Simulation Study of Beam Accumulation with Moving Barrier Bucket and Electron Cooling

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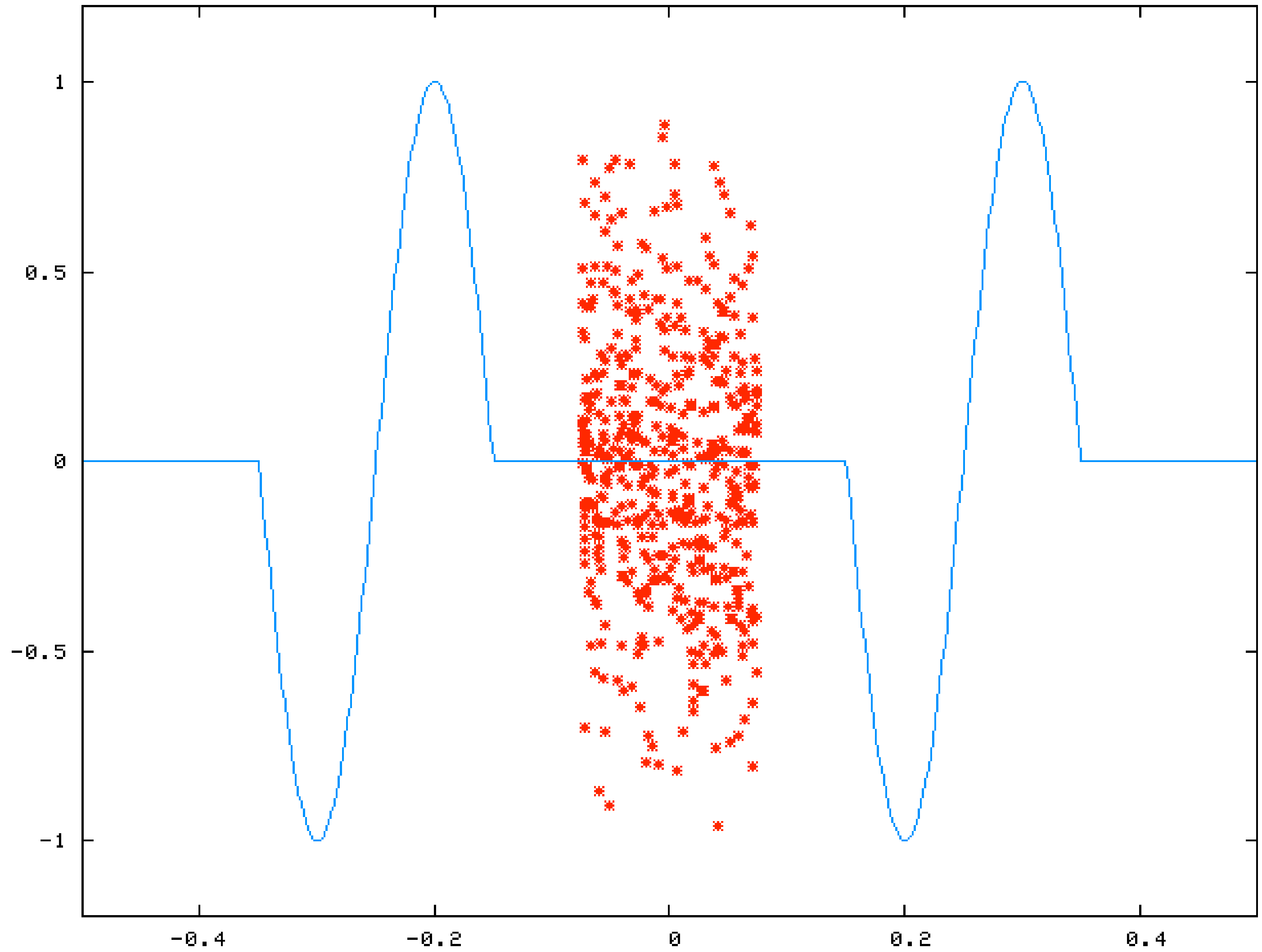
Related paper in the Workshop

