Antiproton Production and Accumulation in Fermilab

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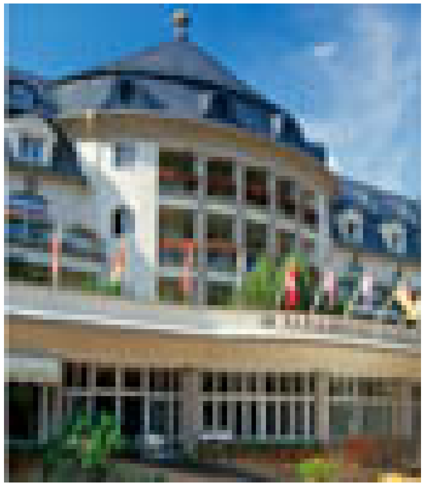


COOL[07]

Bad Kreuznach

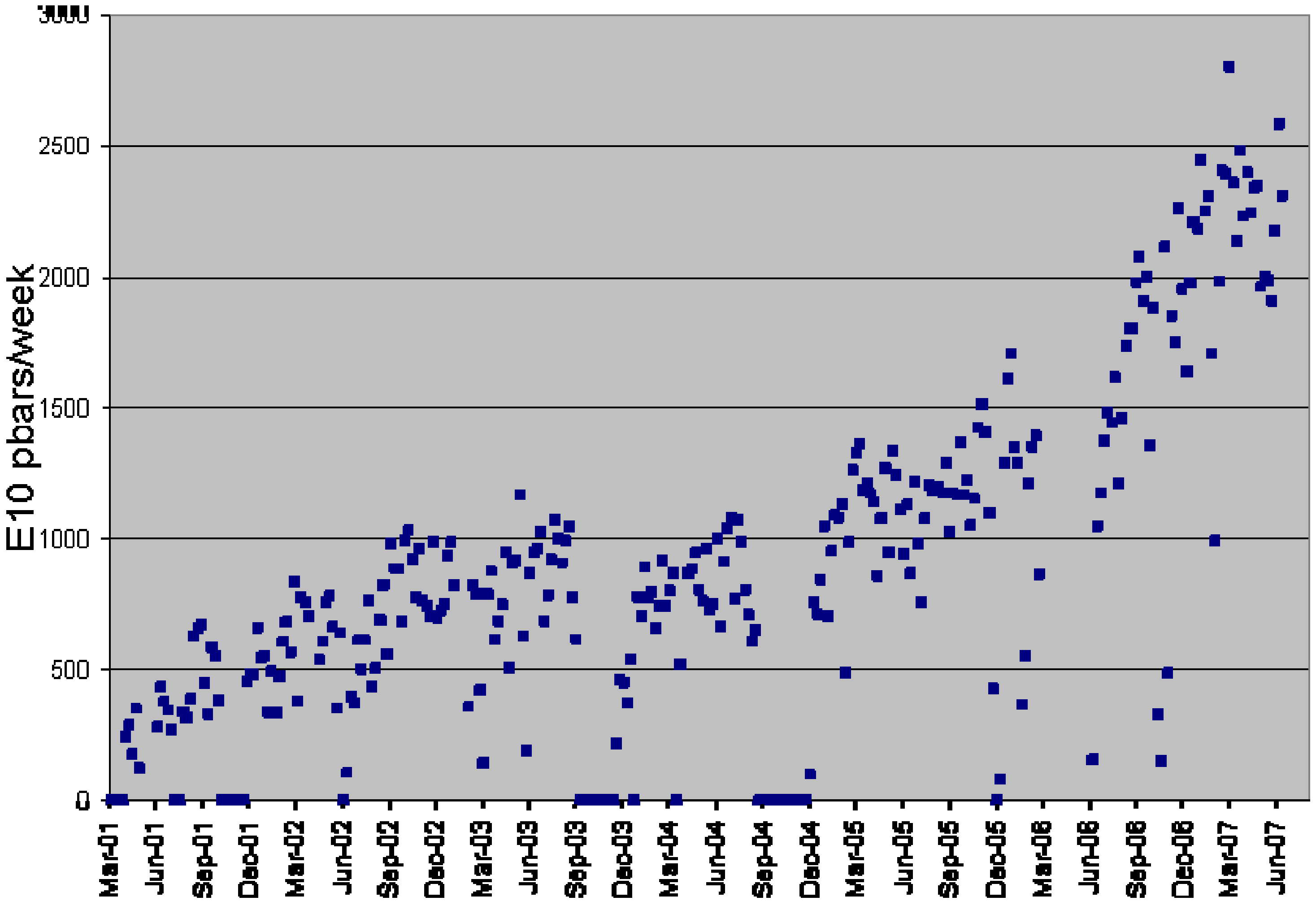
Germany

September 10-14, 2007

Contents

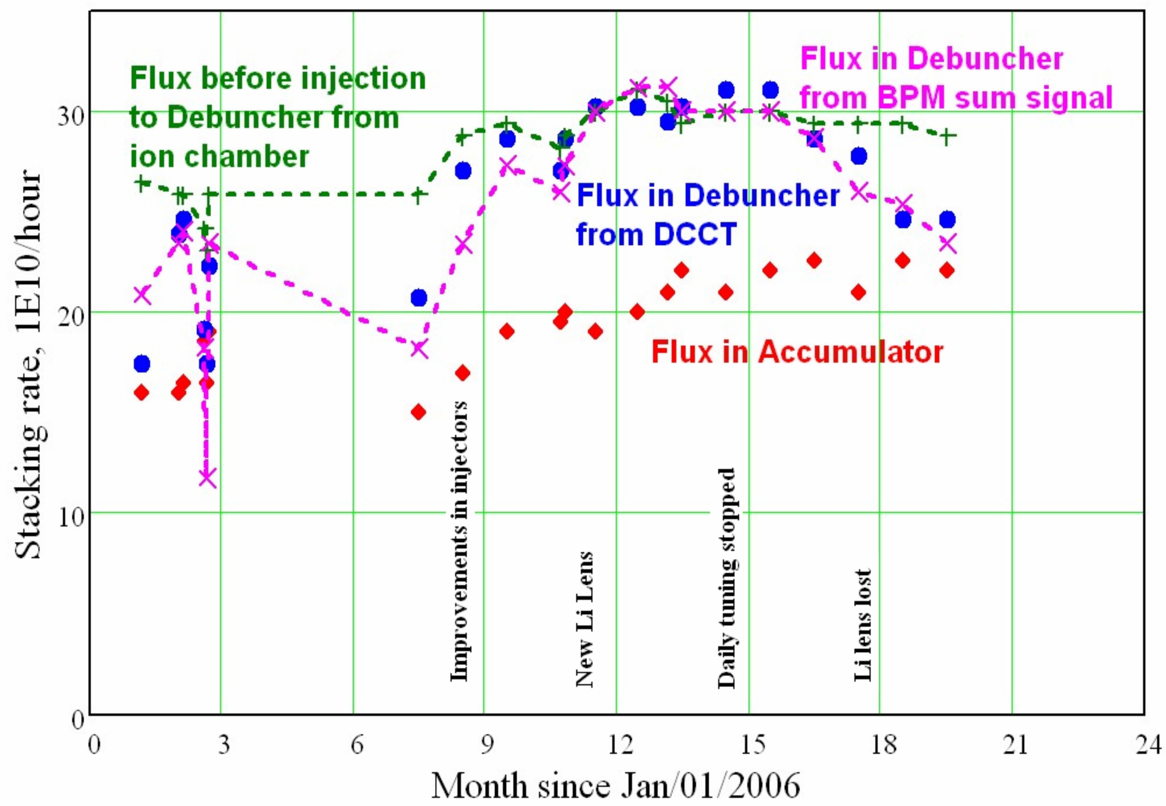
Introduction

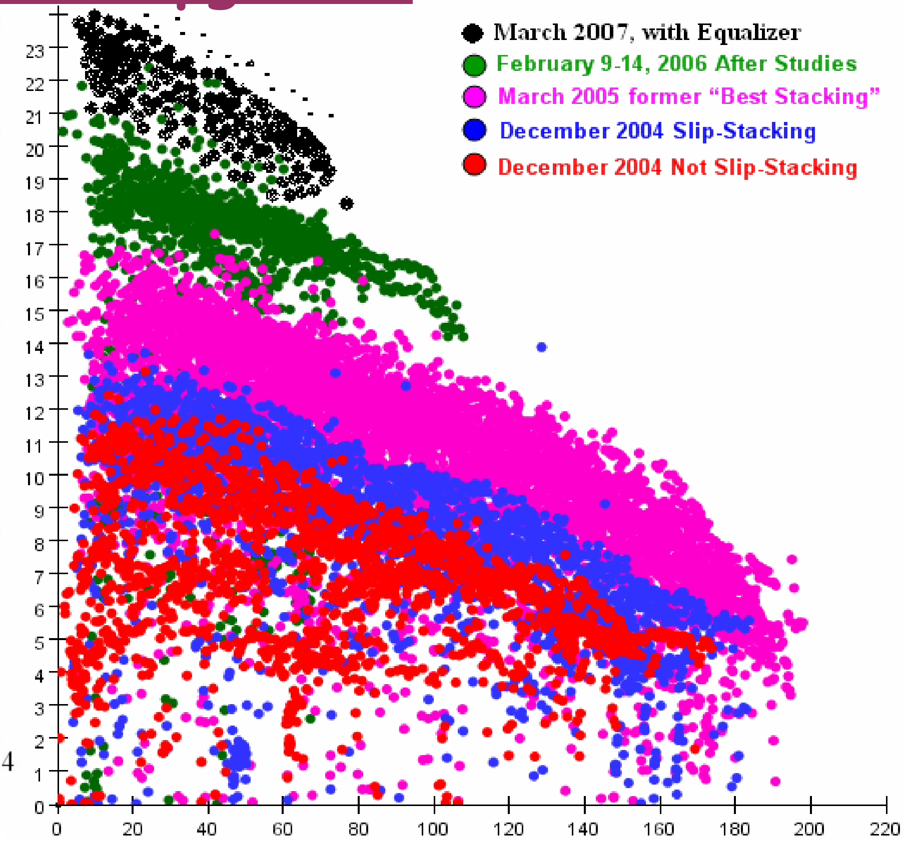
- Great progress with improvement of antiproton production in FY'07
  - ~1.7 times for the weekly production and the weekly luminosity integral
- Two major constituents
  - Stacktail improvements
  - Fast Accumulator-to-Recycler transfers



Weekly antiproton production rate during Run II (2001-2007)

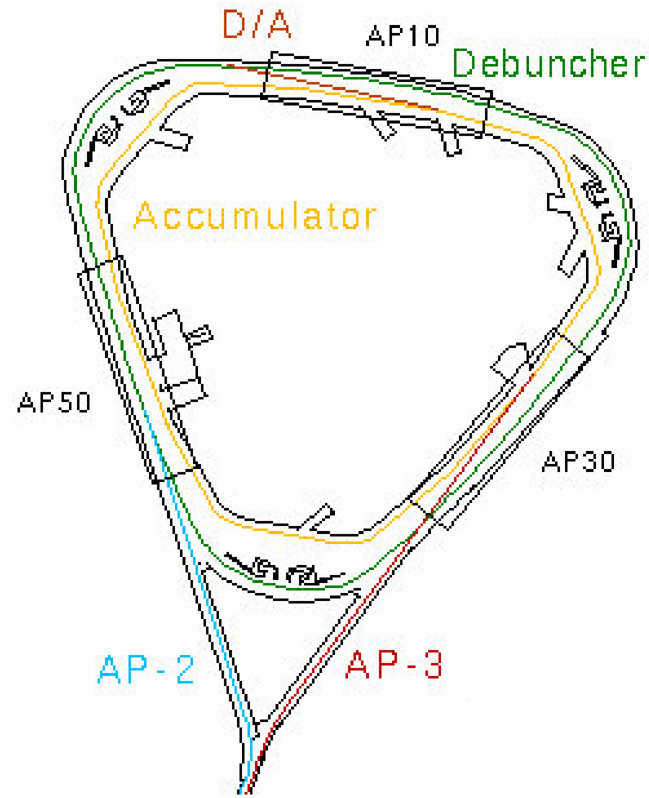
2006 - 2007 stacking improvements and upgrades





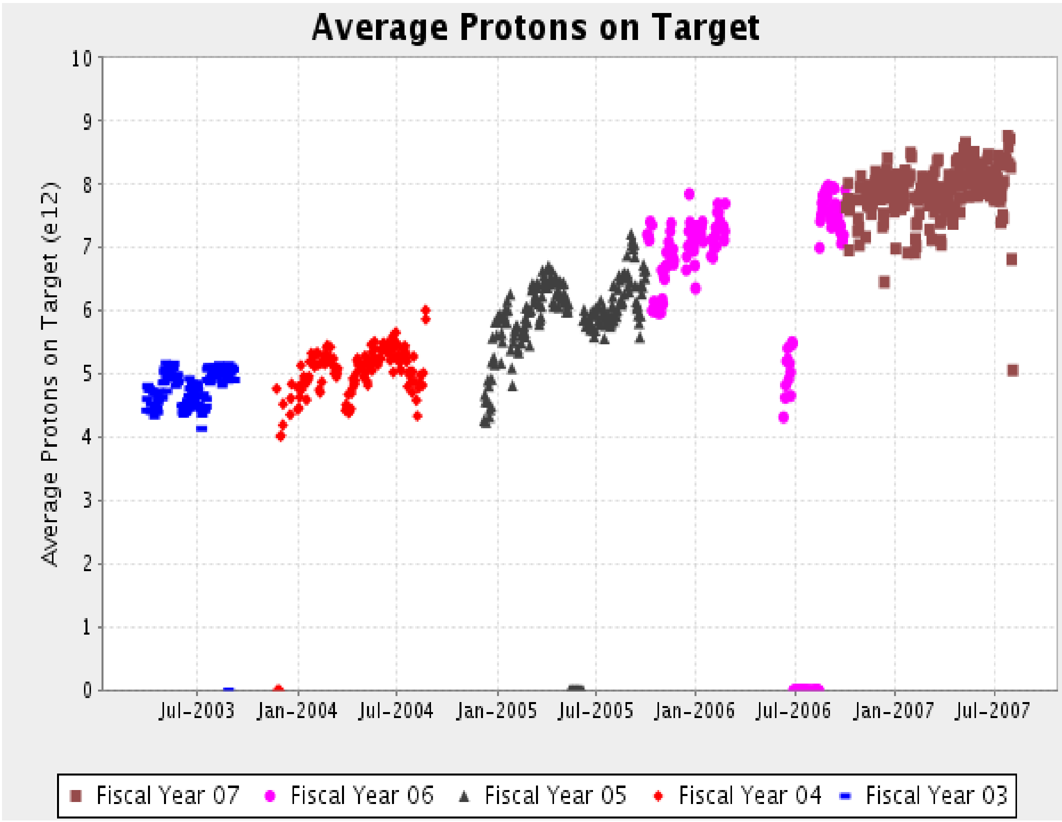
- Dec'05-Deb. optics and steering
- Feb'06 – Larger gain for 4-8 long. core cooling; 18->20 mA/hour
- July–Aug/06 - Tuning injector chain - pre-shutdown param. restored
- Oct. 1 - Stacktail polarity flip ⇒ peak st. rate: 20 ⇒ 22 mA/hour
- Dec. - New Li-lens
- March: Equalizer prototype for stacktail: 22 ⇒ 24 mA/hour
- May: Accumulator optics change
- June: Final Equalizer for stacktail
- July – Notch filter #3: BAW (Bulk Acoustic Wave) ⇒ SC (superconducting)

