Yury Senichev

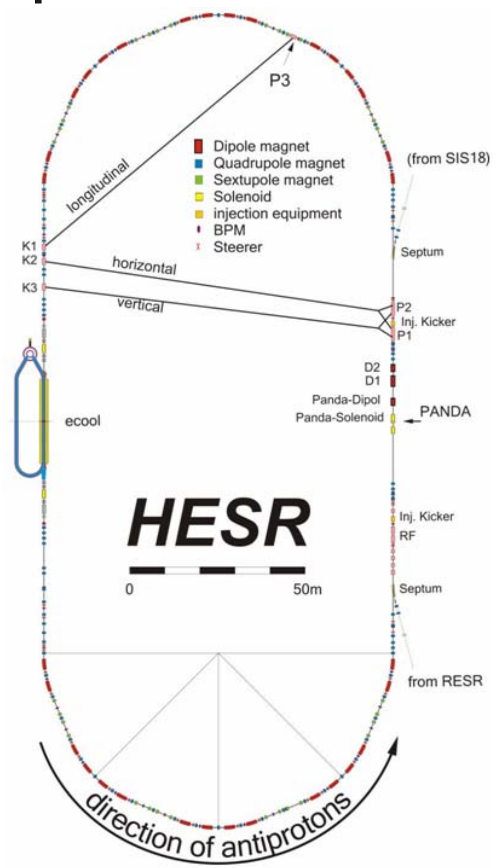
The advanced HESR lattice for improved stochastic cooling

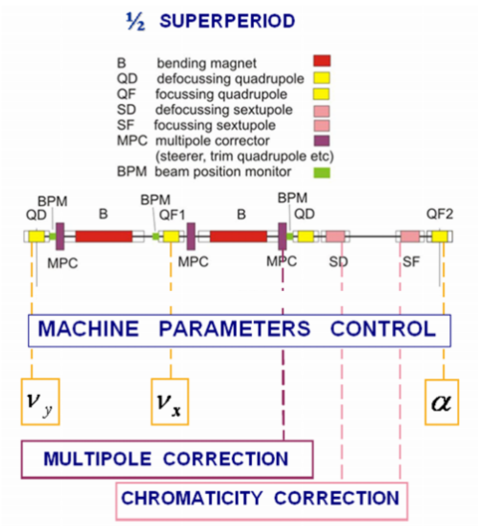
From D. Möhl "Stochastic Cooling" CERN 95-06 v.II:

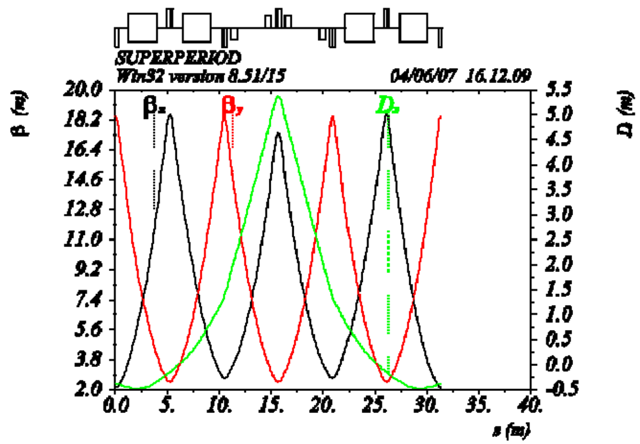
- "By a clever choice of the bending and focusing properties of the storage ring it is possible, in principle, to make ΔTpk->0 independent of momentum, and ΔTkp large to approach the desired situation of 1/Mpk=0 and Mkp=1.

  But this complicates the storage ring lattice. The compromise adopted in existing ring is to sacrifice some of the desired re-randomisation in order too much unwanted mixing"

HESR lattice with curvature and gradient modulation







Arcs with different slip factor: $\eta = 1/\gamma_{tr}^2 - 1/\gamma^2$ and $\eta = -1/\gamma_{tr}^2 - 1/\gamma^2$

- From the article: Yu. Senichev, A "resonant" lattice for a synchrotron with a low or negative compaction factor, KEK Preprint 97 40, 1997

$$K(\tau) = K_0(\tau)\left[1 + \sum_{k=0} g_k \cos kS\tau\right]$$

$$\frac{1}{\rho(\tau)} = r_0 + \sum_{k=1} r_k \cos kS\tau$$

$$\alpha = \frac{1}{\gamma_{tr}^2} = \frac{1}{\nu^2}\left\{1 + \frac{1}{4\cdot(1-kS/\nu)}\cdot\left[\left(\frac{\overline{R}}{\nu}\right)^2 \frac{g_k}{[1-(1-kS/\nu)^2]} - r_k\right]^2\right\}$$

- The lattice has the remarkable feature: the gradient and the curvature modulation amplify each by other if they have opposite signs , and on the contrary they can compensate each other when they have the same sign.

