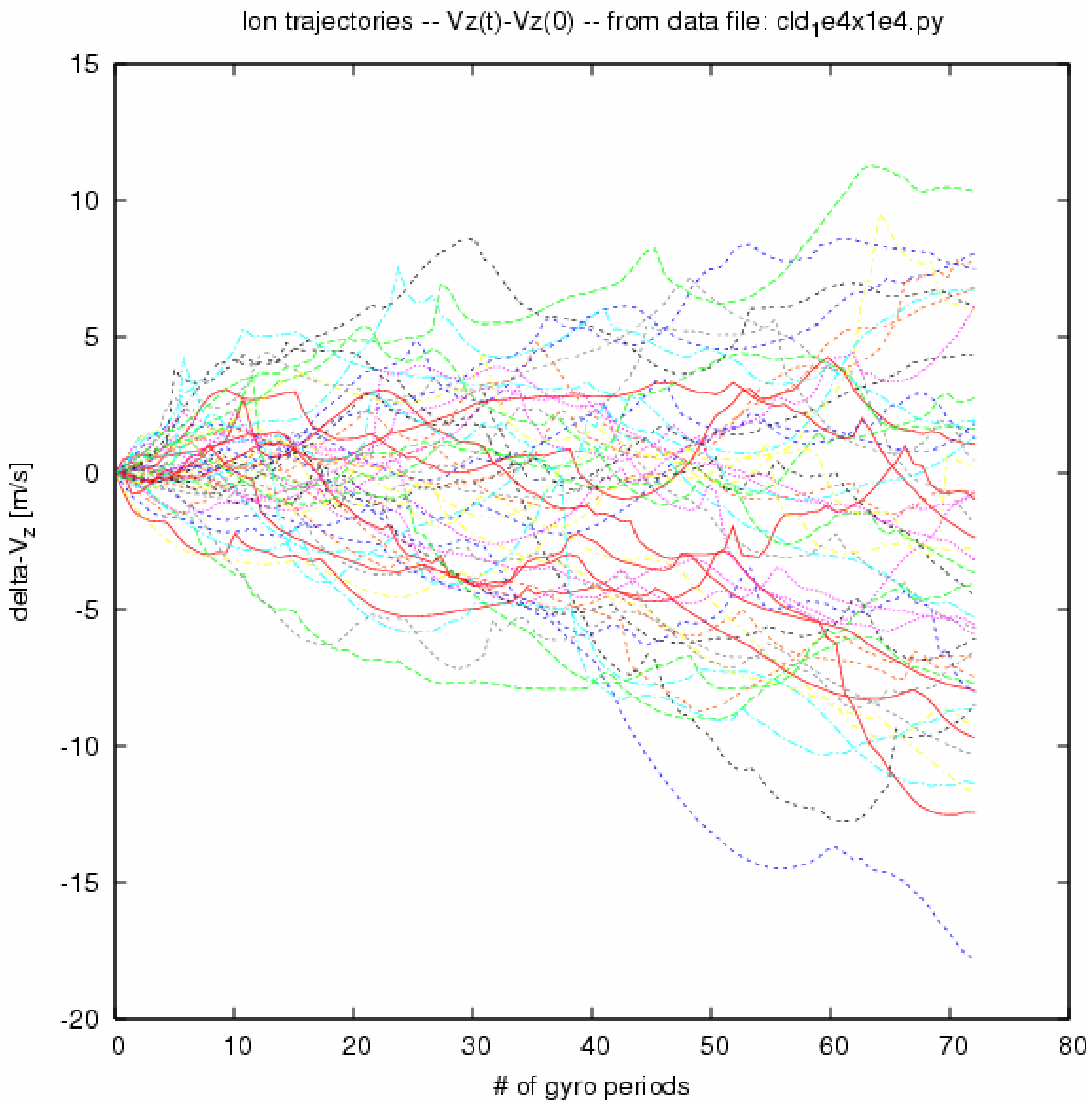
Numerical Approaches for Electron Cooling Simulations

- Langevin approach to solve Fokker-Planck equation
  - uses Rosenbluth potential (or Landau integral)
- Fast multipole method (FMM) and tree-based algorithms
  - requires constant time step; inefficient for MD with a few close collisions
- 4th-order predictor-corrector "Hermite" algorithm
  - taken from astrophysical dynamics community
  - generalized to include solenoid field
  - used successfully in molecular dynamics (MD) approach with a few ions
  - didn't parallelize well, so we used a task farming approach
    - astrophysicists use special "Grape" hardware to parallelize
- Semi-analytic binary collision model
  - also MD approach; very close connection to "Hermite" algorithm above
  - accurately models arbitrarily strong Coulomb collisions
  - arbitrary external fields included via 2nd-order operator splitting
  - scales well up to ~128 processors
- Electrostatic particle-in-cell (PIC)
  - very difficult to capture close Coulomb collisions (fine mesh, noise)
  - can rely on PETSc/Aztec00 for effective use of petascale hardware
  - possibility to combine with "binary collision" model

Molecular dynamics approach – model each e-

Diffusive spreading of ion trajectories obscures any velocity drag due to dynamical friction.