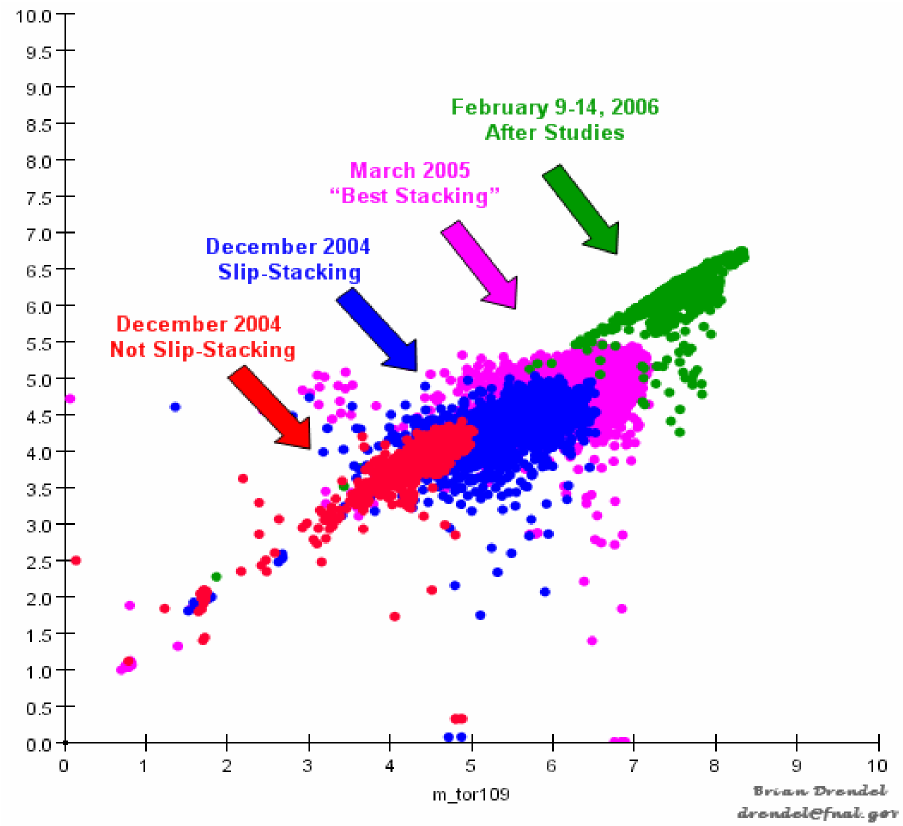
Simplified Review of Operations



- Every 2.4 s $8\cdot10^{12}$ protons at 120 GeV from MI sent to the target of ~10 cm length
- LI lens located at ~30 cm from target (center-to-center) reduces initially large angular spread
- 8 GeV (±2.5%) antiprotons and other secondaries (μ^-, π^-, …) are transported to Debuncher, N_{pbar}~$2\cdot10^8$, acceptance $\varepsilon\approx$35 mm mrad
- After 6D stochastic cooling in Debuncher antiprotons are sent to Accumulator
- 4 stochastic cooling systems (stacking, long. core, H and V) are used to stack and cool antiprotons
- After storing ~$50\cdot10^{10}$ antiprotons in Accumulator (~3 hour) they are sent to Recycler
- ~$300\cdot10^{10}$ antiprotons are stored in Recycler (~24 hour) and then sent to Tevatron

Protons on Target



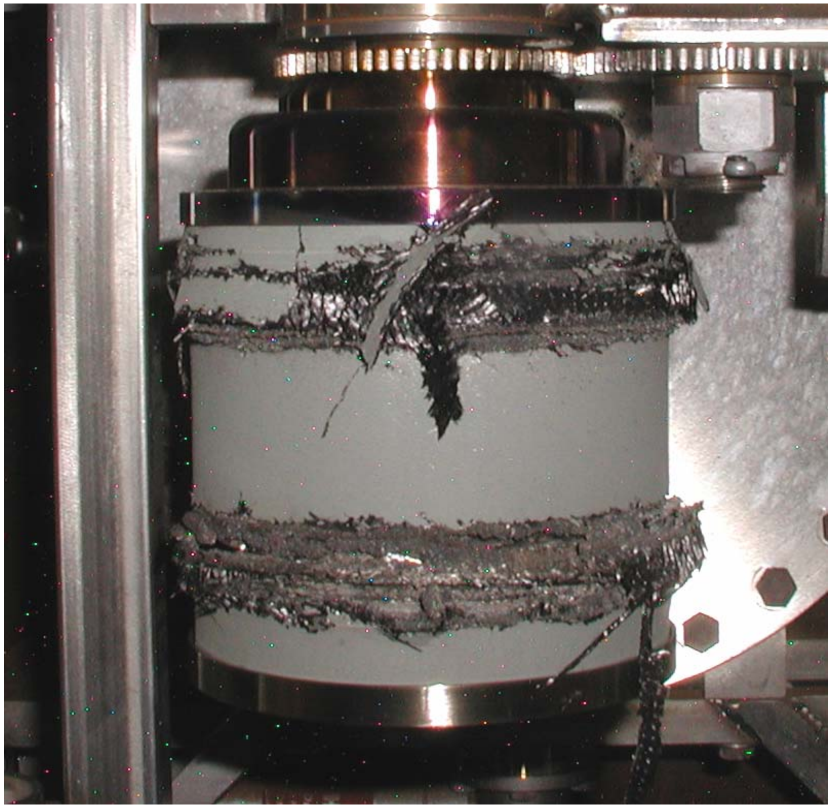


N_{pbar} at Debuncher entrance (arb. units) on number of protons on target (units 10^{12})

- Booster intensity is limited by space charge and coherent beam stability to ~$4.5 \cdot 10^{12}$
- Slip-stacking in MI allows one to double number of protons per bunch
- Strong effort to improve operation of proton accelerators (Jun.05-Feb.06)
  - Reduction of long. emittance in Booster

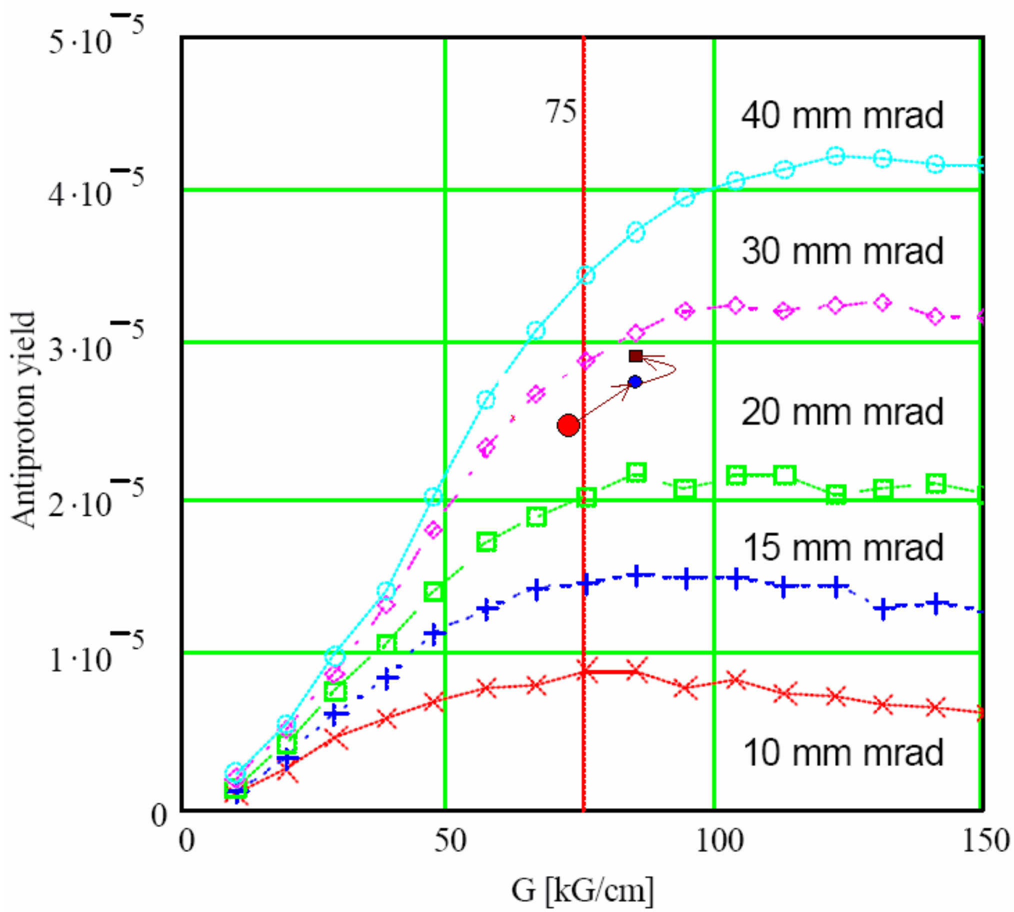
Protons on Target (continue)

- In 2006 we achieved the number of protons on target required by the final Run II parameters (N_p~8·10^{12})
  - It satisfies present and future Run II requirements
- Other improvements
  - Beam position stabilization on target
  - Optics correction in the transfer line for better focusing of protons on the target
    - Rms beam size of ~180 μm is limited by target damage



Result of beam overfocusing on the target

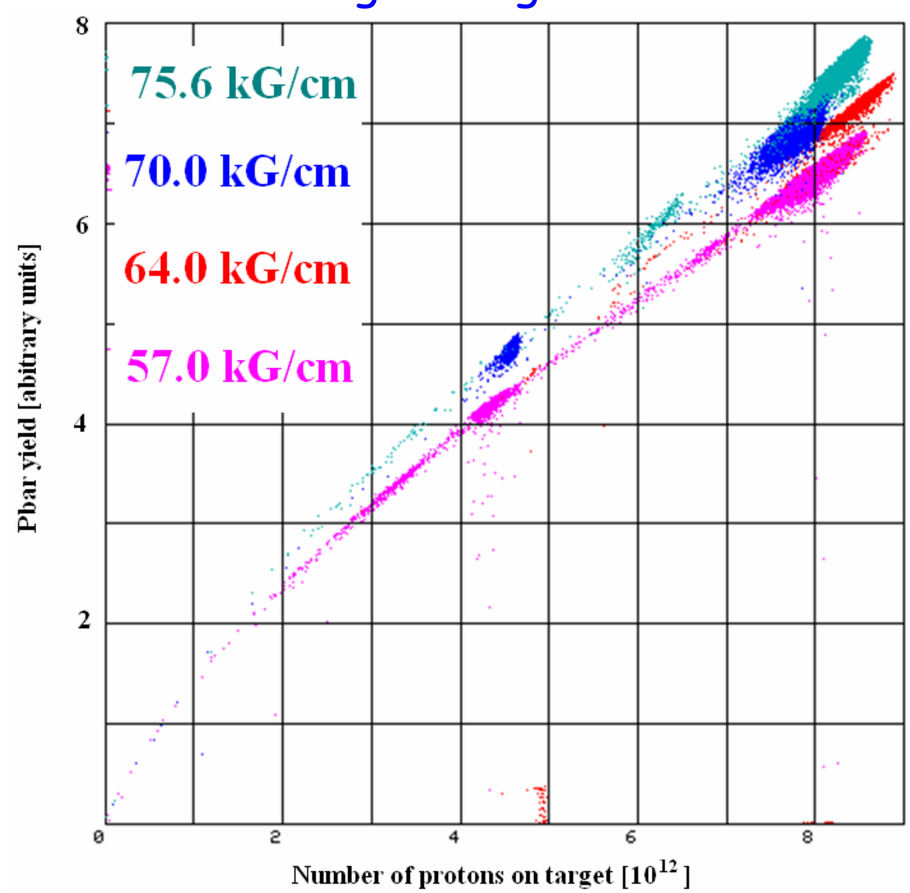
Antiproton Yield to Debuncher



Computed Antiproton yield on Debuncher acceptance and lithium lens gradient (2002)

- Presently:
  - Antiproton yield to Debuncher~25·10^{-6}
  - Number of antiprotons in Debuncher ~2·10^{8} per pulse, every 2.4 s

- Two major factors affect the antiproton yield to Debuncher
  - Debuncher and transfer line acceptance (presently, ε≈34 mm mrad, Δp/p ≈ ±0.022)
  - Focusing strength of Li lens

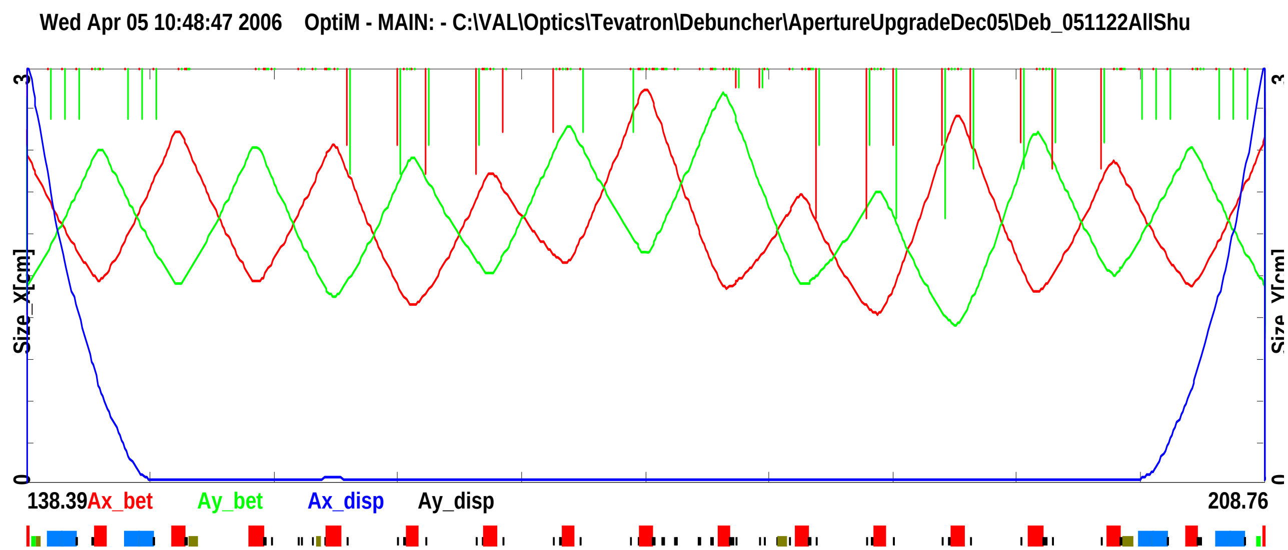


Optics Correction in AP-2 line and Debuncher

- Stacking rate is proportional to $\varepsilon\,\Delta p/p$ (where $\varepsilon = \varepsilon_x = \varepsilon_y$)
- New Optics in Debuncher takes into account aperture limitations
  - Stochastic cooling pickups, extraction kicker and injection septum
  - It would not be possible without accurate optics measurements
  - It also optimizes the stochastic cooling performance

	Initial design	Initial measured	New design	Present measured
ε_x [mm mrad]	34	30	40.5	35.3
ε_y [mm mrad]	31	25	37.5	34.6

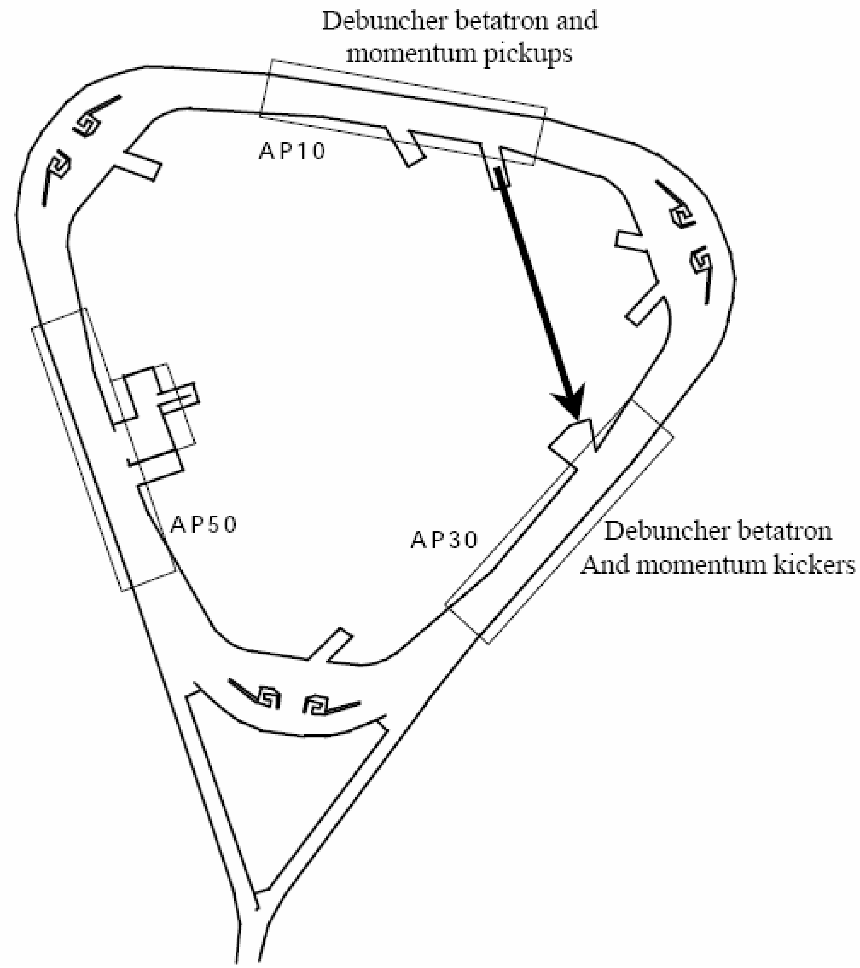
- Next correction will correct pickup-to-kicker $\Delta\nu_y$
  - 5% gain in Λ_y
  - additional shunts to be installed in summer 2007 shutdown