1. C. Dimopoulou et al., “ Longitudinal Accumulation of Ion Beams in the Experimental Storage Ring Supported by Electron Cooling”
2. T. Kikuchi et al., “ Influences of Space Charge Effect during Ion Accumulation using Moving Barrier Bucket Cooperated with Beam Cooling”

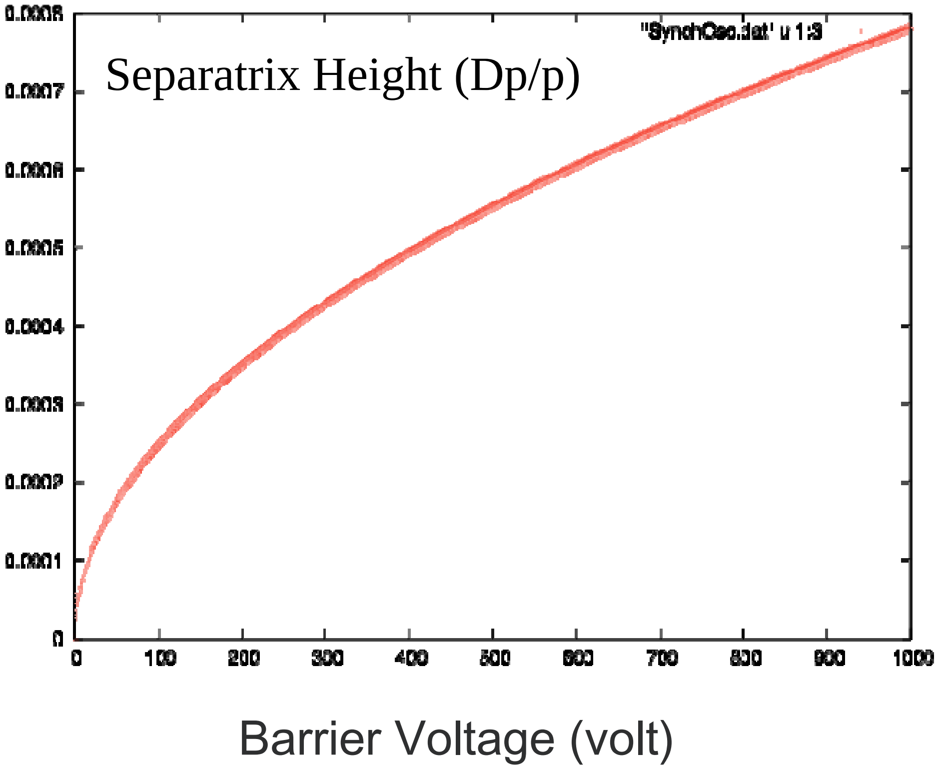
Movie of procedure of proposed accumulation method (Ie=0.5A at ESR)