For many millions of turns, friction forces will dominate diffusion.



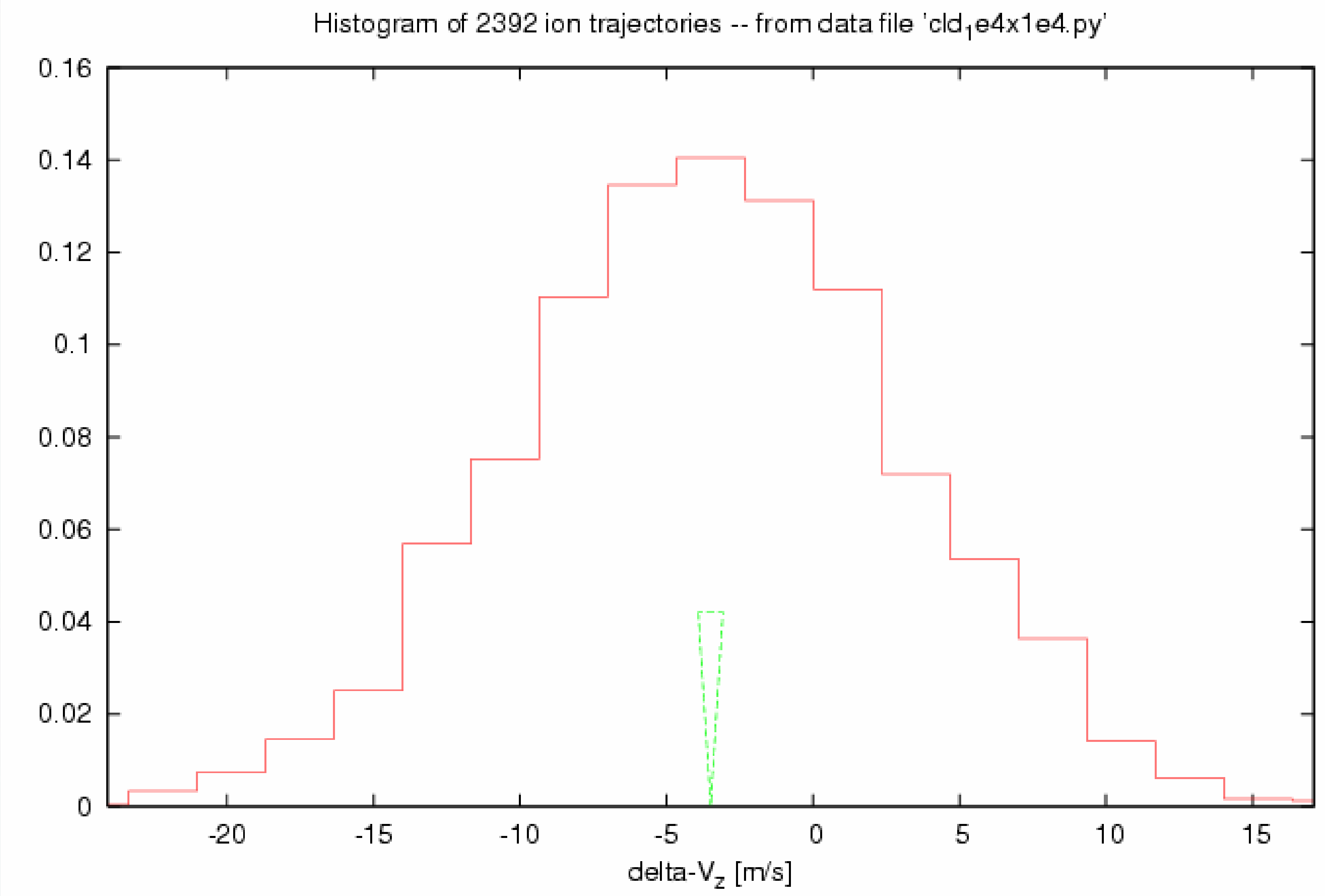
Diffusive dynamics can obscure friction/drag