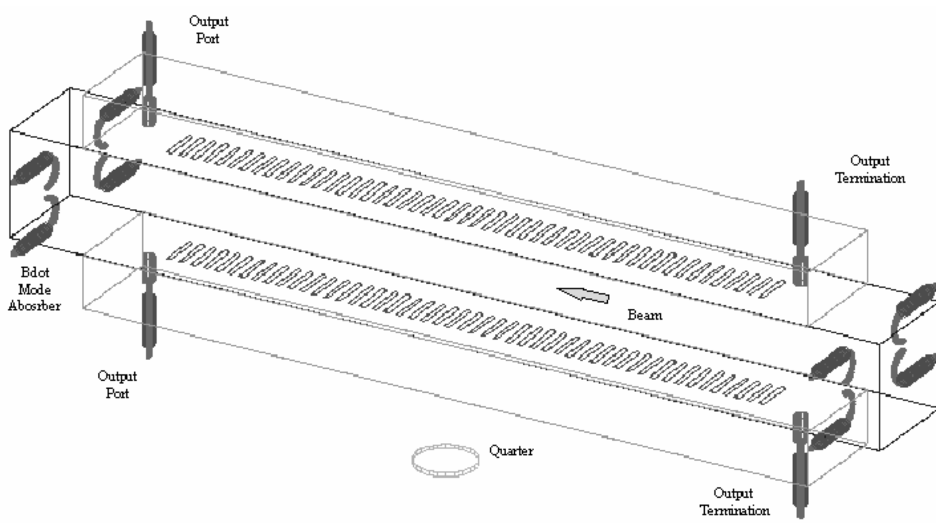


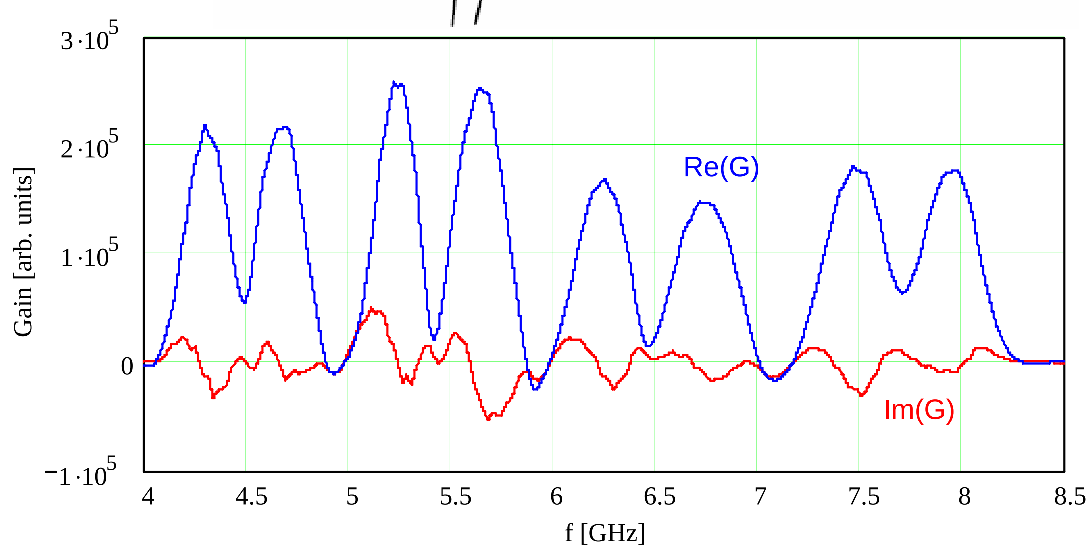
Beam envelopes in the Debuncher AP-10 straight line for H&V acceptances of 40.5 and 37.5 mm

Cooling in Debuncher

- 3 cooling systems (L, H, V)
  - 4-8 GHz band
  - L system uses the same pickups and kickers as H&V but in Σ mode instead of Δ mode (filter cooling)
- Each system has 4 sub-bands
  - Pickups of each subband are split into 2 additional subbands
- Cryogenic wave-guide pickups and preamplifiers

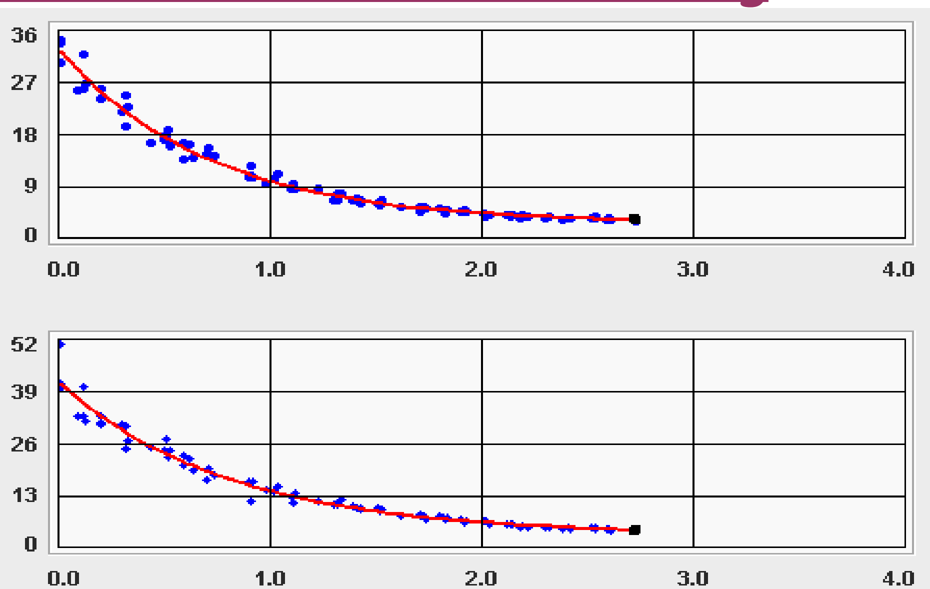






- Debuncher cooling is power limited
  - Wave guide kickers are used. Total power is:
    - ~1.6 kW for each of H&V systems (16 of 100 W TWTs)
    - 3.2 kW for L system (32 of 100 W TWTs)
  - We ramp the gain of transverse systems to keep power at maximum during entire cooling cycle of ~2.4 s

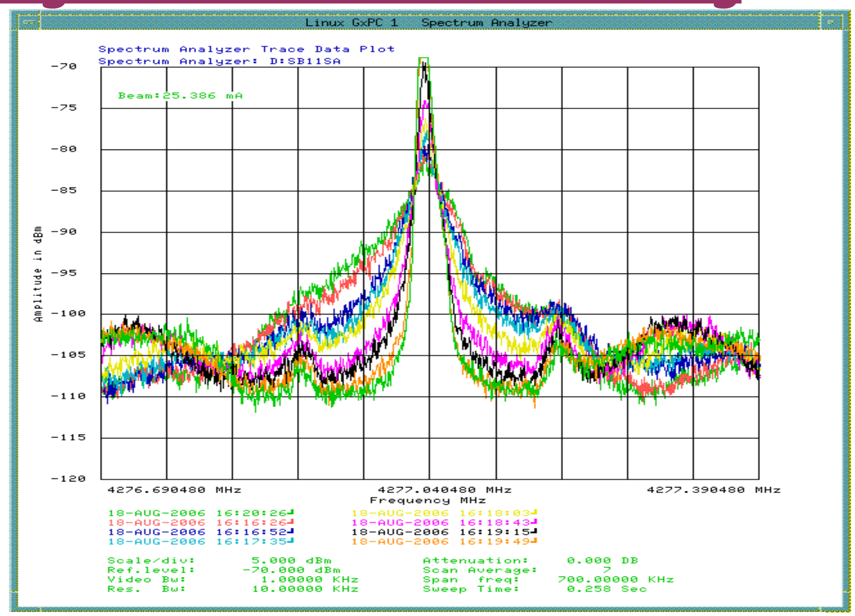
Transverse Debuncher cooling

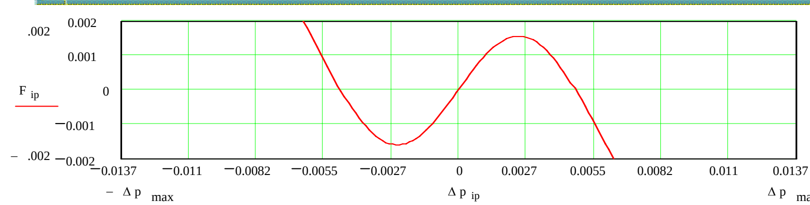


	Initial	Decrement	Base	at 2 sec
X plane:	32.508	0.719	2.06	3.946
Y plane:	40.884	0.811	2.508	5.762

X & Y 95% emittances on time

Longitudinal Debuncher cooling



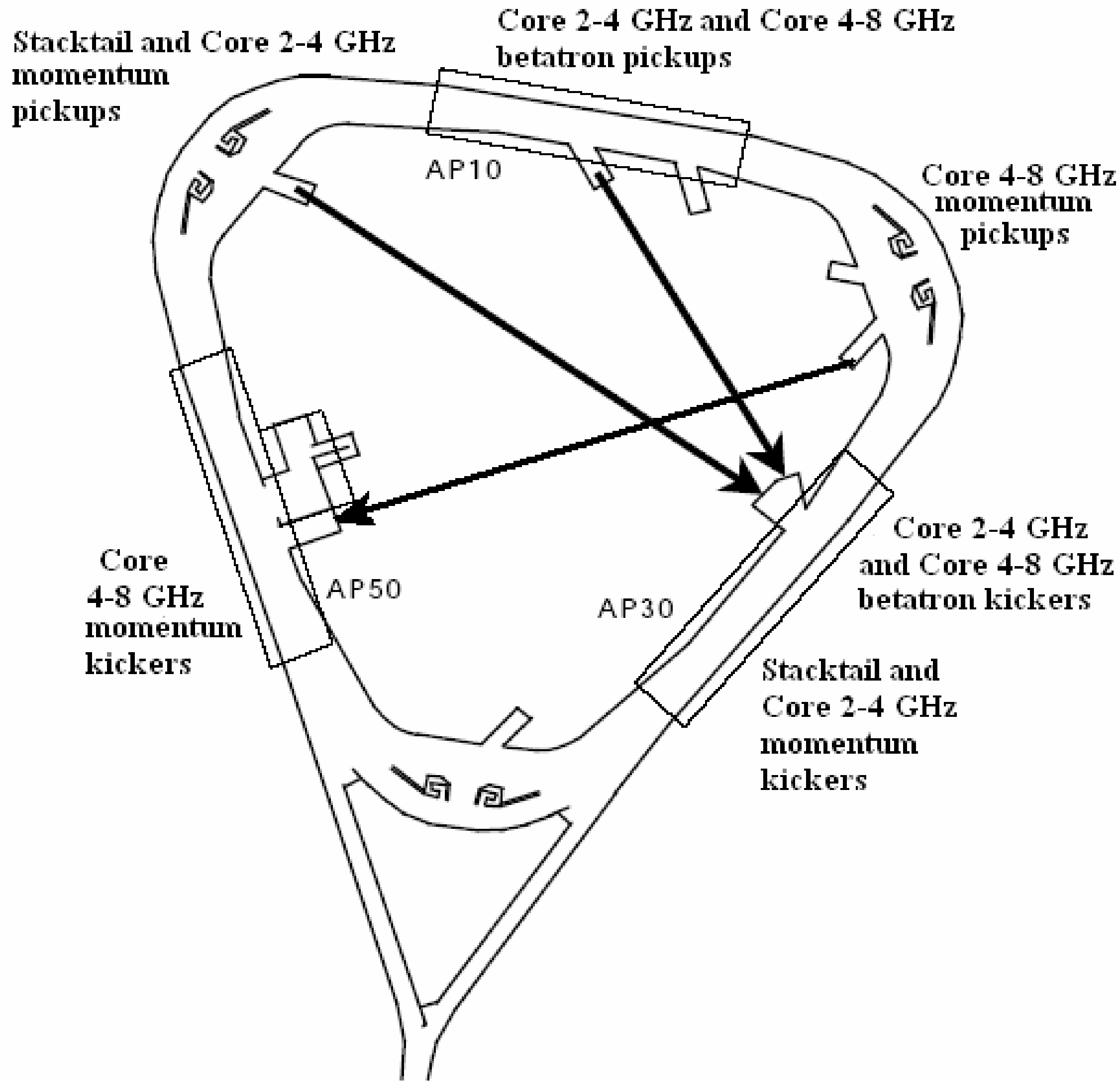


Top: Spectra at 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.6, 2 s

Bottom: Computed dependence of $F_{||}(\Delta p/p)$

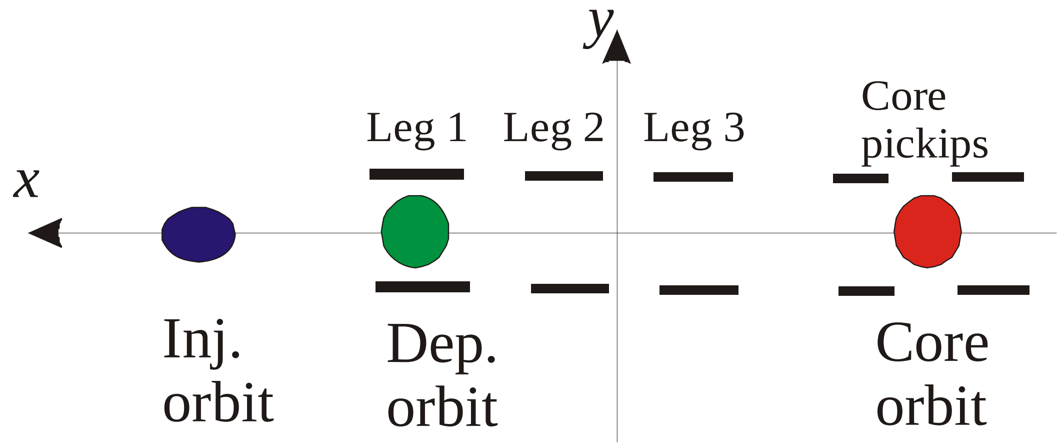
Cooling and Stacking in Accumulator

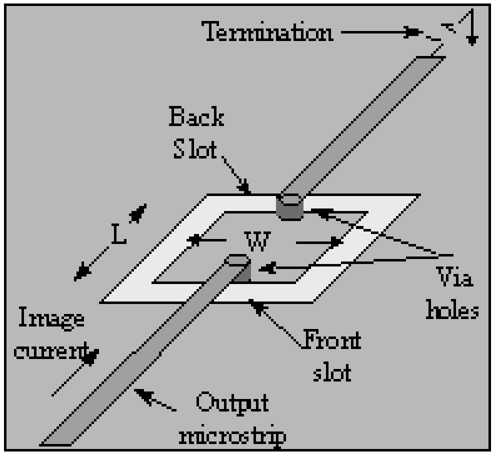
- 5 cooling systems
  - Core cooling
    - H & V – 4-8 GHz
    - Longitudinal: 2-4 GHz and 4-8 GHz
  - Stacktail - 2-4 GHz
- Stacktail system moves injected antiprotons to the core
  - Presently it is a major limitation of stacking rate increase
  - All stacking rate improvements of the last two years are related to improvements of the Stacktail system (every time it is the last bottle neck to be opened)