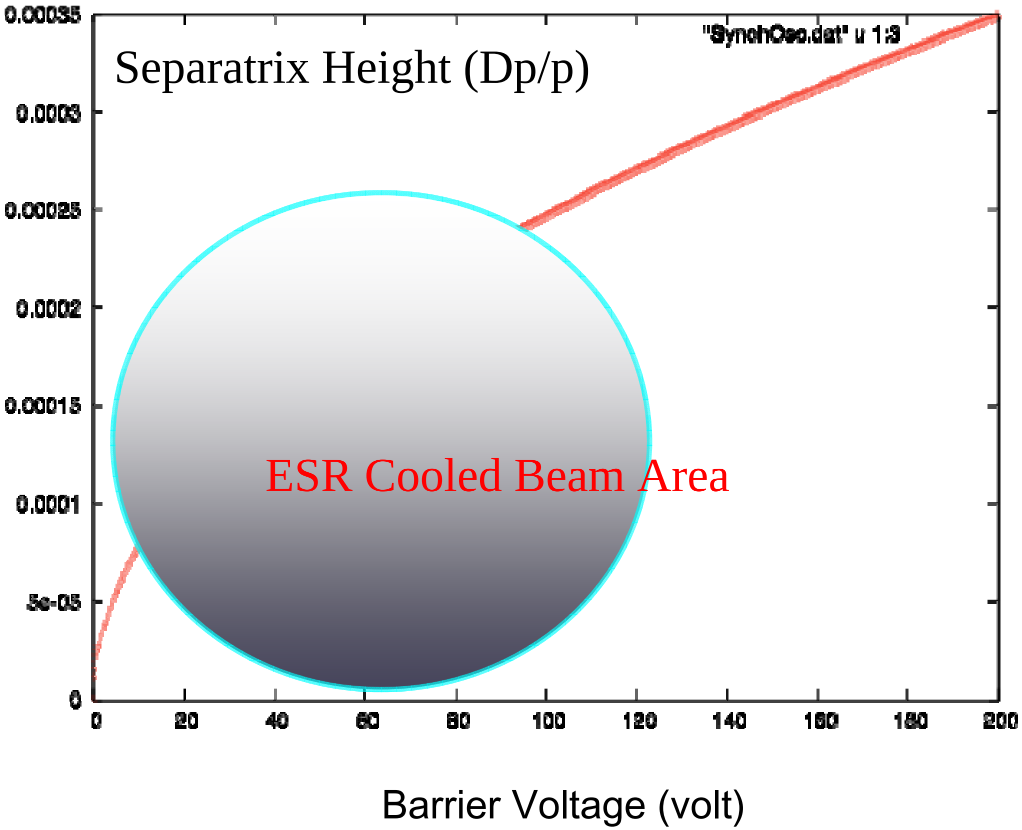
Procedure of Proposed Accumulation Method

1. Beam bunch is injected in the longitudinal empty space prepared by barrier pulses.
2. Barrier voltage is switched off and beam is debunched.
3. Electron cooling is applied continuously and the momentum spread of debunched beam is cooled down within a few seconds to the IBS limit.
4. Barrier voltages are adiabatically switched on and the cooled beam is compressed by moving the barrier pulses, and beam is confined in the longitudinal stacking area
5. Next batch is injected in the empty slot prepared by the barrier pulses.
6. Repeat the process 1~5.