- Numerical trick of e-/e+ pairs can *suppress diffusion*
  - idea came from Alexey Burov
  - simulate with e-/e+ pairs that have identical initial conditions
    - sign of external fields must be flipped for the positrons
  - friction force, independent of sign of charge, is unchanged
  - diffusive kicks are approximately cancelled
- also use ~1,000 trajectories for each electron
  - RMS is reduced by $N_{traj}^{1/2}$ from that of the original distribution
    - from the Central Limit Theorem
- Use of many trajectories *sometimes* changes results !!
  - always true for field free case:

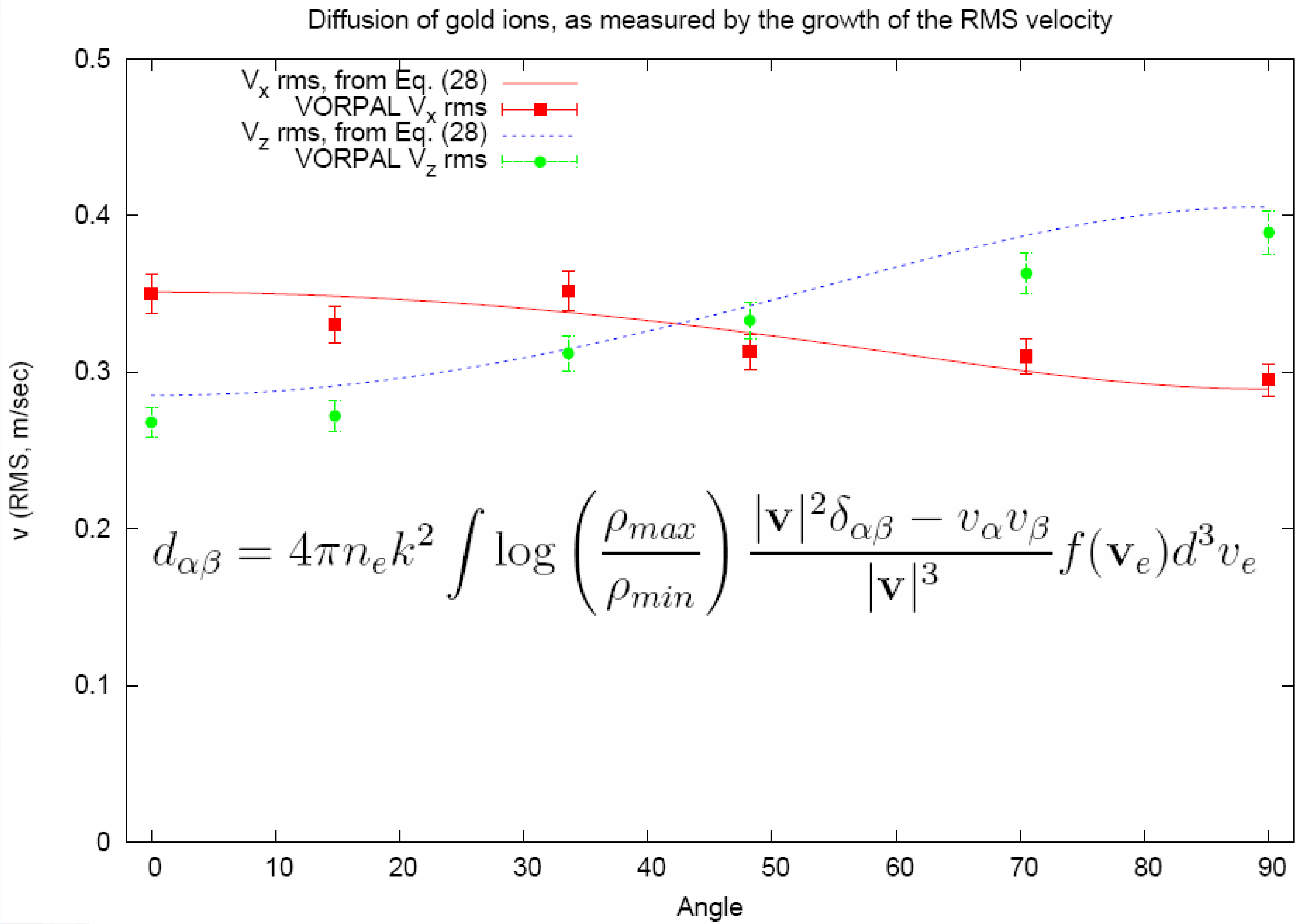
$$\rho_{res,sim} = \sqrt{\frac{N_c / N_{traj}}{\langle v_{rel} \rangle \tau \, \pi \, n_e}} = \rho_{res} \Big/ \sqrt{N_{traj}}$$

  - in presence of external fields
    - some other limitation on ρ_{min} may hide this problem
    - also,higher effective e- velocity can *decrease* ρ_{res}