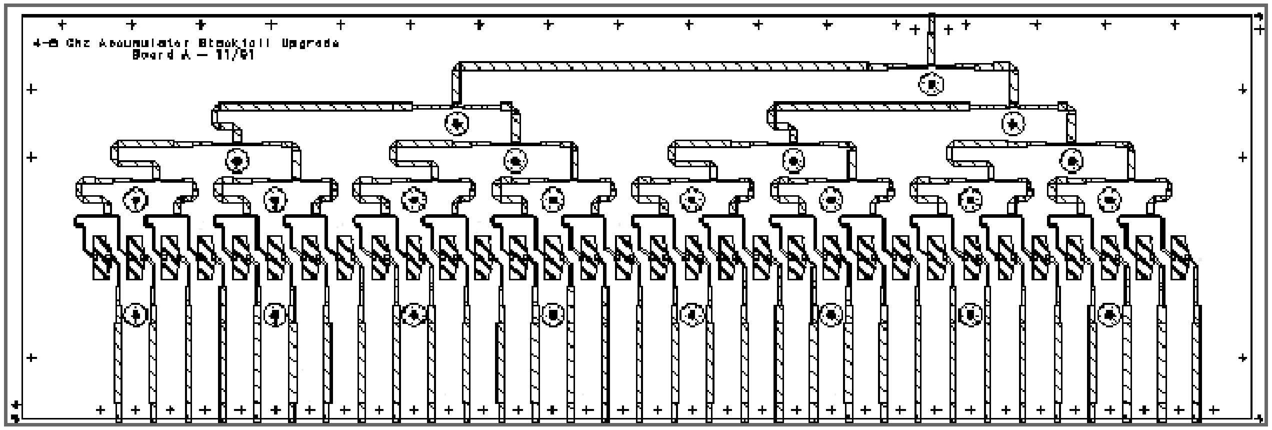


Stacktail

- Pickups are located at large dispersion (~9.1 m) while kickers are at zero dispersion (Palmer cooling)
- Stacktail has 3 pickups located at different radial positions to make desired dependence of gain on the momentum
- Pickups and kickers are built on the same technology
  - Planar loops
  - Printed circuit board technology
  - Works good at small frequencies (f≤4 GHz)
- Outside of pickup aperture its sensitivity drops exponentially. That allows one to form desired gain profile on particle position with small number of pickups
- Notch filters perform additional suppression of the gain on the core (~40 Db dynamic range)







Van der Meer solution for stacktail system

- Assuming that the total gain of the system can be factorized

$$G(x,\omega) = G_x(x)G_\omega(\omega) \ , \quad G_x(x) = G_0 \exp(-x/x_d)$$

- and looking for a static solution with constant flux one arrives that the maximum flux is

$$J_{\max} = |\eta| T_0 W^2 x_d$$

where the effective bandwidth is:

$$W = \sqrt{\frac{\left(\int_0^\infty \mathrm{Re}\left(G_\omega(2\pi f)\right)df\right)^2}{\int_0^\infty \left|G_\omega(2\pi f)\right|^2 \frac{df}{f}}}$$

- For the rectangular gain function

$$G_\omega(2\pi f) = \begin{cases} G_{opt} \ , & f \in [f_{\min}, f_{\max}] \\ 0 \ , & otherwise \end{cases}$$

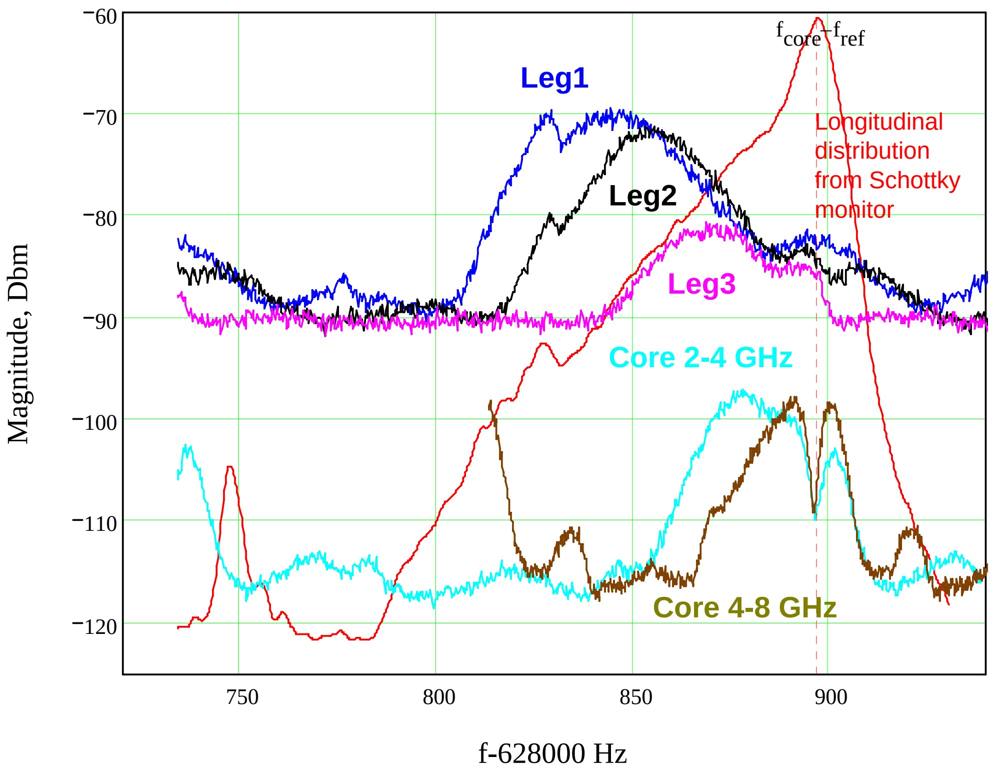
that yields

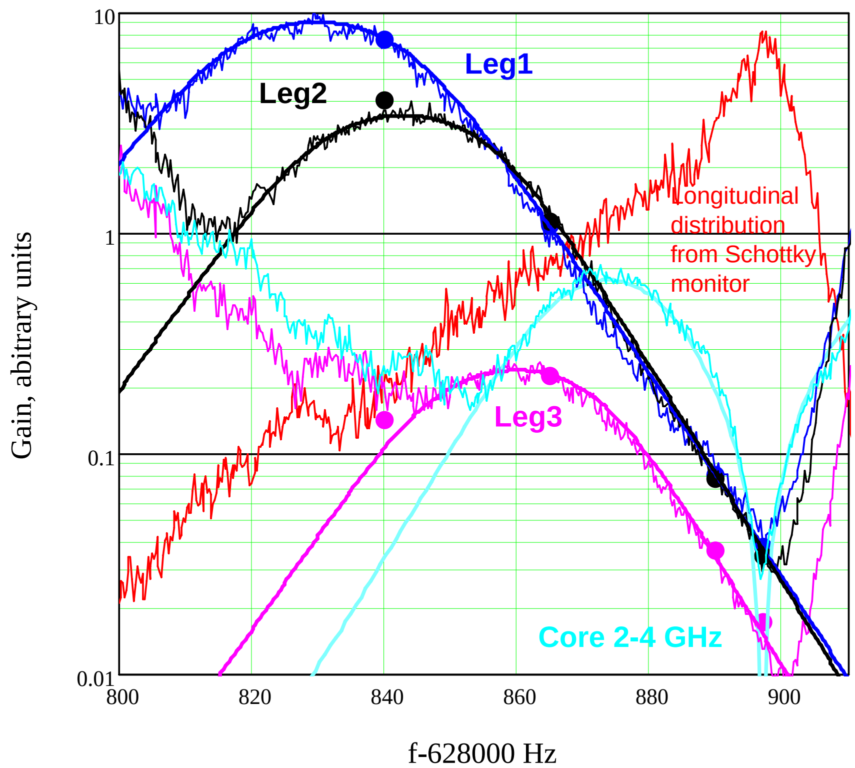
$$W = \frac{\left(f_{\max} - f_{\min}\right)}{\sqrt{\ln\left(f_{\max}/f_{\min}\right)}}$$

- The gain of real system cannot be factorized and is not exponential through entire stacktail region
  - Solving complete Fokker-Plank equation is required to understand the system and to predict its behavior

Numerical model

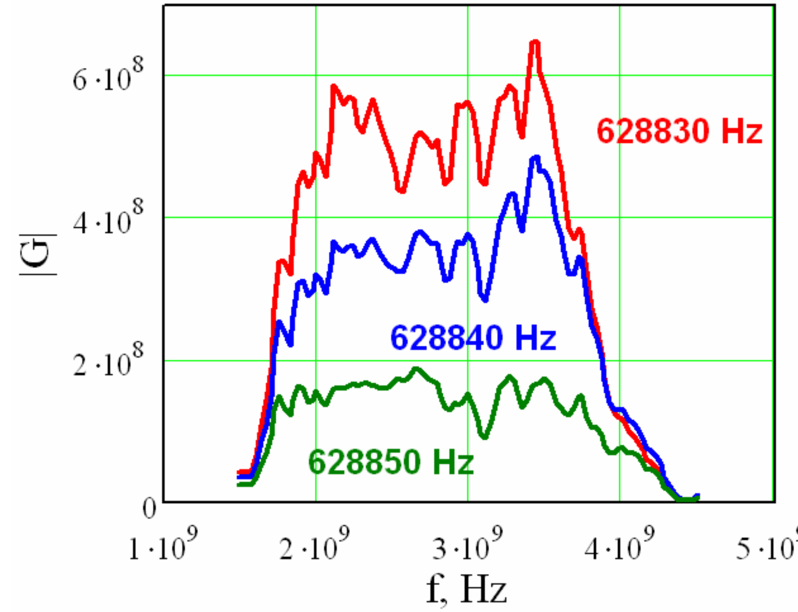
- Numerical model is based on the open loop BTF measurements
- Pickup coordinate response
  - The response is obtained from a ratio of single band Schottky noise of each subsystem to the longitudinal Schottky monitor signal (D_x=0)
  - Results are close to the test-stand measurements
    - $\eta(x)$ variation (~15%) is accounted
    - Measurements are fitted to: $F(x) = \frac{1}{\pi}\left[\operatorname{atan}\left(\sinh\left(\frac{\pi}{h}\left(x - x_0 + \frac{w}{2}\right)\right)\right) - \operatorname{atan}\left(\sinh\left(\frac{\pi}{h}\left(x - x_0 - \frac{w}{2}\right)\right)\right)\right]$

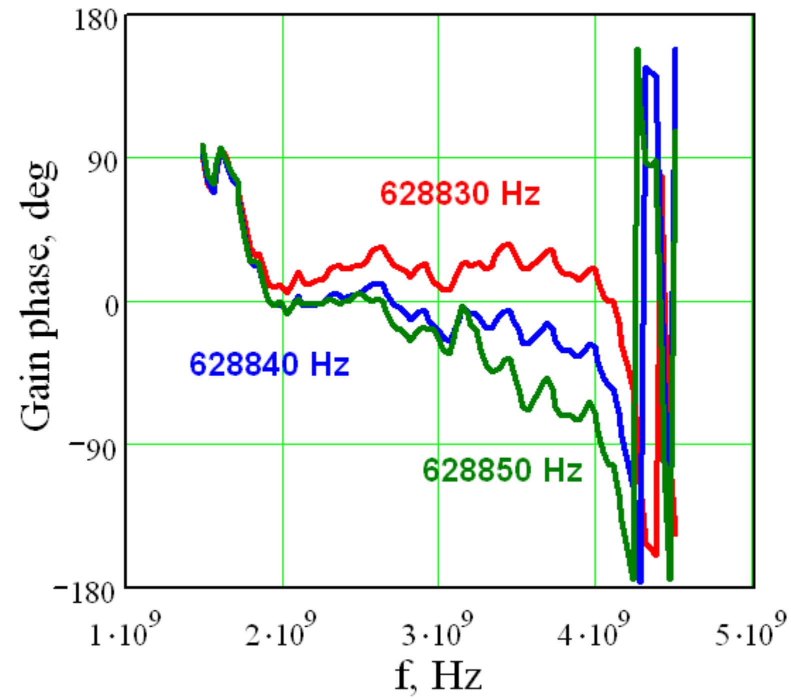




Numerical model (continue)

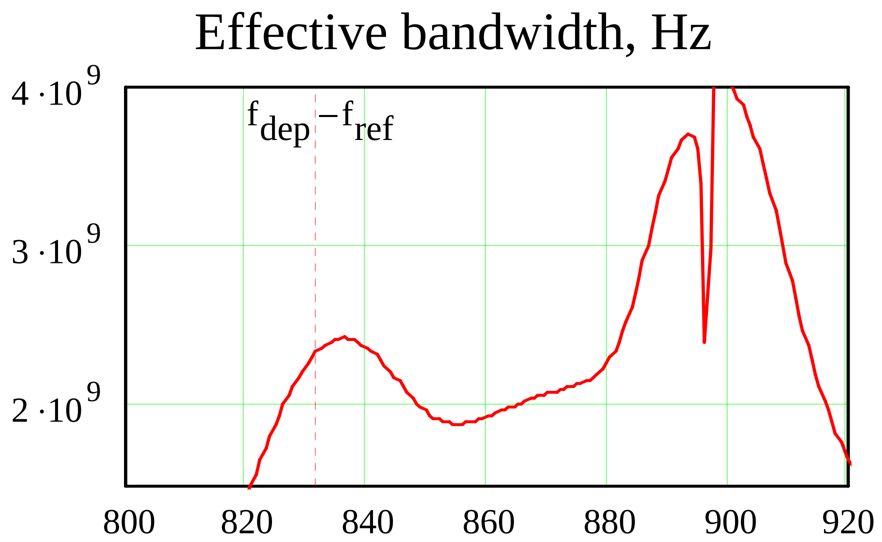
- Dependence of gain on frequency was measured on the revolution frequency harmonics in 1.5 – 5 GHz range with notch filters off
  - Notch filters were measured separately
- Wiring all pieces together (including core cooling) one obtains $G(x,\omega)$
- Static flux model computes
  - cooling force: $G(x)$
  - Inverse rate of cooling force change: $E_d \equiv p\, x_d$
  - Effective bandwidth
  - Van der Meer flux