Separatrix Height vs Sin Wave Barrier Voltage



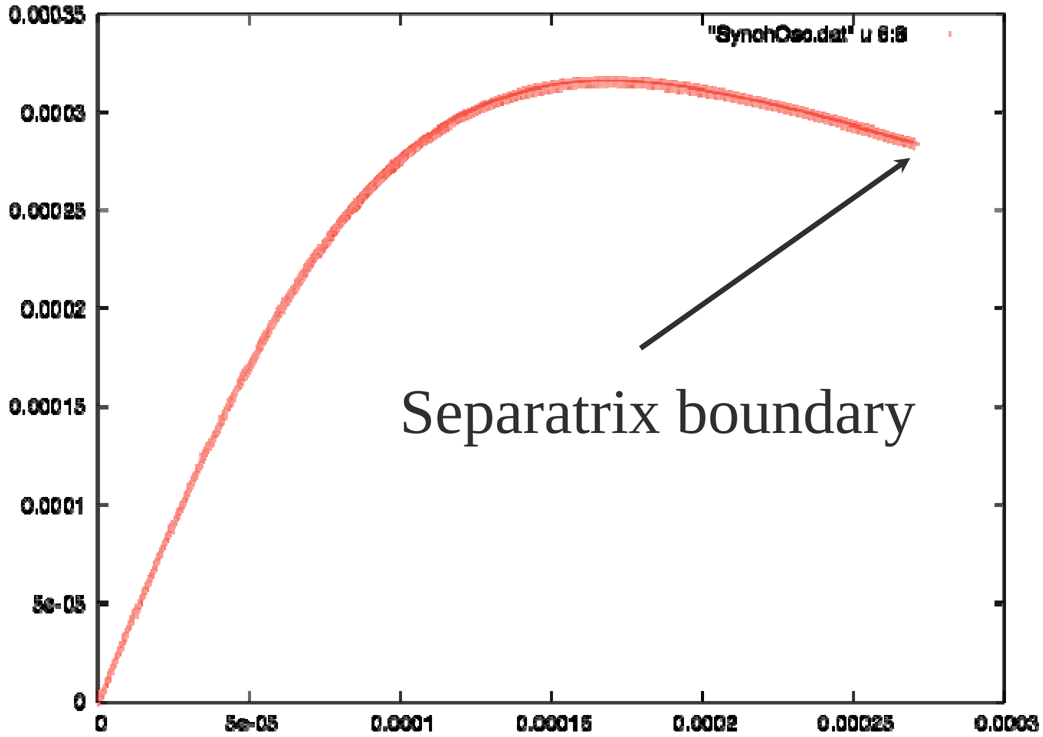
ESR 65 MeV/u 18Ar40



$$\Delta E_b = \left[\frac{QeV_0 T_1 2\beta^2 E_{tot1}}{A\pi\eta T_0} \right]^{1/2}$$

Synchrotron Tune (Barrier voltage=120V)

Nyus=T0/Ts



Momentum spread Dp/p

$$T_s = 2\frac{T_2}{\eta}\frac{\beta^2 E_{tot}}{\delta E} + 4\frac{\delta E}{\varepsilon e V_0} T_0$$

$$\nu_s = T_0 / T_s$$

Key subjects

1. Strong electron cooling.

DeltaP/P has to be reduced from typically 1e-3 to 1e-4 within a few seconds, cycle period.

2. Low barrier pulse voltage.

A small momentum spread 1e-4 after cooling will benefit a low barrier voltage, say less than 2 kV, to confine a cooled beam.

3. Limiting factors.

Intra-Beam-Scattering effects may limit the minimum momentum spread of cooled beam. Space charge repulsion force may significantly decrease the barrier voltage.

4. Kicker magnet.

An injection kicker magnet of fast ramping and falling time, say 100 nsec, with small jitter, is crucial to minimize the particle loss.

How calculate the beam dynamics ?

1. Initial values of momentum spread and transverse emittances are assigned for each injected particle as Gaussian or uniform random.

2. Longitudinal phase equations are solved for the moving barrier voltages.

3. Electron cooling drag force is derived from Parkhomchuk empirical formula for energy spread and transverse emittances, and cooling process are calculated for each particle.

4. IBS heating rates of momentum spread and transverse emittances, are calculated based on the Martini analytical formula. Sigma values of Dp/p and emittances are derived from the informations of phase points of injected and accumulated particles.

5. Longitudinal space charge repulsion force is calculated with PIC method using particle distribution along the ring orbit.

6. In the calculation, LOST particle is labelled when the particle time location at the next beam injection is within the kicker period (+/- 250 n sec) or the energy deviation is larger than the ring acceptance limit.