Three conditions for gradient and curvature modulation:

- the quadrupole gradient is modulated with the opposite sign

$$g_k \cdot r_k < 0$$

- and the value determined by the gradient modulation when the ratio between them is:

$$|r_k| \le \left(\frac{\overline{R}}{\nu}\right)^2 \left| \frac{g_k}{1-(1-kS)^2} \right|$$

- it is desirable to get such ratio:

$$\frac{1}{4(kS/\nu - 1)} \cdot \left(\frac{g_k}{[1-(1-kS/\nu)^2]} - r_k \right)^2 \approx 2$$

Real and Imaginary arcs

- The momentum compaction factor in imaginary and real arcs takes the meaning:

$$\alpha_{kp} = -\frac{1}{4\nu_{arc}^2} \qquad \alpha_{pk} = \frac{1}{4\nu_{arc}^2}$$

- and slip factors:

$$\eta_{pk} = \frac{1}{\gamma^2} - \frac{1}{4\nu_{arc}^2}$$

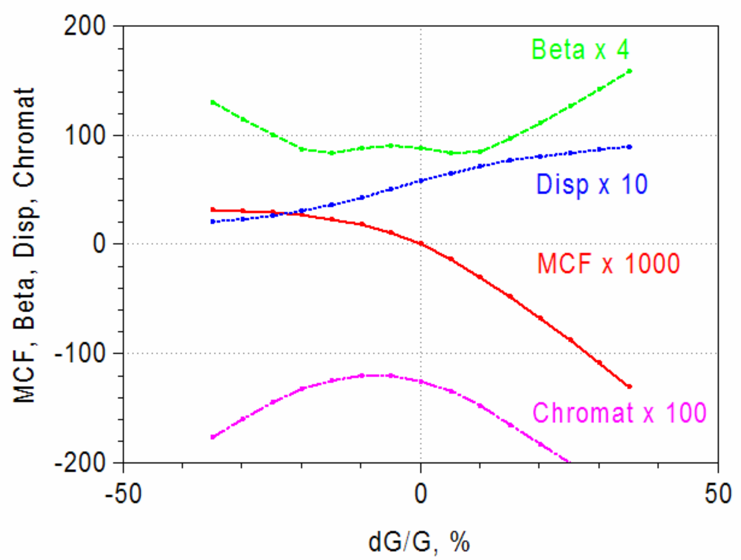
$$\eta_{kp} = \frac{1}{\gamma^2} + \frac{1}{4\nu_{arc}^2}$$

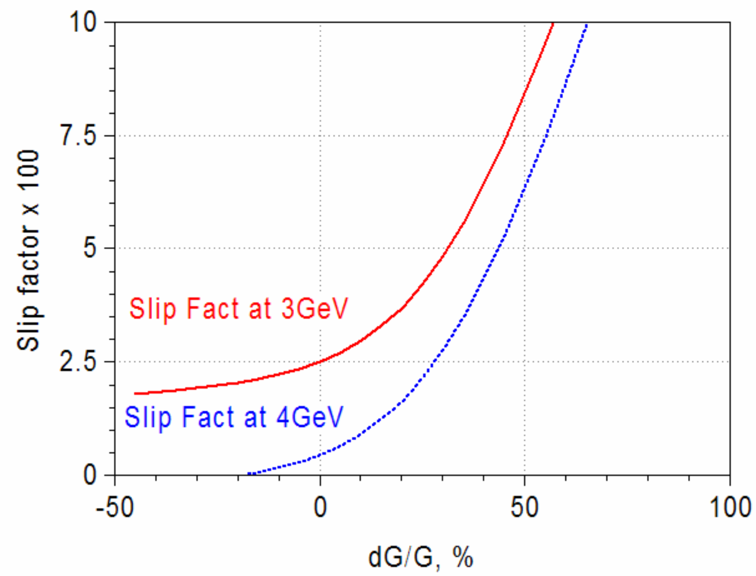
In case $\gamma \approx 2\nu_{arc}$:

the real arc is isochronous $\eta_{pk} \approx 0$

the imaginary arc has a slip factor $\eta_{kp} \approx 1/2\nu_{arc}^2$

The mixed functions modulation

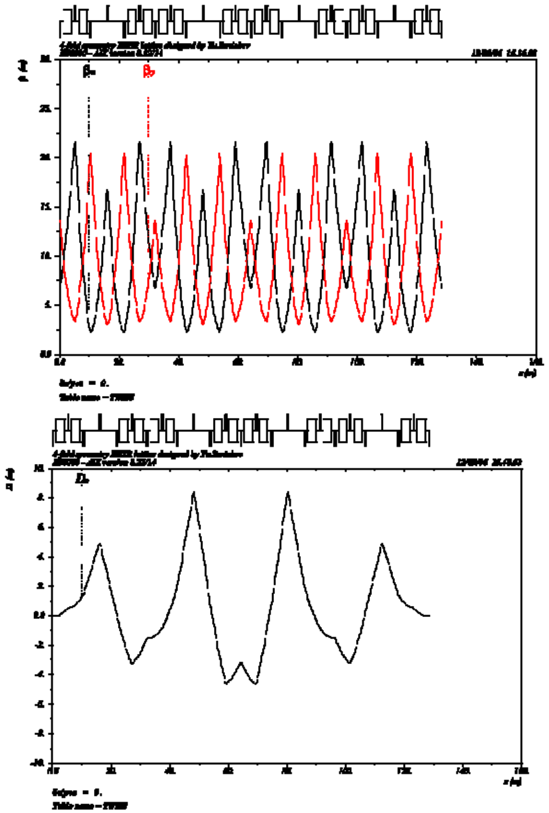




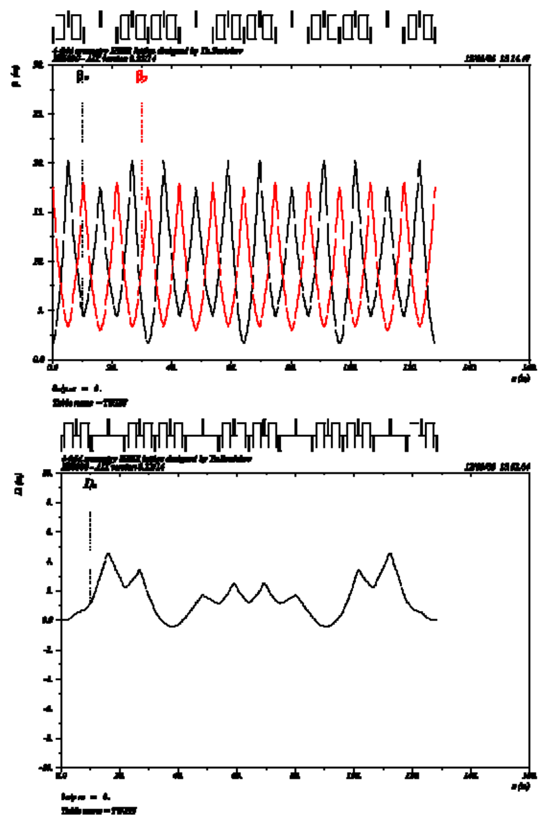
Twiss parameters of 4 fold symmetry arcs