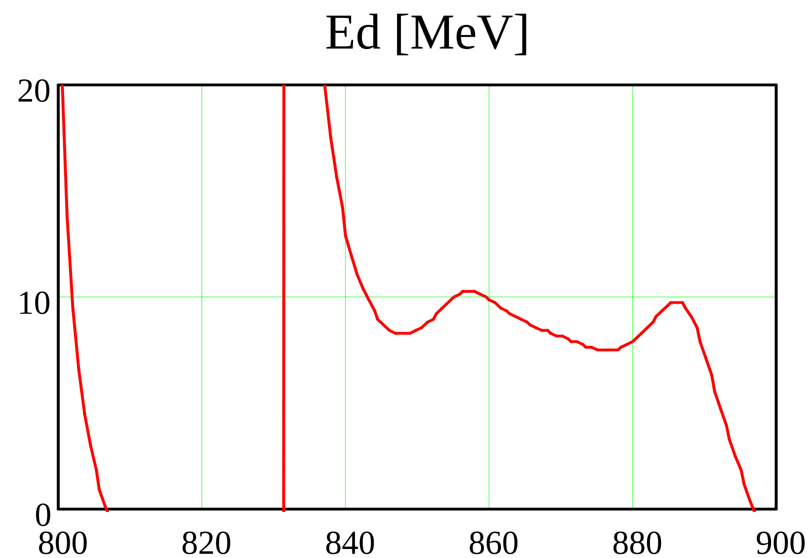


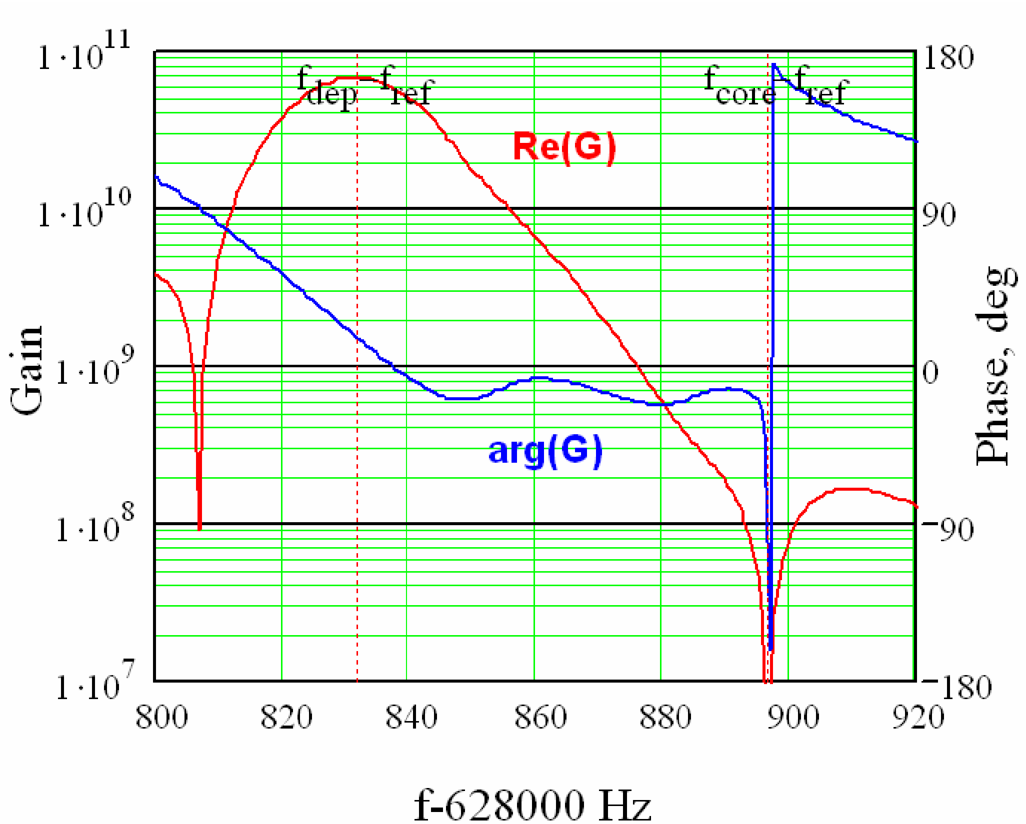


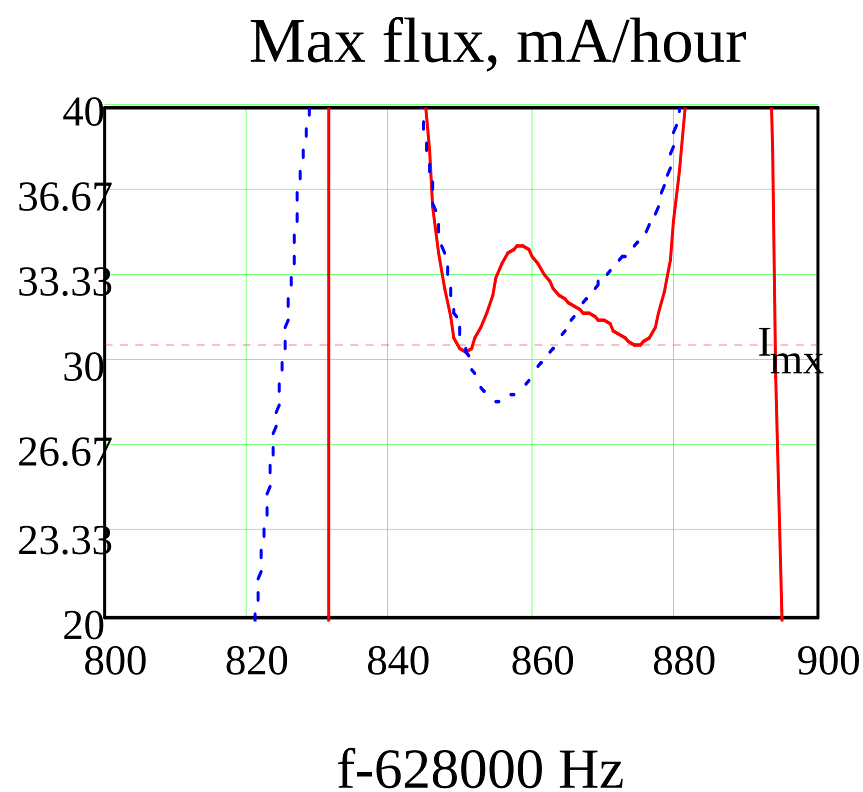
Total system gain after stacktail upgrade

Results of Static Model (for stacktail system after all upgrades)

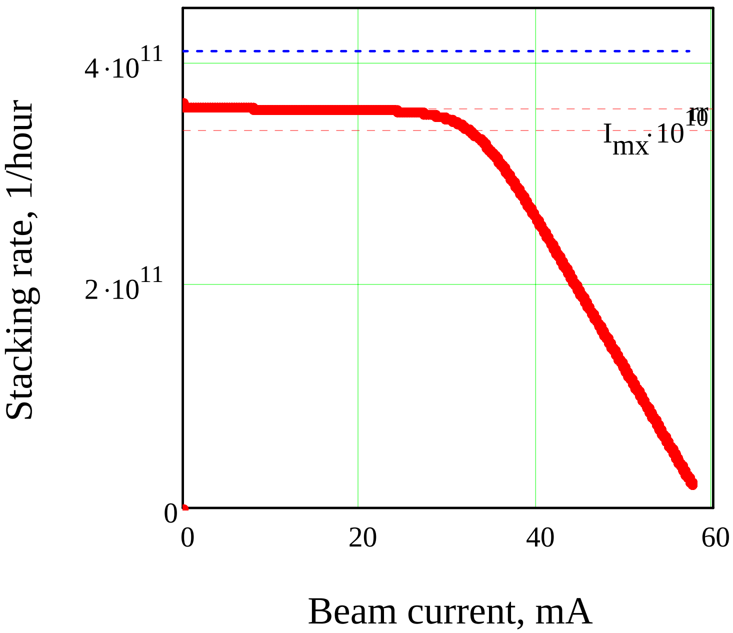




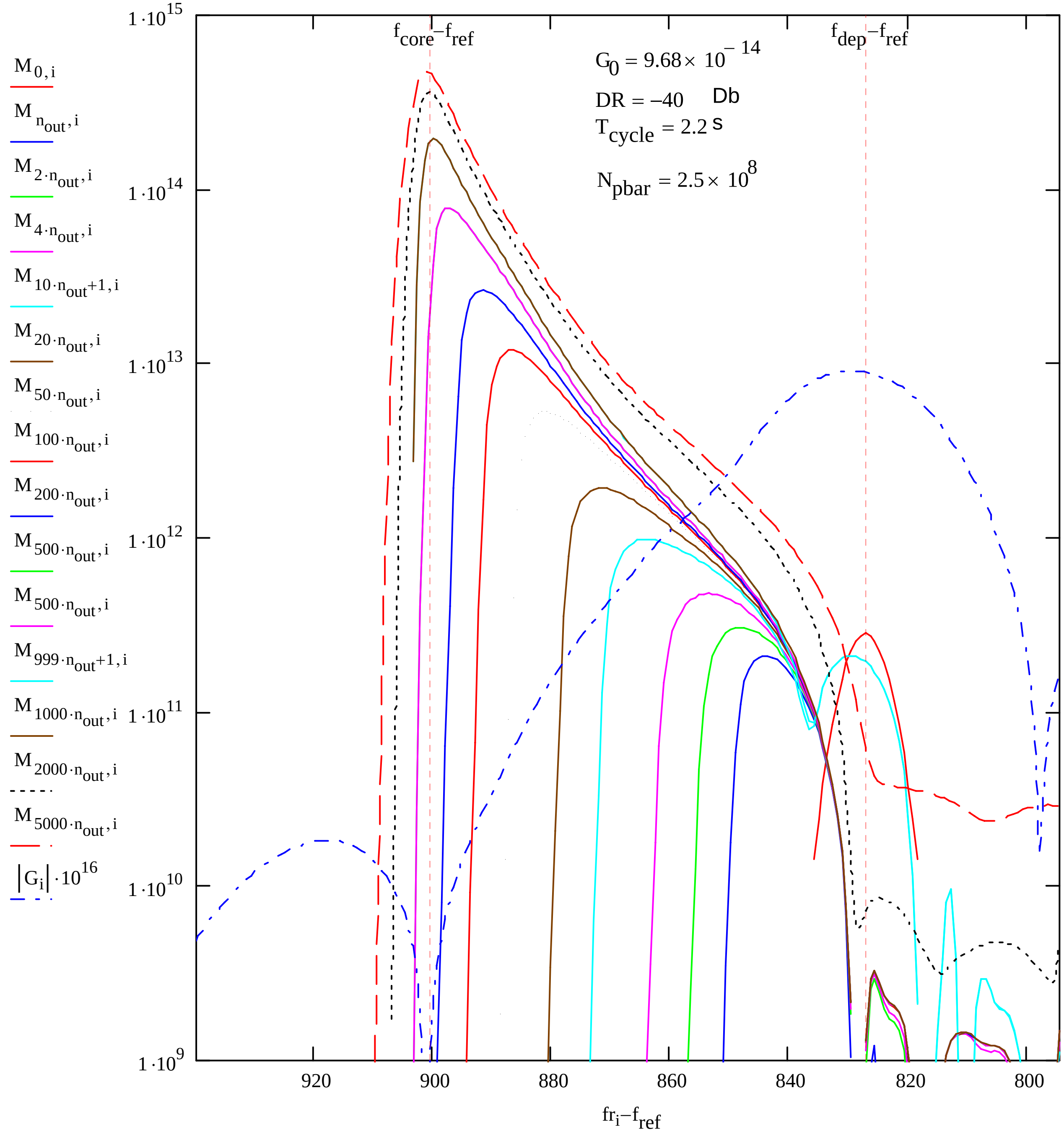




Dynamic Stacktail model predictions

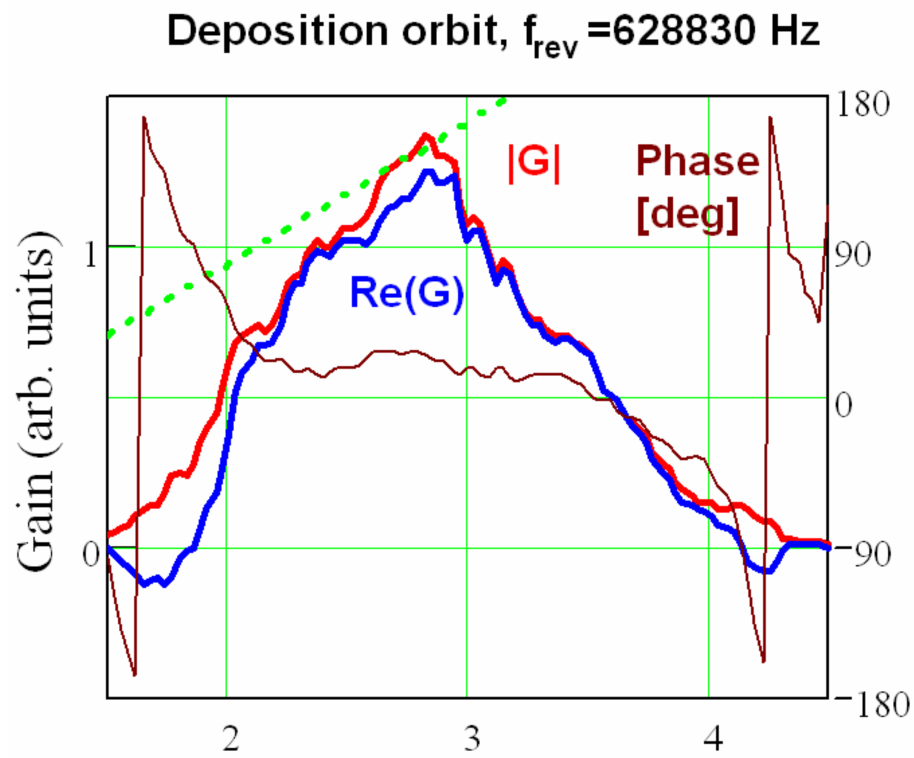


- $\varepsilon(\omega)$ is presently ignored in simulations to accelerate computations



Ways to improve the stacktail throughput (end of 2006 view)

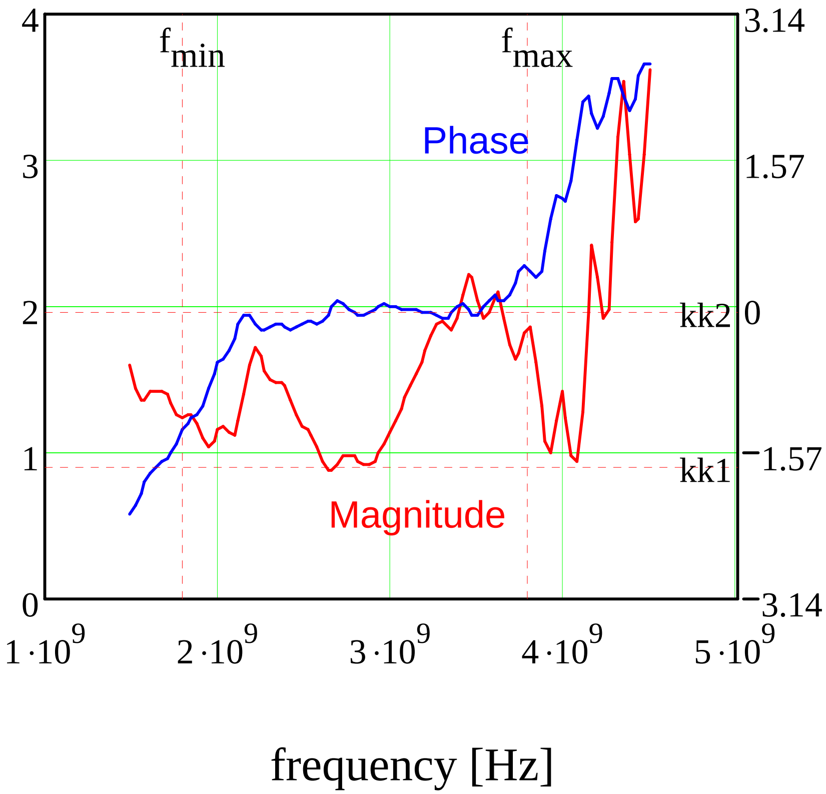
$$J_{max} = T_0 |\eta| W^2 x_d$$



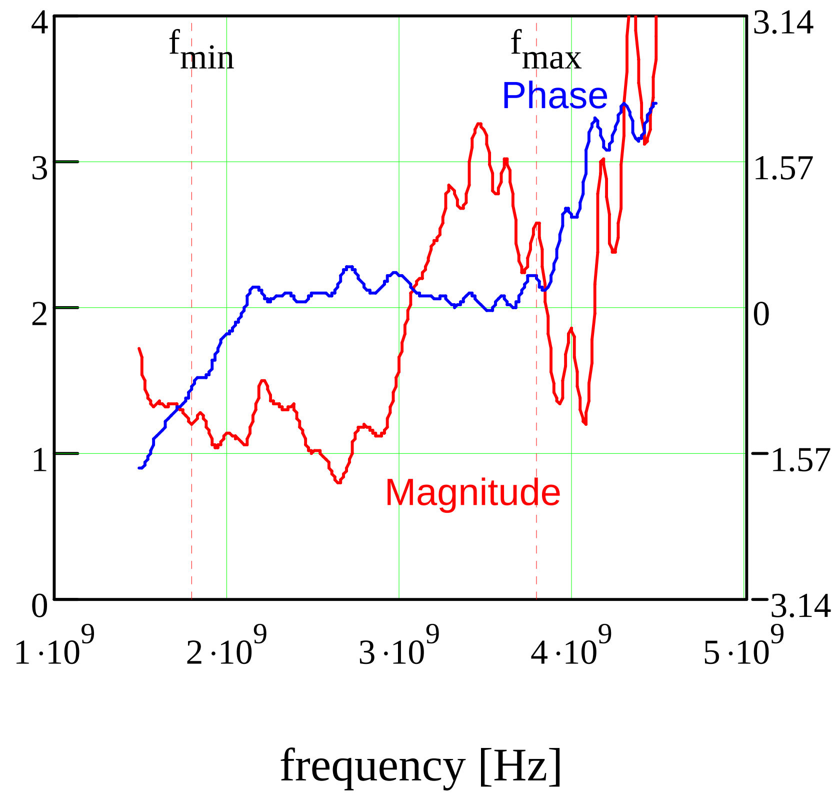
- Bandwidth increase, W
    - Gain is peaked in the band center
    - Large phase varations at band edges
  - ♦ 20 Db signal-to-noise ratio
  - ♦ 10 Db gain correction ⇒ increase of effective bandwidth by ~20% ⇒ 40% increase of stacking rate.
- Increase of x_d would require more power which we have not had
- Increase of the slip factor, η, by ~20% looked attractive
  - ♦ Further increase is limited by the band overlap at high frequencies
  - ♦ It required optics change in Accumulator
- Feb.'06 stacking rate coincided with the model predictions
  - ♦ At the end of 2006 it looked very probable that $30 \cdot 10^{10}$/hour will be achieved by summer of 2007
    - Nevertheless, other complications were coming soon

What Equalizer does?