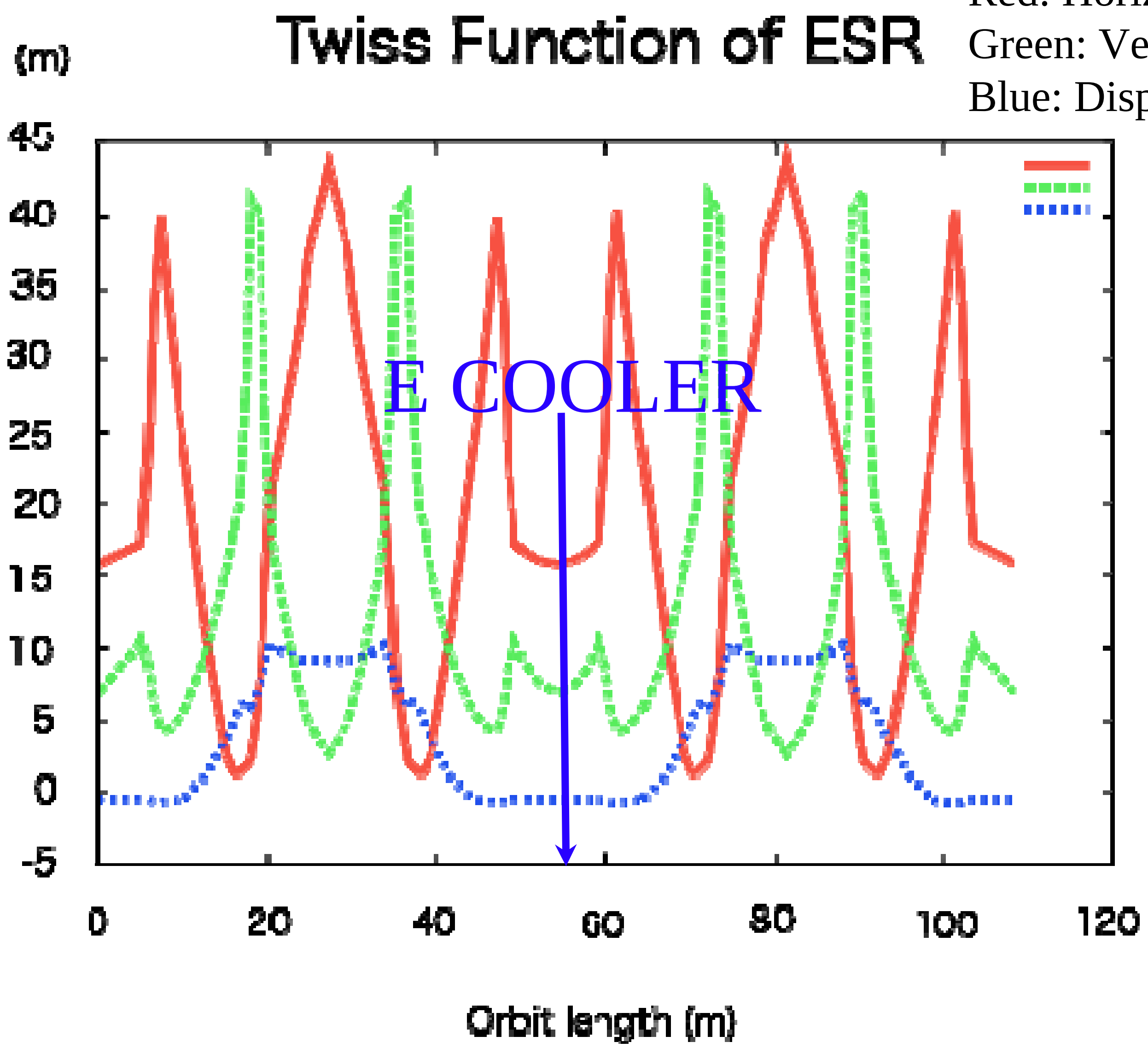
Calculation of Electron Cooling and IBS at ESR

- Ar beam, Kinetic Energy=65.3 MeV/u, Q/A=18/40, Horizontal/Vertical emittance (1 sigma)=1.35e-6 m.rad, Momentum spread (1 sigma)=8.25e-4.

- Cooler length=1.8 m, Electron diameter=0.05 m, Effective electron temperature=1e-3 eV, Beta function at cooler (H/V)=16/6.88 m, Electron current=0.1~0.3 A

- Calculation has been performed with two methods, Single particle calculation and Multi-particle calculation.

IBS effects are averaged over around the ring orbit.