CLT is used to pull <F> and error bars from binned data



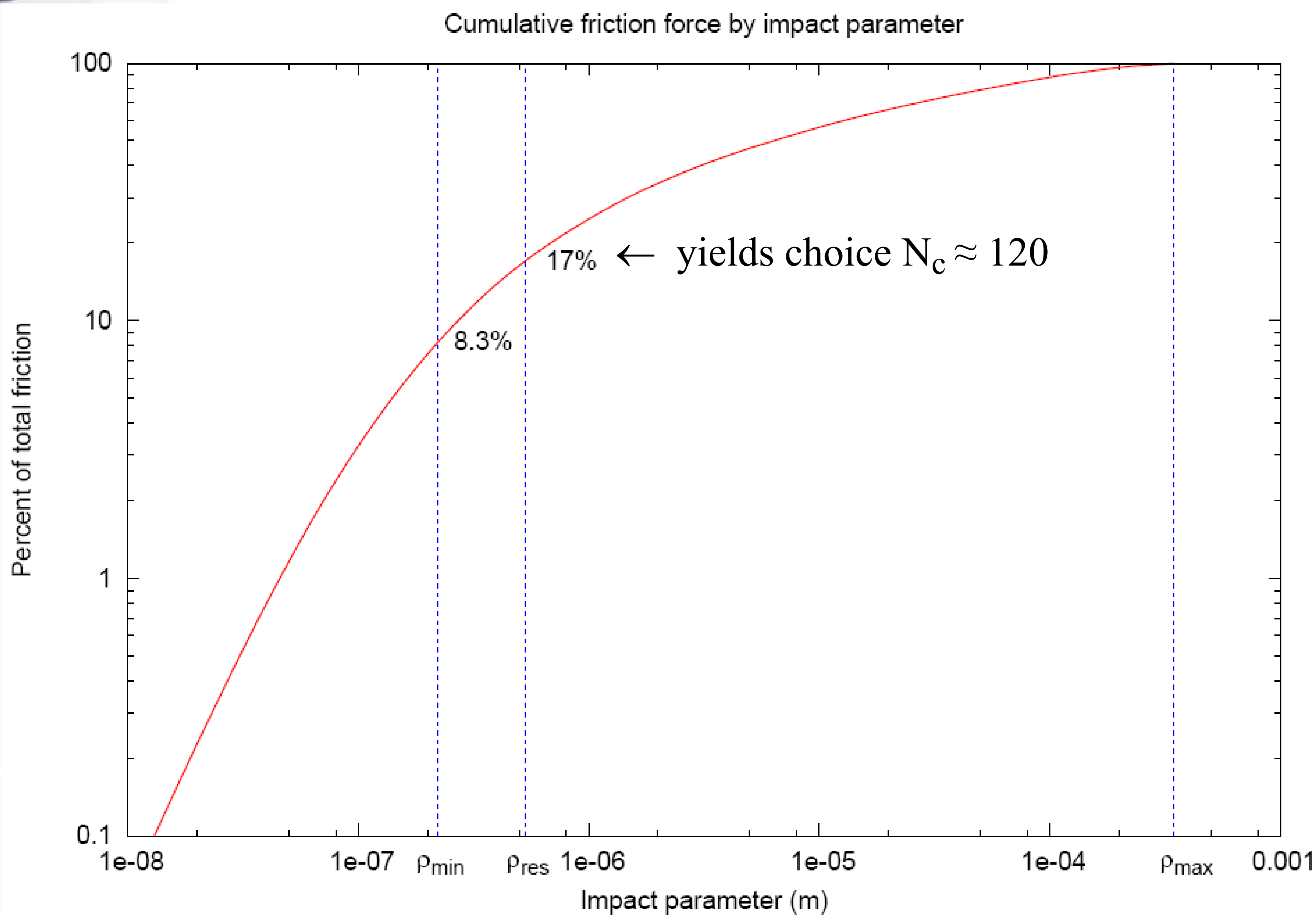
Diffusion vs. Dynamical Friction



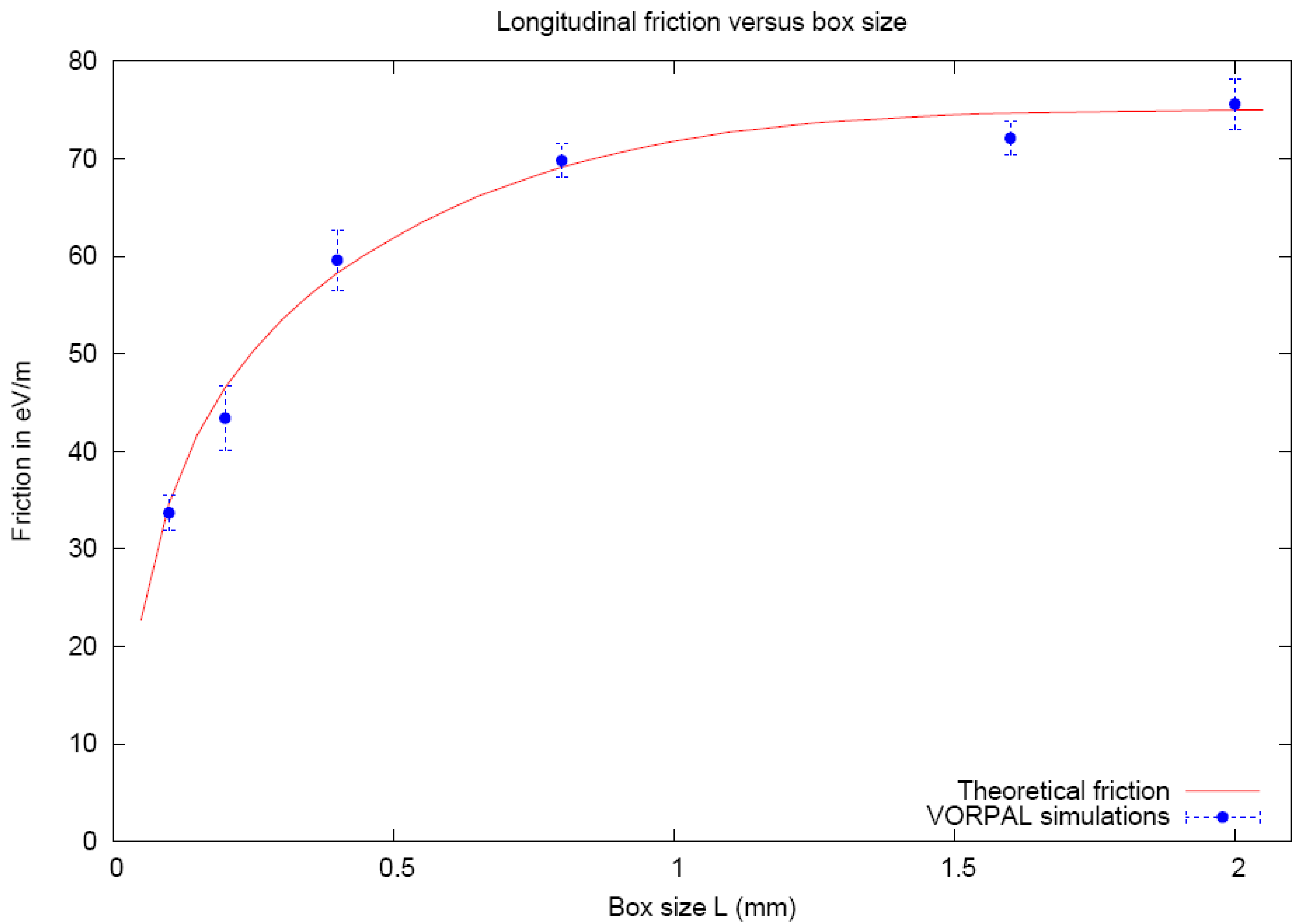
$$d_{\alpha\beta} = 4\pi n_e k^2 \int \log\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{|\mathbf{v}|^2 \delta_{\alpha\beta} - v_\alpha v_\beta}{|\mathbf{v}|^3} f(\mathbf{v}_e) d^3 v_e$$

Finite-time effects on field-free simulations

Failure to sample small ρ reduces friction force



Numerical effects of domain size are understood



Effects of Helical Undulator

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RHIC II

eRHIC

Unmagnetized high-energy cooling w/ undulator

- Purpose of the helical undulator magnet
  - provides focusing for electrons
  - suppresses e-/ion recombination
    - Modest fields (~10 Gauss) effectively reduce recombination via 'wiggle' motion of electrons:

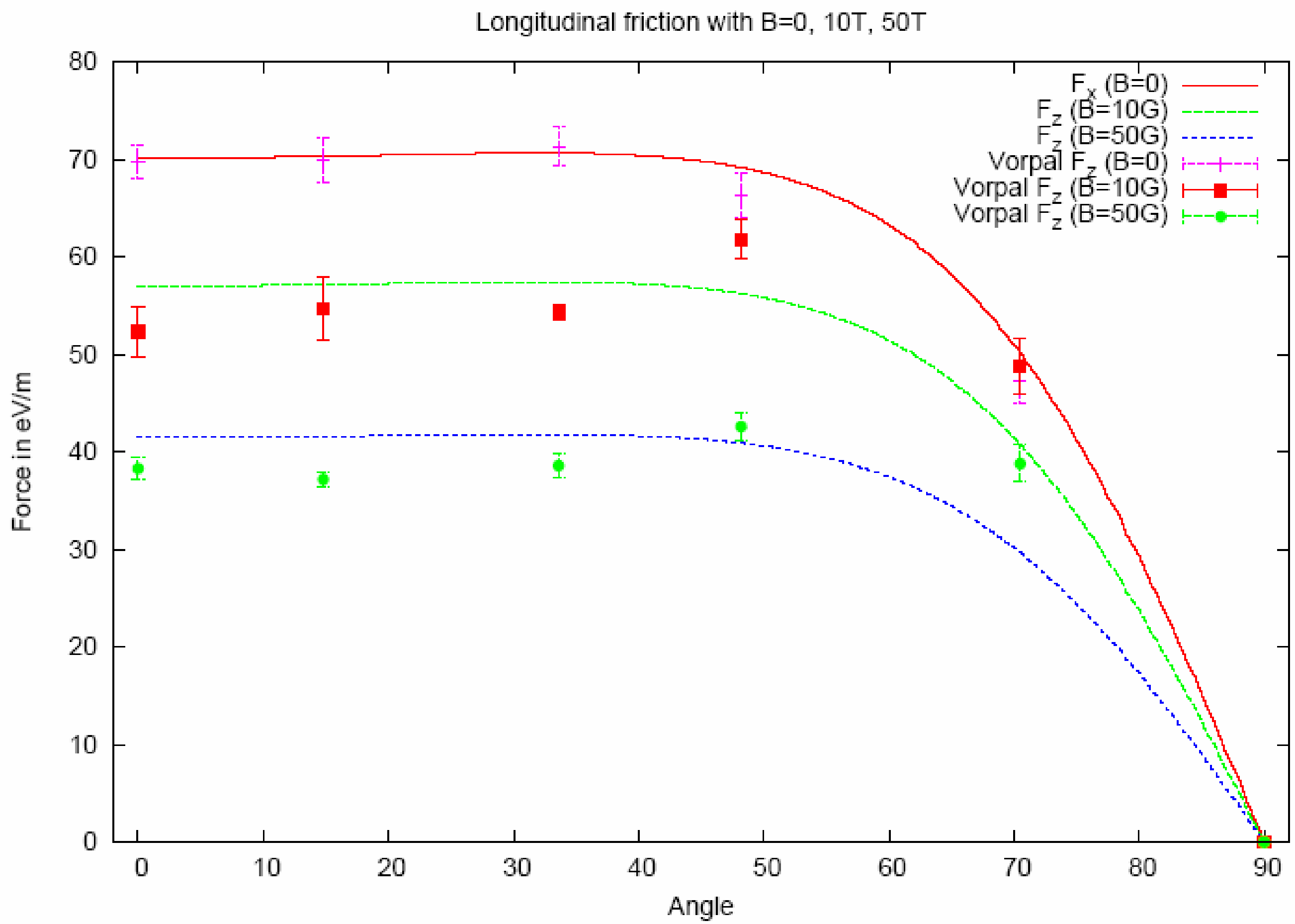
$$\rho_{osc} = \frac{\Omega_{gyro}}{k_w^2 v_{beam}} \sim 1.4x10^{-3} \lambda_w^2[m] B_w[G] / \gamma$$

- What's the effect of 'wiggle' motion on cooling?
  - increases minimum impact parameter of Coulomb log

$$\rho_{\min} \rightarrow \max(2\rho_{osc}, \rho_{res})$$

VORPAL simulations verify effect of undulator

- Coherent e- wiggle motion
  - increases the effective minimum impact parameter
  - dynamical friction force is only reduced logarithmically



Effects of Magnetic Field Errors

Simple model of single-wavelength error fields

$$B_y(x,y,z) \approx B_{err}\sin(2\pi z/\lambda)$$

$$M \equiv \max\left[\int_{-\infty}^{z} dz\, B_y(x,y,z)\right] = \lambda B_{err}/2\pi$$