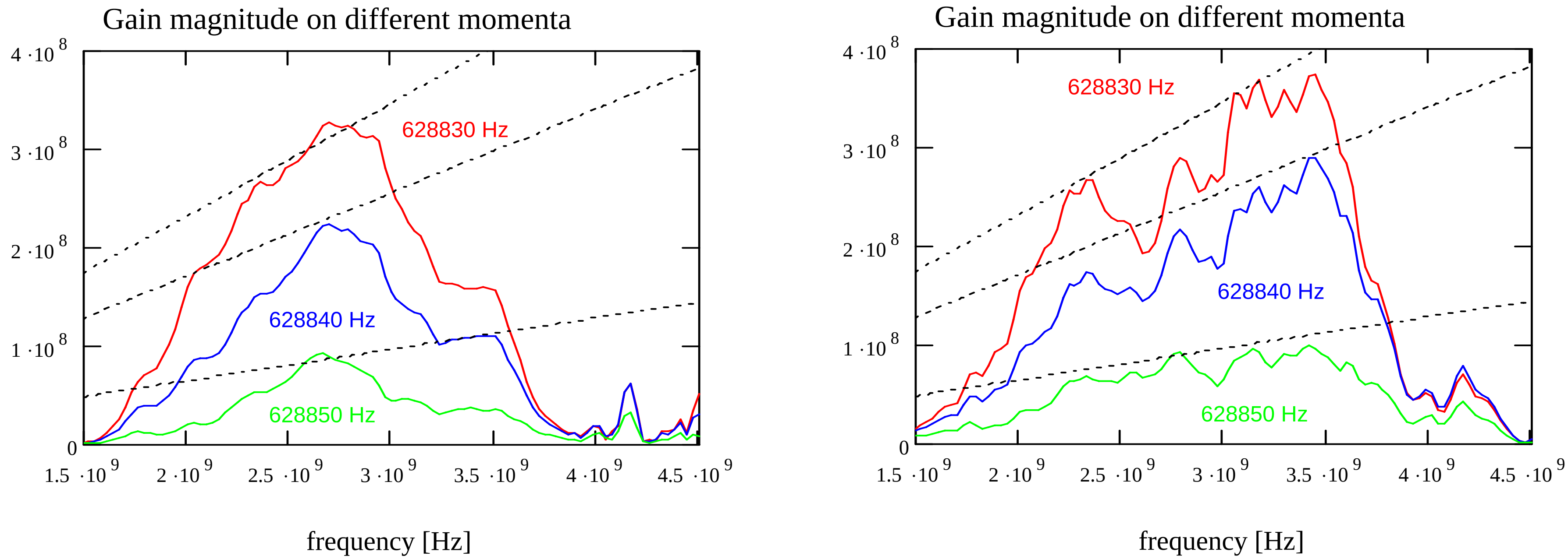
- It corrects phase and magnitude of the gain so that to achieve maximum bandwidth



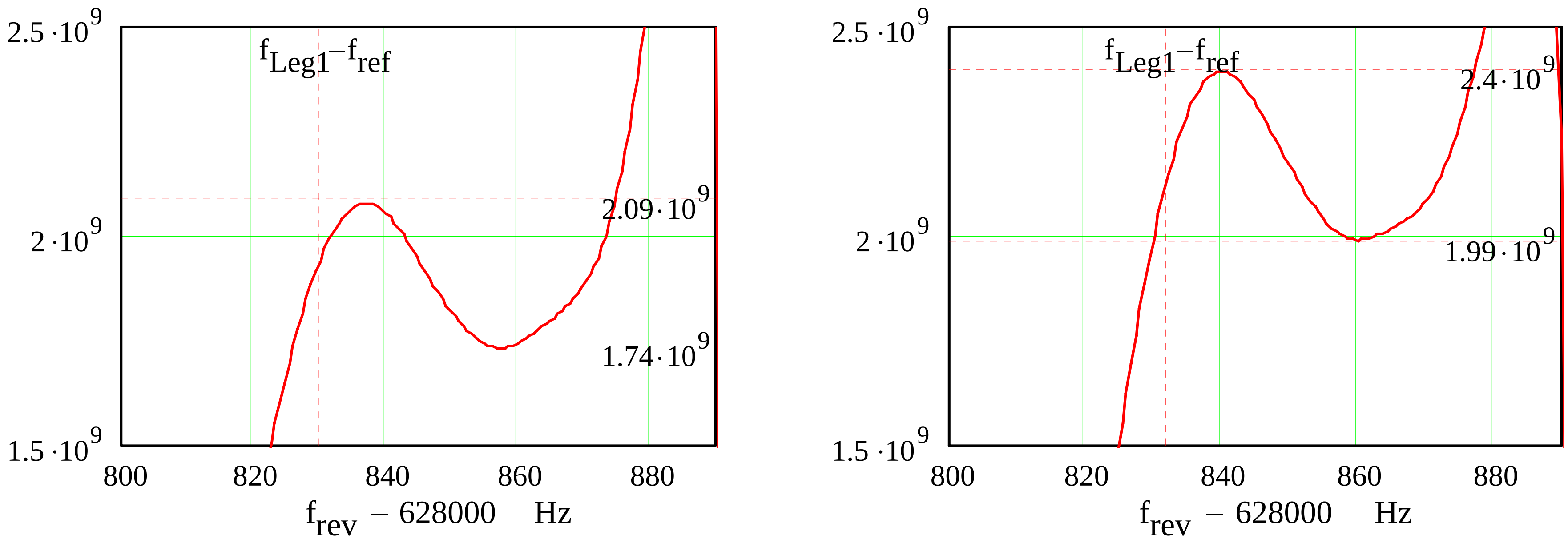
Equalizer with reduced gain at high frequency. It was tested first.



Present equalizer

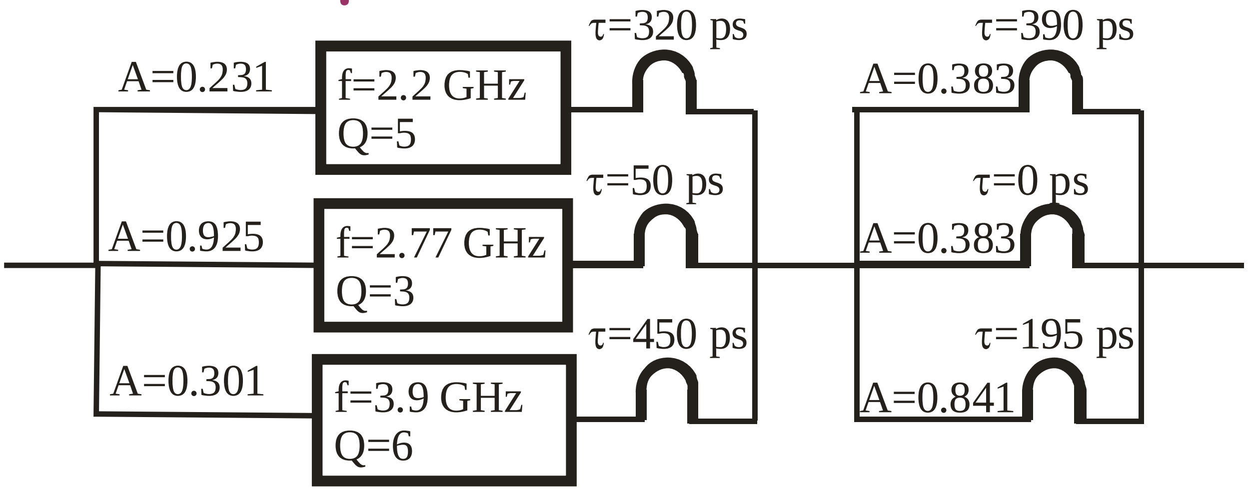


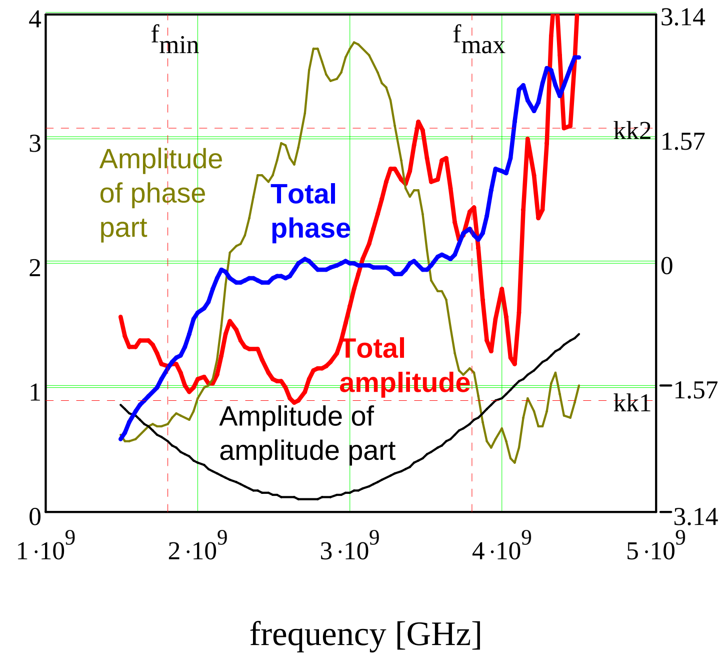
Dependence of stacktail gain on frequency before and after installation of the equalizer



Dependence of effective bandwidth before and after installation of the equalizer (~15% growth)

How the equalizer is built





Prototype Equalizer specifications

- Phase part corrects phase
- Amplitude part corrects amplitude so that to get the total amplitude as desired

$$K_i(\omega) = \frac{A_i}{1 + iQ_i \dfrac{\omega^2 - \omega_i^2}{\omega \omega_i}}, \quad i = 1,2,3$$

$$K_A(\omega) = 1 + 0.91\cos(\omega\tau), \quad \tau = 195\ ps$$

$$K_{tot}(\omega) = K_A(\omega)\left(K_1(\omega) + K_2(\omega) + K_3(\omega)\right)$$

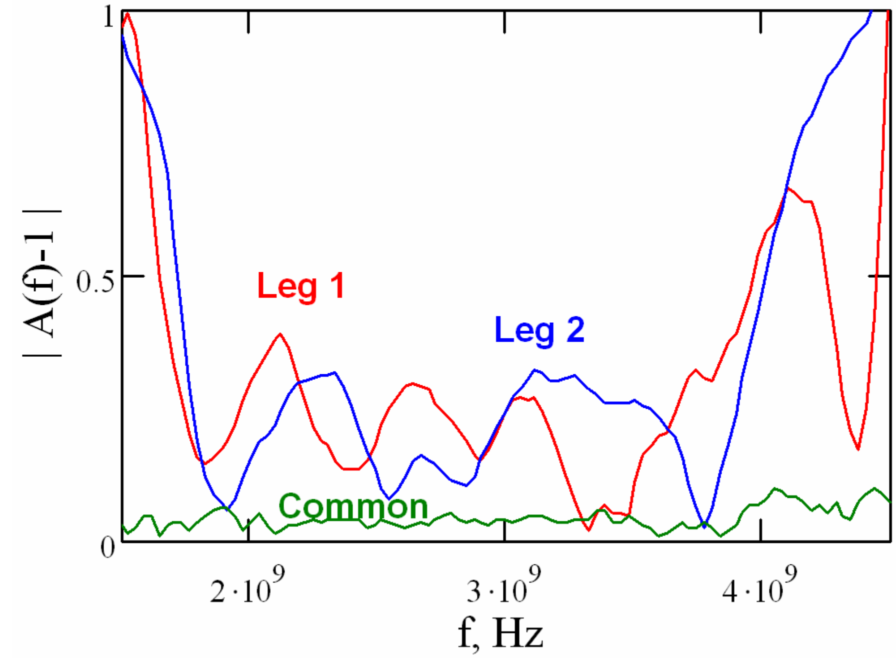
- Final equalizer has 5 resonators and two-stage amplitude correction