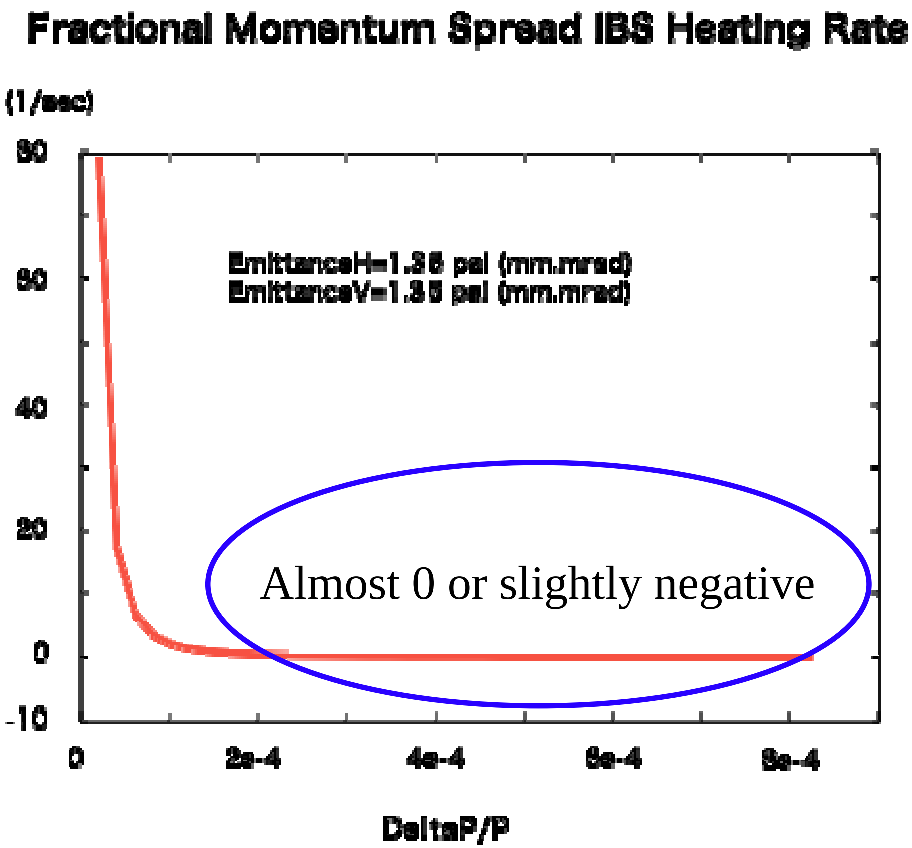
Red: Horizontal beta function.
Green: Vertical beta function
Blue: Dispersion

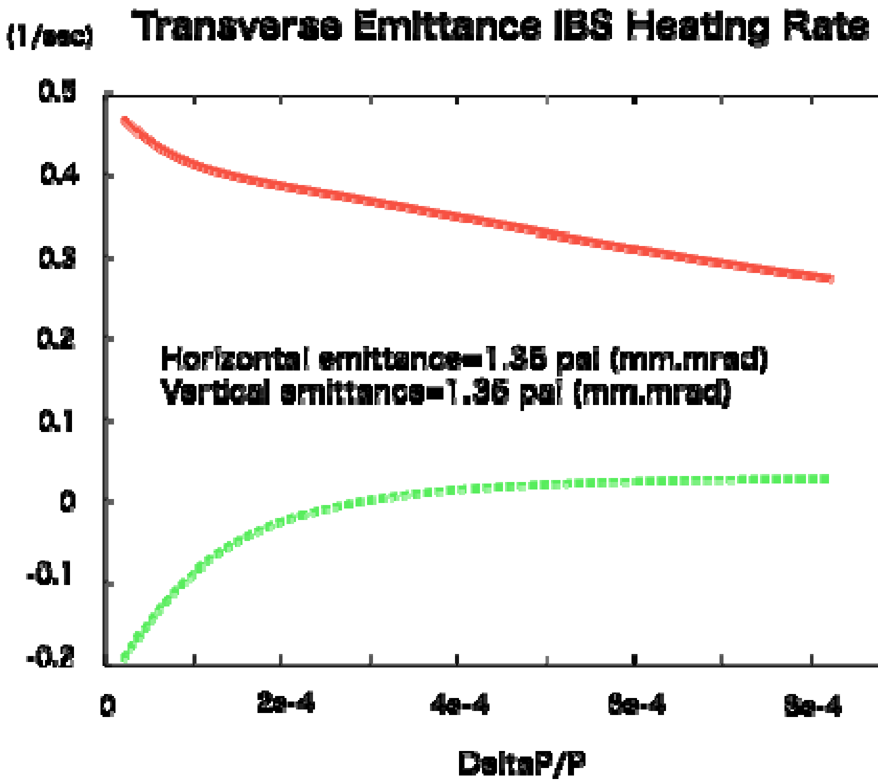
At the electron cooler
BetaH=16m
BetaV=7m
Dispersion= 0 m