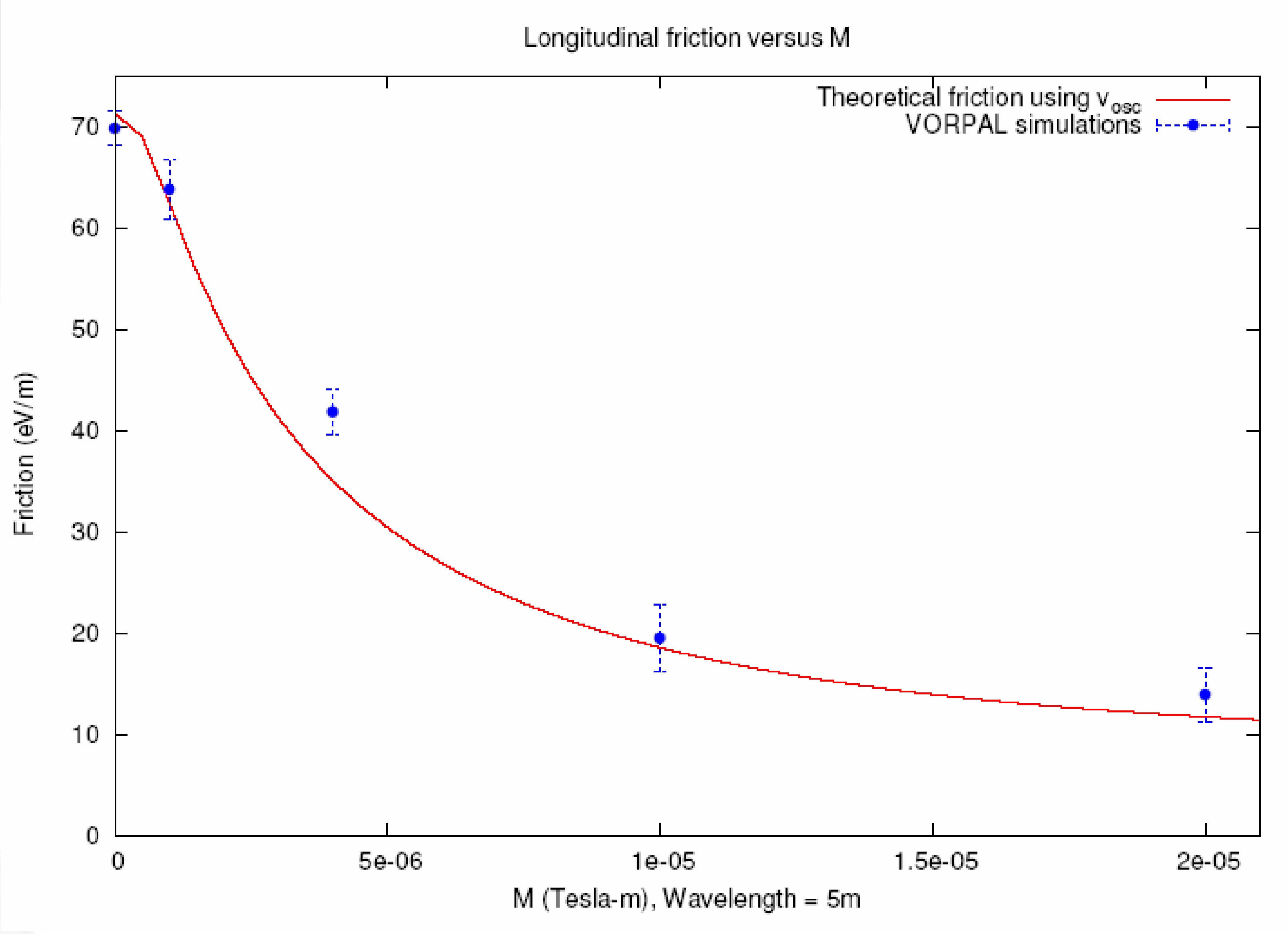
$$B_{err} = 2\pi\, M/\lambda \qquad 10\,\mathrm{cm} < \lambda < 80\,\mathrm{m} \qquad 10^{-6}\,Tm < M < 2x10^{-5}\,Tm$$

Lorentz transform to beam frame: $E'_x(x,y,t) \approx \beta\gamma c B_{err}\sin(2\pi\beta ct/\lambda)$