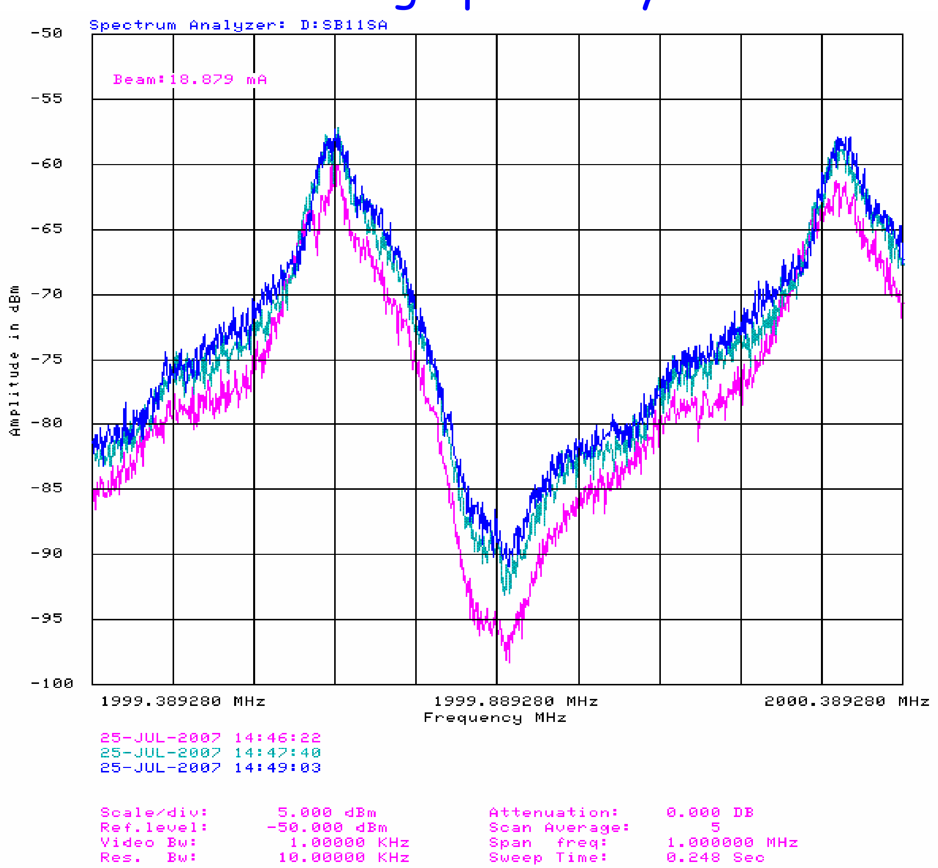
Other stacking improvements

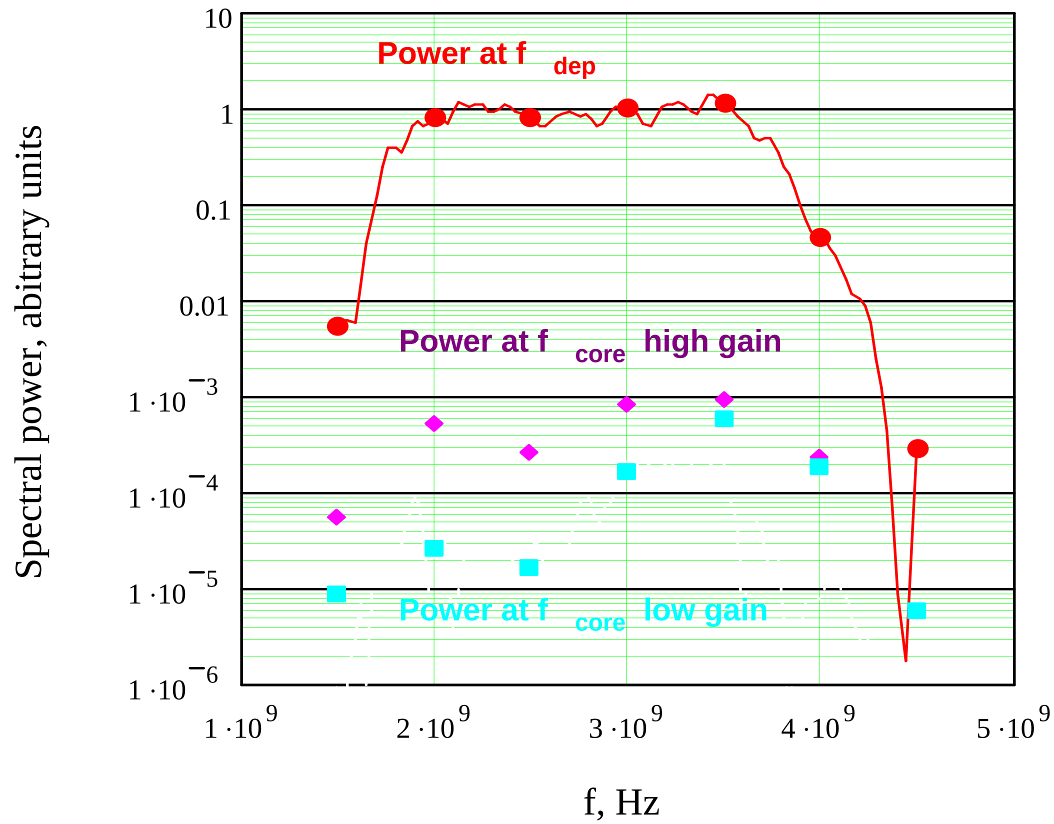
- With equalizer fully commissioned we found that we cannot run system with sufficiently high gain because of ⊥ heating
  - Basically the same stacking rate but 2 times smaller power
    $\Rightarrow$ 1.6 kW $\rightarrow$ 0.8 kW
  - Optics change in Accumulator addressed this problems (see below)
- But then the longitudinal heating set another limitation
  - Replacement of BAW (bulk-acoustic wave) notch filter 3 by SC notch filter addressed the longitudinal core stability but longitudinal heating still has been too large
  - Future (after shutdown) steps
    - Equalizer for the 4-8 GHz longitudinal core cooling will improve damping by ~1.6 times
    - Reduction of stack size (50 $\rightarrow$ 30 mA) yields another factor of 2
    - Gain ramps will reduce power variations)beginning-to-end of cycle)
    - Combined these measures have to be sufficient to bring the stacktail power back to 1.6-1.8 kW level
    - and, consequently, the stacking rate in the range (25-27)$\cdot 10^{10}$/hour

Longitudinal heating

- Longitudinal core blowup is excited by Stacktail noise on harmonics of core revolution frequency
- Notch filters for additional suppression
  - ~35 Db dynamic range is set by noise of preamplifier
    - at high power by intermods







Transverse heating

- Transverse emittance growth is excited by the stacktail because of non-uniformity of longitudinal kick across the aperture

$$U(x,y)=U_0\left(1\pm\frac{x^2-y^2}{2a_{eff}^{\ 2}}\right)\quad\Rightarrow\quad E_x\propto\frac{dU(x,y)}{dx}=U_0\frac{x}{a_{eff}^{\ 2}}\ ,\ a_{eff}\approx1.87\ \text{cm}$$

  - Kicker offset
    - Average offsets are kept sufficiently well, <
    - But, electrical center position varies with frequency
  - Parametric heating
    - Excited by noise at sidebands of doubled betatron tune, $2\nu_x+n$
  - Non-zero dispersion at kicker location

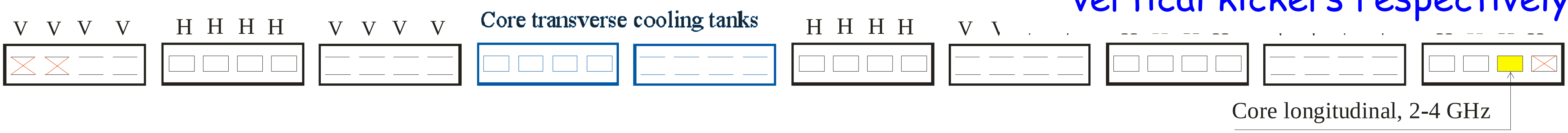
Emittance growth due to parametric heating

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt} = \frac{v_0^{\ 2}\beta_{eff}^{\ \ 2}}{4\pi\omega_0 a_{eff}^{\ \ 4}} \sum_{k,m=-\infty}^{\infty} \frac{\psi(\tilde{p}_{km})}{\left|\eta k^3\right|} \left|G(\tilde{p}_{km},\tilde{\omega}_{km})\right|^2 \left|\left(1 - A(\tilde{\omega}_{km})e^{-i\omega_{km}T_0}\right)\right|^2 , \quad p \equiv \frac{\Delta P}{P}$$

$$\beta_{eff}^{\ \ 2} = \frac{1}{\left(\sum_{i=1}^{N}\left|\kappa_i\right|\right)^2} \sum_{i,j=1}^{N} \kappa_i \kappa_j \left(\beta_0^{\ 2} + 2 s_i' s_j' + \frac{s_i'^2 s_j'^2}{\beta_0^{\ 2}} - \left(s_i' - s_j'\right)^2\right)$$

s_i' - long. coordinate of *i*-th kicker relative to the point of minimum beta-function, β_0

$\kappa_i = \pm 1$ for horizontal and for vertical kickers respectively



- Kicker rearrangement to reduce parametric heating
  - Core 2-4 GHz kicker is moved to the end
  - Two kickers at each end are off
- Effective β-function is reduced from 2.3 m to 0.6 m
  - 15 times reduction resulted negligible effect from parametric heating

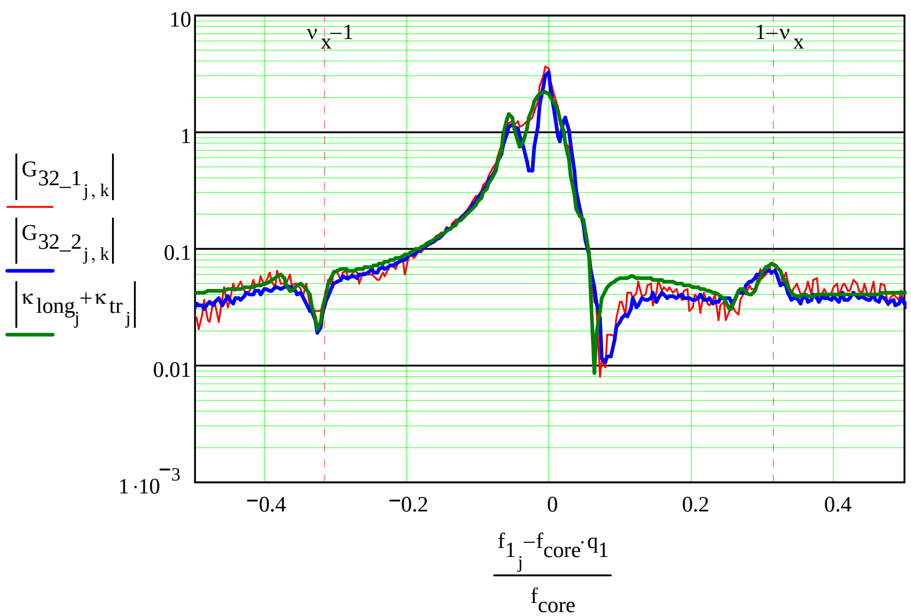
Emittance growth due to offset of kickers

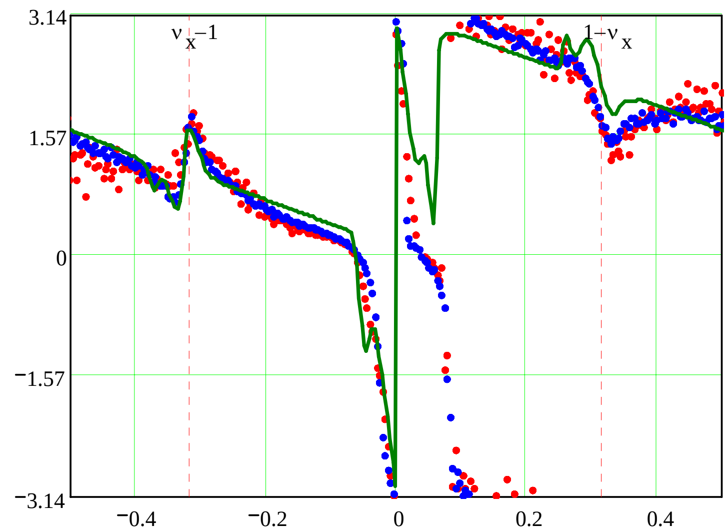
$$\frac{d\varepsilon(p)}{dt} = \frac{\omega_0 \beta_{kick}}{4\pi} \sum_{k,m=-\infty}^{\infty} \frac{\psi(p_{km})}{|\eta k|} \left|G(p_{km},\omega_{km})\right|^2 \left|D_{eff}(\omega_{km})\right|^2 \left|\left(1 - A(\omega_{km})e^{-i\omega_{km}T_0}\right)\right|^2 ,$$

$$D_{eff}(\omega) = \frac{v_0 X(\omega)}{\omega a_{eff}{}^2} + \frac{D'_{kick}\beta_{kick} + \alpha_{kick} D_{kick} - iD_{kick}}{\beta_{kick}}{}^2 .$$

$X(\omega)$ - position of kicker electrical center relative to the beam center

- Open loop stacktail measurements exhibited that the kicker electrical center depends on frequency
  - Resonance at 3.25 GHz, x_0=2 mm, Q=27,
- It results in emittance growth. It cannot be suppressed by kicker centering
  - Δ-kickers with correct amplitude and phase response will be used if required



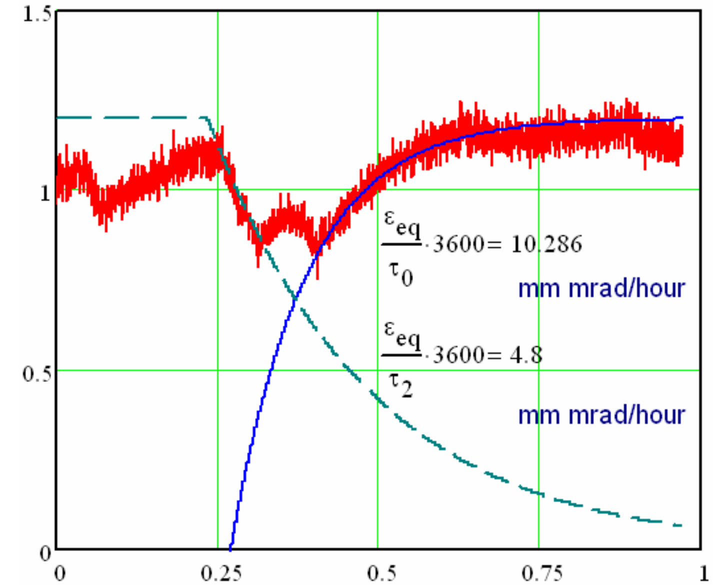


Narrow band open loop measurements of stacktail at 3.2 GHz (Apr. 10/2007)

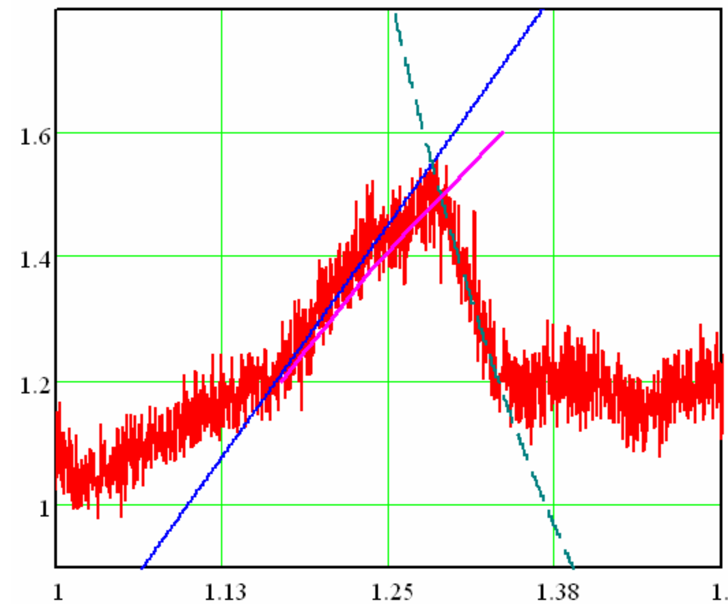
Directly Measured Heating Terms before Accumulator optics correction

- Analysis of H. emittance changes during stacking interruption and switching off all cooling systems results in
  - Core cooling time is 7 min
  - Direct measurements of heating terms before Accumulator optics correction

Heating mechanisms	mm mrad/hour
IBS heating at 50 mA	~3
Stacktail heating	5-6
Noise of core systems	~2
Total heating	~10



Data taken during stacking interruption

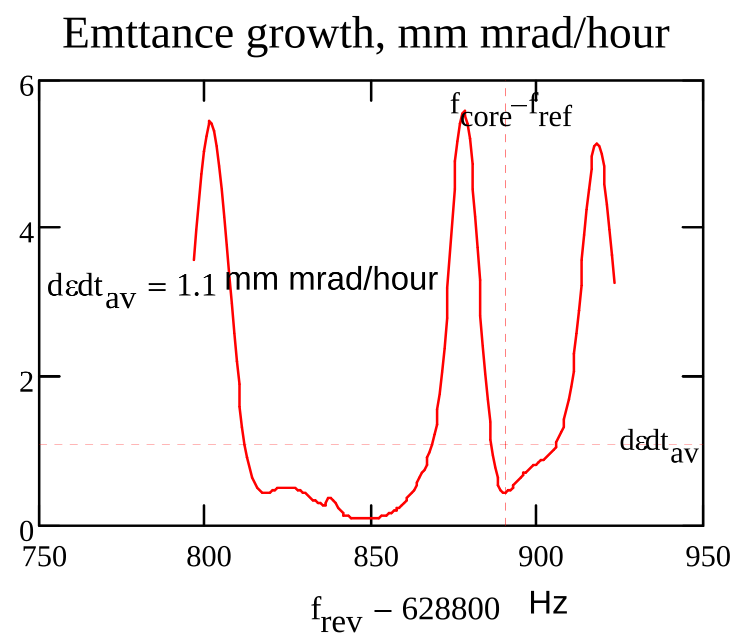


Data taken during stack-tail Schottky noise measurements; all cooling is off for ~10 min,

Estimate of Stacktail heating based on other measurements

Heating mechanisms	mm mrad/hour
Parametric heating	~0.25
Dispersion mismatch	~2.4
Kicker offset (res. at 3.25 GHz)	~1.2 - 2.2
Unaccounted*	~1.1 mm

* Most probably it is heating due to geometric kicker offset



Numerical simulation for measured kicker parameters:

x_0=2 mm, Q=27, $\omega_0/2\pi$ = 3.25 GHz

Accumulator Optics Change

- Accumulator optics correction reduced slip-factor, η, by ~20% and resulted in an acceptable $\perp$ emittances
- It reduced heating due to
  - IBS – smaller dispersion invariant
  - Dispersion match – smaller dispersion in kickers
  - Parametric heating
  - Kicker offset heating
    - the core sets between two peaks generated by Q and (1-Q) sidebands
- It also improved core cooling