- The β-function and dispersion on the imaginary, the real 4-fold symmetry arcs

IMAGINARY

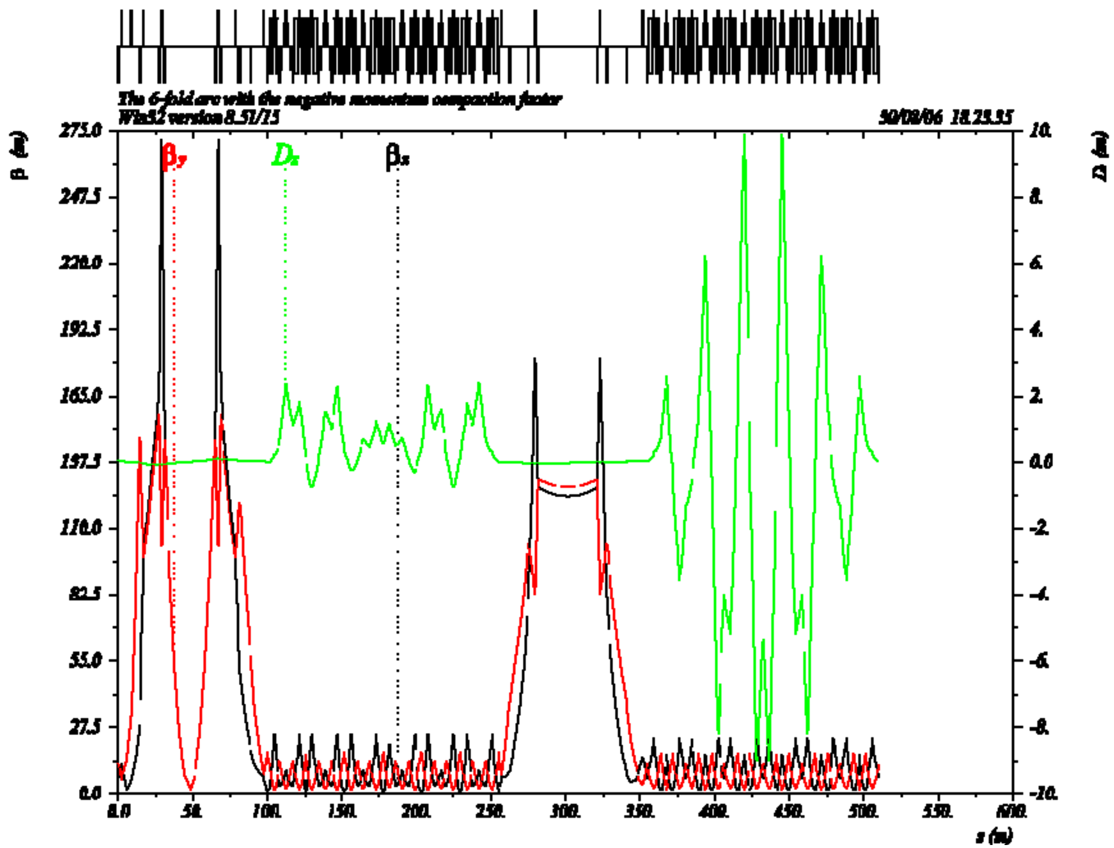


REAL



The total momentum compaction factor of whole ring is negative.

It satisfies to the Keil-Schnell stability criteria



Main advantage of this lattice

- In each arcs the integral of errors -> 0 and each arc is absolutely decoupled with another
- Due to the same geometry of both arcs the set of the structure resonances is not changed

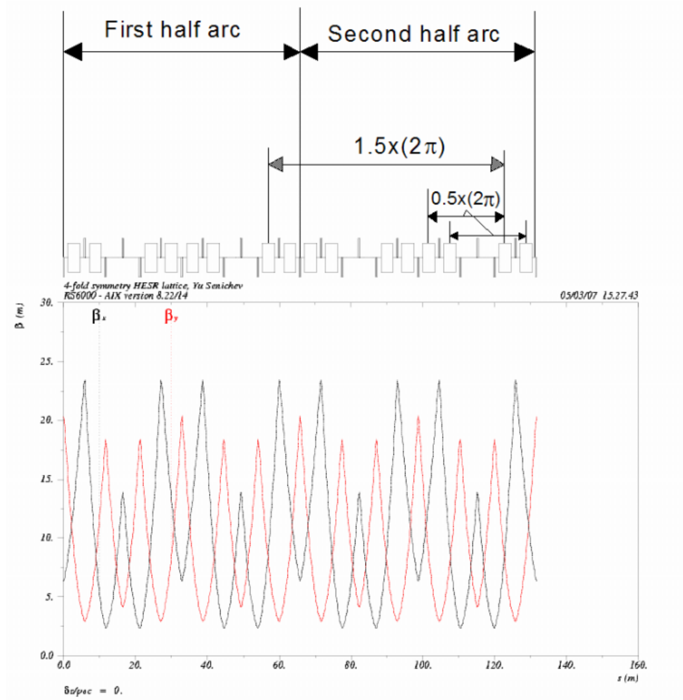
$$\left\langle h_{k_x,k_y,p} \right\rangle \propto \int_0^C \beta_x^{k_x/2} \beta_y^{k_y/2} \frac{\partial^{(k_x+k_y-1)} B_{y,x}}{\partial(x,y)^{(k_x+k_y-1)}}(s) \exp i(k_x\mu_x + k_y\mu_y) ds$$