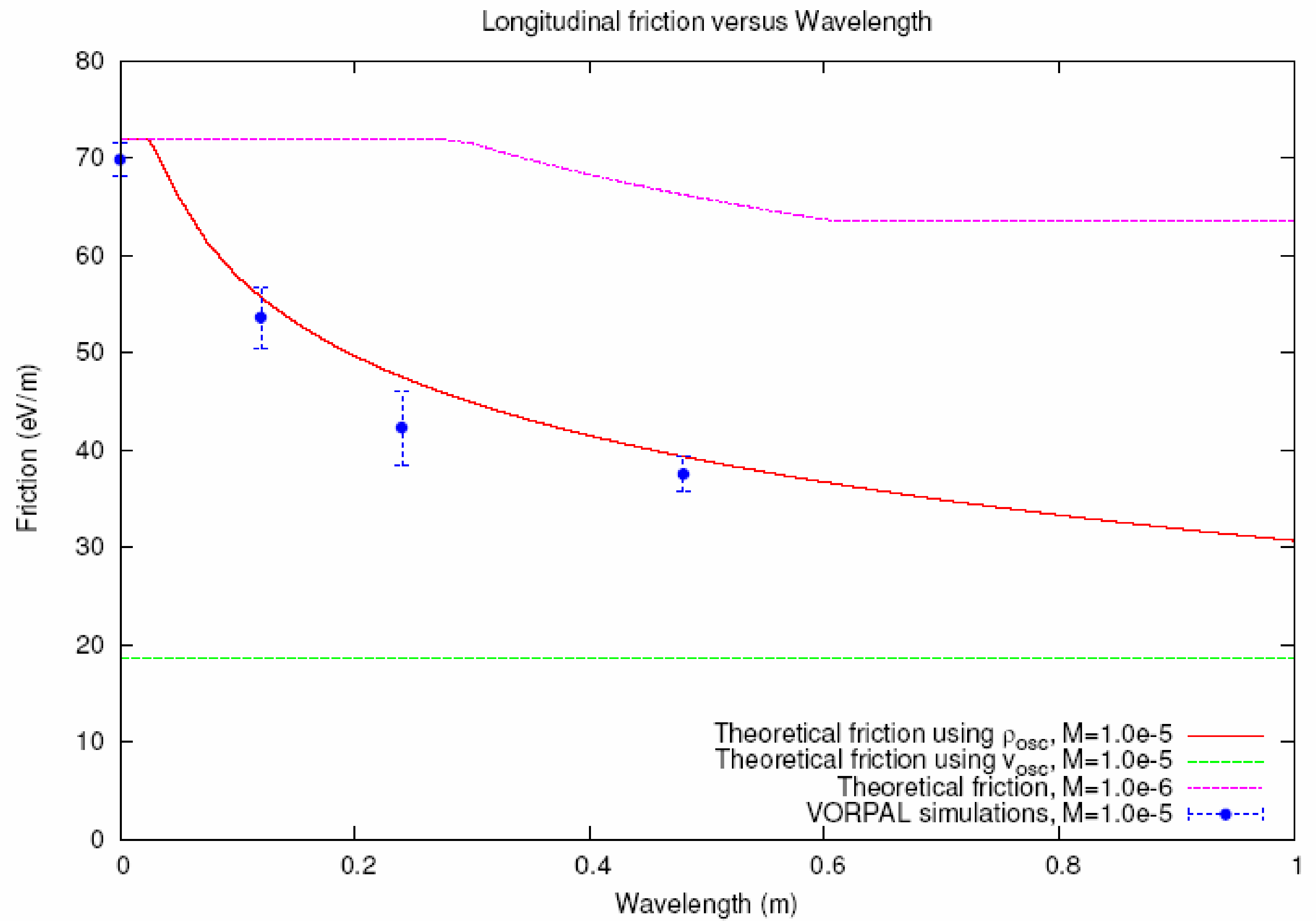
e- oscillations: $v_{err} = \dfrac{e}{m_e} M \qquad \left(\approx 2x10^{5}\ m/s\ \text{ for } M = 10^{-6}\,Tm\right)$

$$x_{err} = \frac{\lambda}{2\pi\beta\gamma c} v_{\mathrm{err}} \qquad \left(\approx 9x10^{-7}\,m\ \text{ for } M = 10^{-6}\,Tm,\ \lambda = 1\,m\right)$$

Long wavelengths increase effective e- temp



Short wavelengths increase effective ρ_{min}



Conclusions and Future Work

- Finite-time effects reduce field-free friction force
  - modified Coulomb log should be used
    - approximately equivalent to choosing $\rho_{max} \sim v_{rel}\, \tau / 2$
    - effective ρ_{max} is limited by simulation box size (understood)
  - $\rho_{eff} \rightarrow \max(\rho_{min}, \rho_{res}, 2\rho_{osc}, 2x_{err}, \ldots)$
    - effect of ρ_{res} was not understood previously
    - >20% effect for RHIC II parameters
    - ρ_{res} artificially decreased by use of many trajectories in simulations
      - this complication needs to be removed by improved noise reduction
- Effect of helical undulator magnets is understood
  - friction reduced logarithmically; $\rho_{min} \rightarrow \max(\rho_{res}, 2\rho_{osc})$
- Single-wavelength error field simulations
  - short wavelengths: $\rho_{min} \rightarrow 2x_{err}$ (logarithmic)
  - long wavelengths: $v_{e,rms} \rightarrow \sqrt{v_{e,rms}^2 + v_{err}^2}$
- New understanding should be included in BETACOOL

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