Conclusions and Further plans

- The work carried out during last 2 years resulted in
  - Much better understanding of system operation
  - All important upgrades have been introduced
    - Few final ones will be finished during shutdown (Oct. 2007)
- During last two years:
  - peak stacking rates: $(15.5 \Rightarrow 23.1)\cdot 10^{10}$/hour
    - further increase to $(25\text{-}27)\cdot 10^{10}$/hour is expected in 2008
  - Average stacking rate (weekly peak): $(8.3 \Rightarrow 16.5)\cdot 10^{10}$/hour
    - further increase to $(20\text{-}22)\cdot 10^{10}$/hour is expected in 2008
- Optimal tuning of Antiproton source is the highest priority for 2008
  - Shortening the cycle time from 2.4 to 2.2 s
  - Reducing stack size from $50\cdot 10^{10}$ to $30\cdot 10^{10}$ (fast transfers)
  - New equalizer in Accumulator longitudinal core system
  - Better transverse and longitudinal cooling in Debuncher
    - Additional notch filters
    - Debuncher optics correction
- Further cooling and beam stability improvements in Recycler are required to support the planned antiproton production growth
  - They are planned to be introduced within next few months

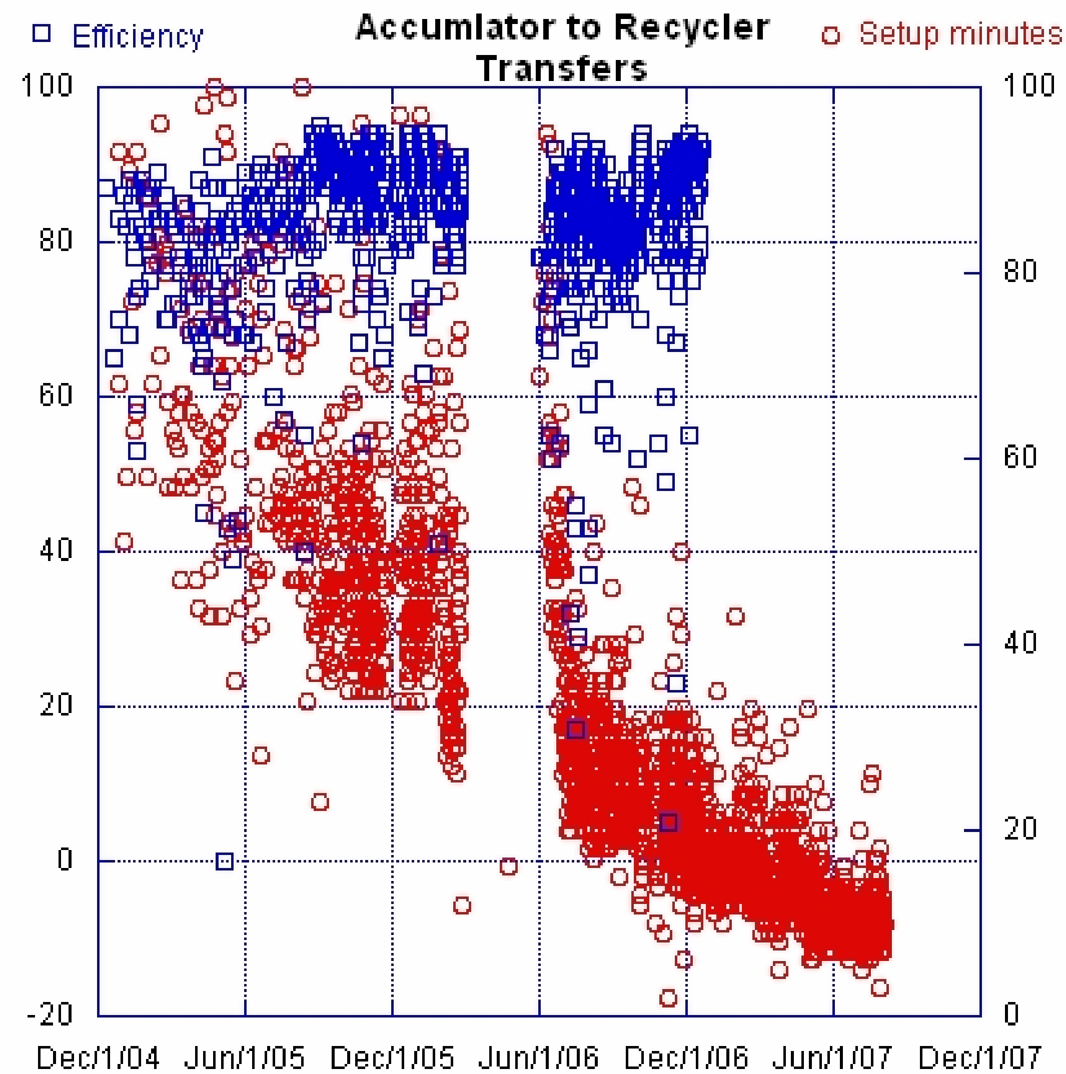
Backup transparencies

Sequence of major events for the Stacktail upgrade

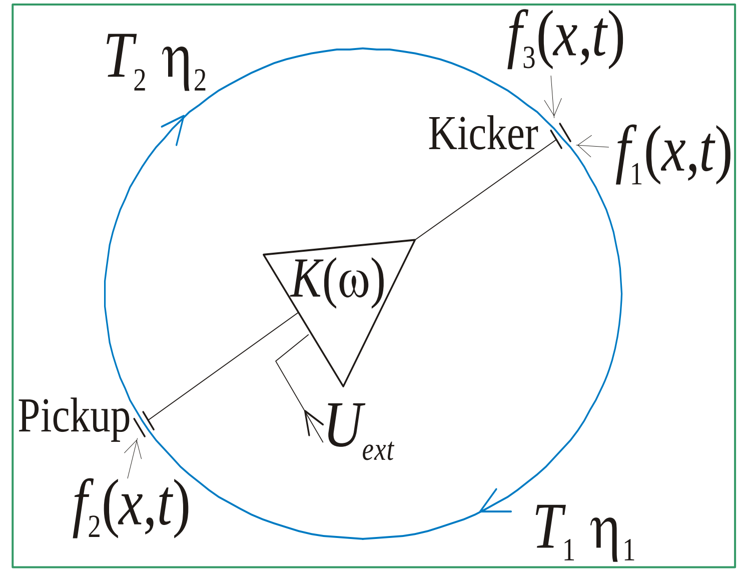
Hybrid flip	– October 1, 2006
Equalizer prototype installation	
First attempt	- March 12, 2007
Installation wit reduced gain at high f	- March 19, 2007
Final installation	- March 23, 2007
Stacking record, 23.1 mA/hour	– March 22, 2007
Legs 2 & 3 pulled away	- April 3, 2007
Accumulator lattice upgrade	– May 16, 2007
Leg 3 is fully operational	- May 4, 2007
New lithium lens lost	– May 24, 2007
Final equalizer installation	- June 4 , 2007
SC notch filter 3 installed	- July 18, 2007

Fast Transfers

- Acceleration of Accumulator-to-MI antiproton transfers played important role in increasing staking rate
- Decreasing stack size requires further improvements
- Transfers on t-clock event are planned to be introduced shortly after shutdown
  - Stacking cycle time $3 \rightarrow 1$ hour
  - Transfer time $9 \rightarrow 1$ min



Theory: stacktail & L. stochastic cooling



- Evolution of particle distribution is described by Fokker-Planck equation

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial x}\left(F(x)\psi\right) = \frac{1}{2}\frac{\partial}{\partial x}\left(D(x)\frac{\partial \psi}{\partial x}\right), \qquad x \equiv \frac{\Delta p}{p}$$

$$F(x) = f_0 \sum_{n=-\infty}^{\infty} \frac{G_1(x,\omega_n)}{\varepsilon(\omega_n)}\left(1 - A(\omega_n) e^{-i\omega_n T_0}\right) e^{i\omega_n T_2 \eta_2 x}$$

$$D(x) = \sum_{n=-\infty}^{\infty} \frac{1}{\left|\varepsilon(\omega_n)^2\right|}\left[\frac{2\pi e^2 P_{Unoise}(\omega_n)}{T_0^{\,2}\left(\gamma\beta^2 mc^2\right)^2}\left|\frac{Z_k(\omega_n)}{Z_{ampl}}\right|^2 + f_0 \left|G_1(x,\omega_n)\left(1 - A(\omega_n) e^{-i\omega_n T_0}\right)\right|^2 \frac{\psi(x)}{|n\eta|}\right] \ .$$

$$\varepsilon(\omega) = 1 + \left(1 - A(\omega) e^{-i\omega T_0}\right) \int_{\delta \to 0_+} \frac{d\psi(x)}{dx} \frac{G_1(x,\omega) e^{i\omega T_2 \eta_2 x}}{e^{i\omega T_0 (1+\eta x)} - (1-\delta)} dx \ ,$$

where: $\int \psi(x) dx = N \quad , \quad \omega_n = n\omega_0 (1 - \eta x)$

- Taking into account that the betatron size in the pickup is much smaller than the synchrotron size we can factorize the gain for each system or leg:

$G_1(x,\omega) = G_x(x) G_\omega(\omega)$, so that $\mathrm{Im}(G_x(x)) = 0$

Stacktail Pickup Coordinate Response

- Pickup coordinate response coincides well with following formula

$$F(x) = \frac{1}{\pi}\left[\operatorname{atan}\left(\sinh\left(\frac{\pi}{h}\left(x - x_0 + \frac{w}{2}\right)\right)\right) - \operatorname{atan}\left(\sinh\left(\frac{\pi}{h}\left(x - x_0 - \frac{w}{2}\right)\right)\right)\right]$$

- Parameters used in the fitting are

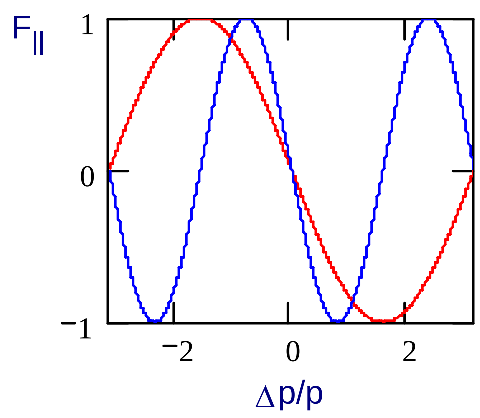
	x_0[cm]	h [cm]	w[cm]
Leg1	1.07	3.2	3.2
Leg2	-0.77	3.1	
Leg3	-2.4	3.1	
Core 2 - 4 outer	-3.5	3	2
Core 2 - 4 inner	-8.5		
Core 4 - 8 outer	-4.88	3.3	1
Core 4 - 8 inner	-7.08		

- Dispersion at pickup is 910 cm (nonlinearity of dispersion on momentum is neglected

Cooling in Debuncher

- Presently, performance of the system does not limit the overall performance
  - Further improvements of stacktail in Accumulator will require improvements of Debuncher cooling
- Both transverse and longitudinal systems will have minor upgrades in May-June of 2007
  - Bands 3 & 4 of transverse systems do not have notch filters to suppress common modes (10-20% improvements for transverse cooling)
    - Installation of filters will results in a factor of 2 gain in power
      - ⇒ this results in reduction of $\Delta p/p$ cooling range therefore filters will be engaged in the second half of the cooling cycle
      - ⇒ The gain in power is mainly due to thermal noise reduction which dominates total power at the end of cooling cycle
  - Installation of additional 2 turn delay notch filter for longitudinal systems
    - Similar to $\perp$ systems will be used in the second half of the cycle

Factor of 2 increase in the gain for the same power



Emittance growth due to parametric heating

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt}=\frac{v_0^{\,2}\beta_{eff}^{\,2}}{4\pi\omega_0 a_{eff}^{\,4}}\sum_{k,m=-\infty}^{\infty}\frac{\psi(\tilde{p}_{km})}{\left|\eta k^3\right|}\left|G(\tilde{p}_{km},\tilde{\omega}_{km})\right|^2\left|\left(1-A(\tilde{\omega}_{km})e^{-i\omega_{km}T_0}\right)\right|^2,\quad p\equiv\frac{\Delta P}{P}$$

where

$$\tilde{p}_{km}=p-\frac{2\nu+m}{\eta k},\quad \tilde{\omega}_{km}=\omega_0\left(2\nu+m+k(1-\eta x)\right)$$

η - slip factor;

$G(p,\omega)$ - single particle gain;

$\psi(p)$ - particle distribution, $\int\psi(p)dp=N$

v_0 - particle velocity; $T_0=1/f_0=2\pi/\omega_0$ is the revolution time

a_{eff} - determines dependence of longitudinal kick on transverse coordinates: $U(x,y)=U_0\left(1\pm\frac{x^2-y^2}{2a_{eff}^{\,2}}\right)$, $a_{eff}\approx 1.87$ cm

$A(\omega)$ takes into account the notch filter response: $A(\omega)=1$ - for ideal notch filter.

$$\beta_{eff}^{\,2}=\frac{1}{\left(\sum_{i=1}^{N}\left|\kappa_i\right|\right)^2}\sum_{i,j=1}^{N}\kappa_i\kappa_j\left(\beta_0^{\,2}+2s_i's_j'+\frac{s_i'^2 s_j'^2}{\beta_0^{\,2}}-\left(s_i'-s_j'\right)^2\right)$$

s_i' - the longitudinal coordinate of k-th kicker relative to the point of minimum beta-function, β_0

$\kappa_i=\pm 1$ for horizontal and for vertical kickers respectively

Emittance Growth due to Offsets of Stack-tail Kickers and Non-zero Dispersion in Kickers

$$\frac{d\varepsilon(p)}{dt}=\frac{\omega_0\beta_{kick}}{4\pi}\sum_{k,m=-\infty}^{\infty}\frac{\psi(p_{km})}{|\eta k|}\left|G(p_{km},\omega_{km})\right|^2\left|D_{eff}(\omega_{km})\right|^2\left|\left(1-A(\omega_{km})e^{-i\omega_{km}T_0}\right)\right|^2$$

where:

$$D_{eff}(\omega)=\frac{v_0 X(\omega)}{\omega a_{eff}{}^2}+\frac{D'_{kick}\beta_{kick}+\alpha_{kick}D_{kick}-iD_{kick}}{\beta_{kick}}$$

$$p_{km}=p-(\nu+m)/\eta k\,,\quad \omega_{km}=\omega_0\left(\nu+m+k(1-\eta p)\right)$$

Sum over kicker offsets $x_i(\omega)$ yields their effective strength

$$X(\omega)=\left(\sum_{i=1}^{N_{kicker}}\kappa_i x_i(\omega)+\frac{\alpha_{kick}-i}{\beta_{kick}}\sum_{i=1}^{N_{kicker}}\kappa_i s_i x_i(\omega)\right)\Bigg/\sum_{i=1}^{N_{kicker}}|\kappa_i|$$

$\beta_{kick},\alpha_{kick},D_{kick},$ and D'_{kick} are the beta- and alpha-functions, and dispersion and dispersion prime in the kicker section center

s_i - the longitudinal coordinate of *k*-th kicker relative to the kicker section center

Open Loop response for Stack-tail with Offsets of Kickers and non-zero Dispersion in Kickers

- Longitudinal response is

$$S_{\parallel}(\omega) = -\left(1 - A(\omega)e^{-i\omega T_0}\right) \int_{\delta \to 0_+} \frac{df_0(p)}{dp} \frac{G(p,\omega)e^{i\omega T_2 \eta_2 p}}{e^{i\omega T_0 (1+\eta\, p)} - (1-\delta)} dp$$

where

T_2 is the pickup-to-kicker time

η and η_2 are slip-factor and partial pickup-to-kicker slip factor ($\Delta t = T_2\ \eta_2\ p$)

- Transverse response is excited by kicker offset and is

$$S_{\perp}(\omega) = i\frac{\sqrt{\beta_{pickup}\beta_{kick}}}{2\,D_{pickup}}\left(1 - A(\omega)e^{i\omega T_0}\right)D_{eff}(\omega) \int_{\delta \to 0_+} \frac{G(p,\omega)e^{i\omega T_2 \eta_2 p}\left[e^{-i\omega T_0 (1+\eta p)} \sin(2\pi\nu_2) + \sin(2\pi\nu_1)\right]}{\cos(\omega T_0 (1+\eta p)) - \cos(2\pi\nu) + i\delta \sin(\omega T_0 (1+\eta p))} \frac{df_0(p)}{dp} dp$$

where D_{pickup} and β_{pickup} are the dispersion and beta-function in the pickup

ν, ν_1 and ν_2 are the betatron tune and the partial kicker-to-pickup and pickup-to-kicker tunes, correspondingly

- The total response is

$$S(\omega) = S_{\parallel}(\omega) + S_{\perp}(\omega)$$