Compensation of sextupole non-linearity

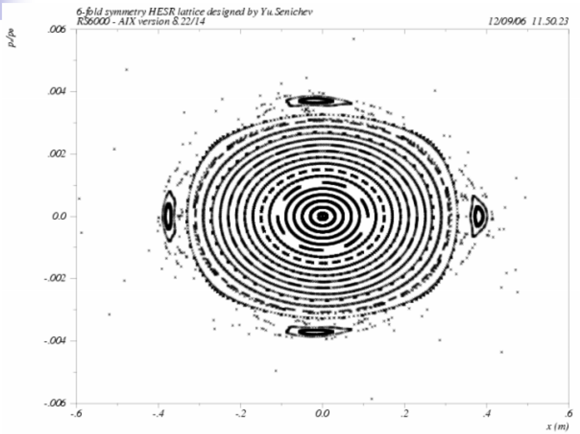
- the total multipole of third order is canceled:

$$M_3^{total} = \sum_{n=0}^{N} S_{x,xy} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0$$

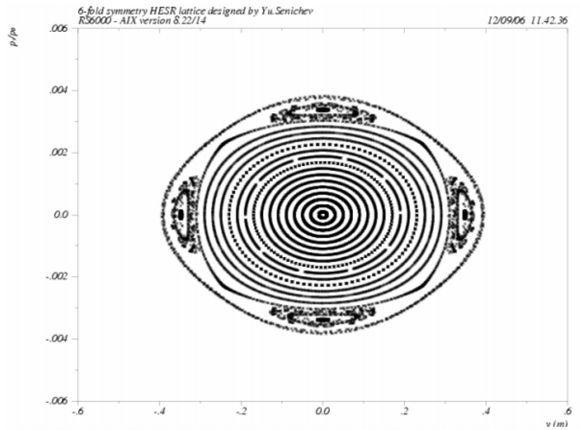
- If super period number is even and arc tunes are odd then we have an exact condition for compensating each sextuplet's non-linear action by another one.



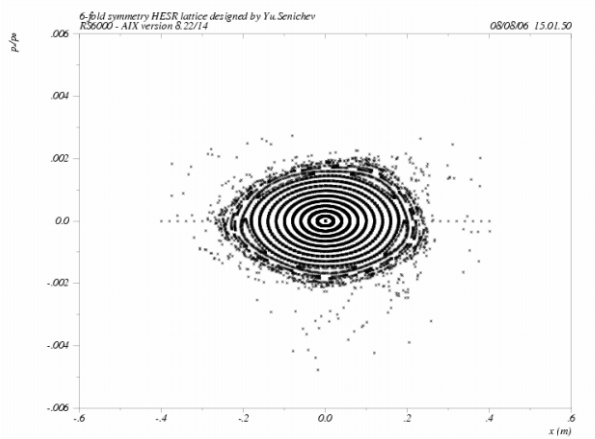
Dynamic aperture of structure with similar and non similar arcs



Similar arcs:
DA=900 mm mrad

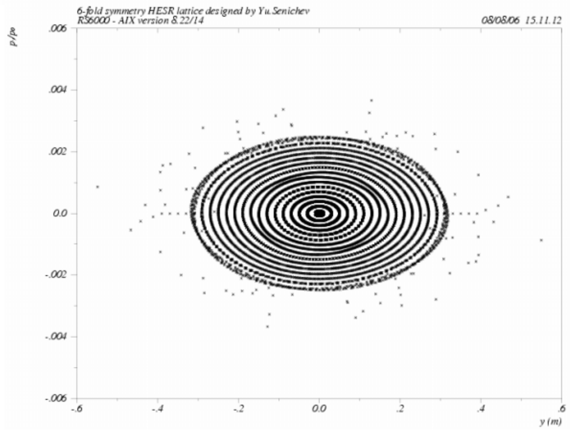


Physical
acceptance of arcs
~ 100 mm mrad



Non-similar arcs:

DA=400 mm mrad



As conclusion: The advanced lattice for the stochastic cooling with imaginary and real arcs was developed with features

- ability to achieve the negative momentum compaction factor using the resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_{tr}=\nu_x$ to $\gamma_{tr}=i\nu_x$ with quadrupole strength variation only;
- integer odd 2π phase advance per arc with even number of superperiod and dispersion-free straight section;
- independent optics parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and as consequence low sensitivity to multipole errors and a large dynamic aperture