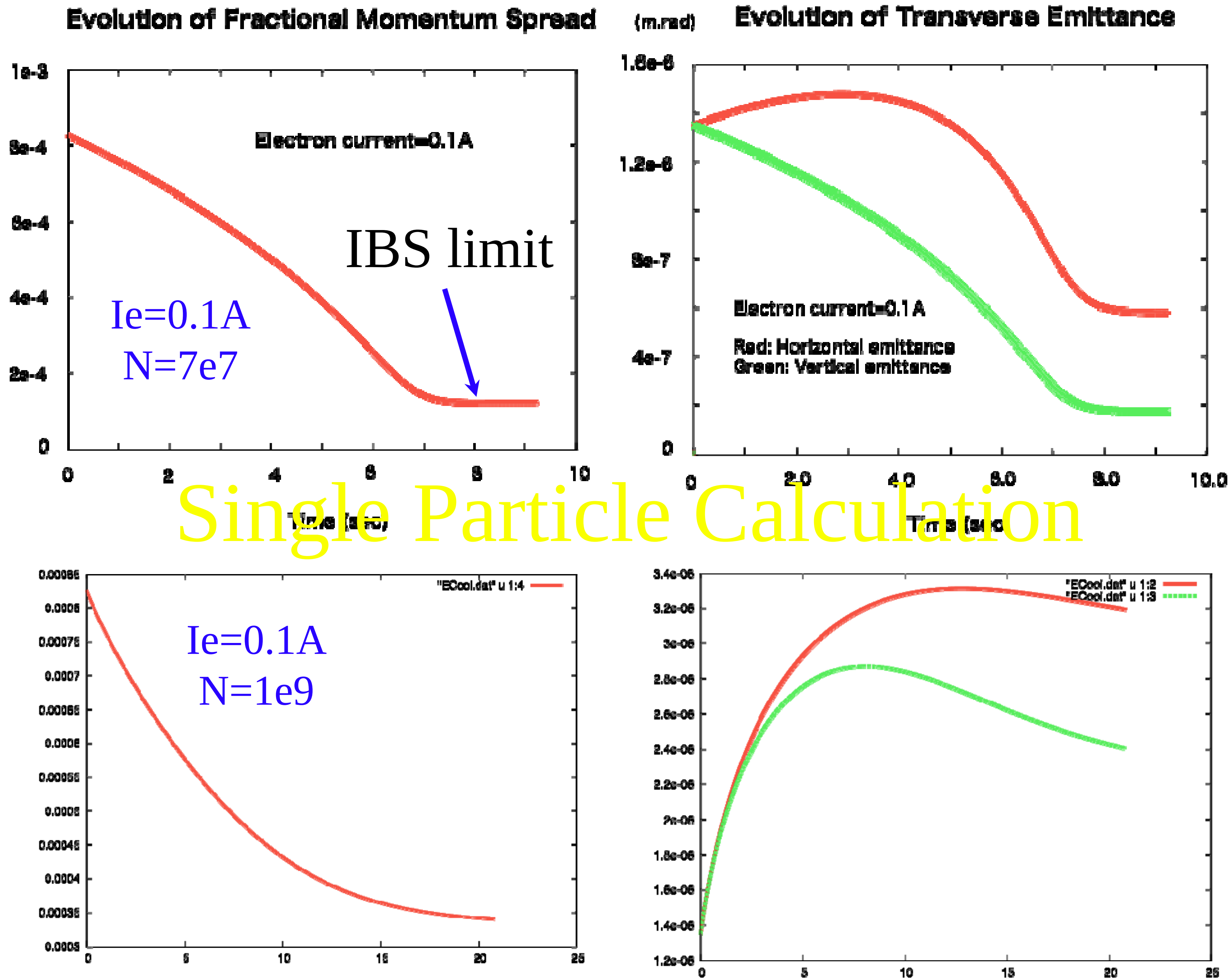
Tune values
Qh=2.268
Qv=2.273

IBS Heating Rate at ESR

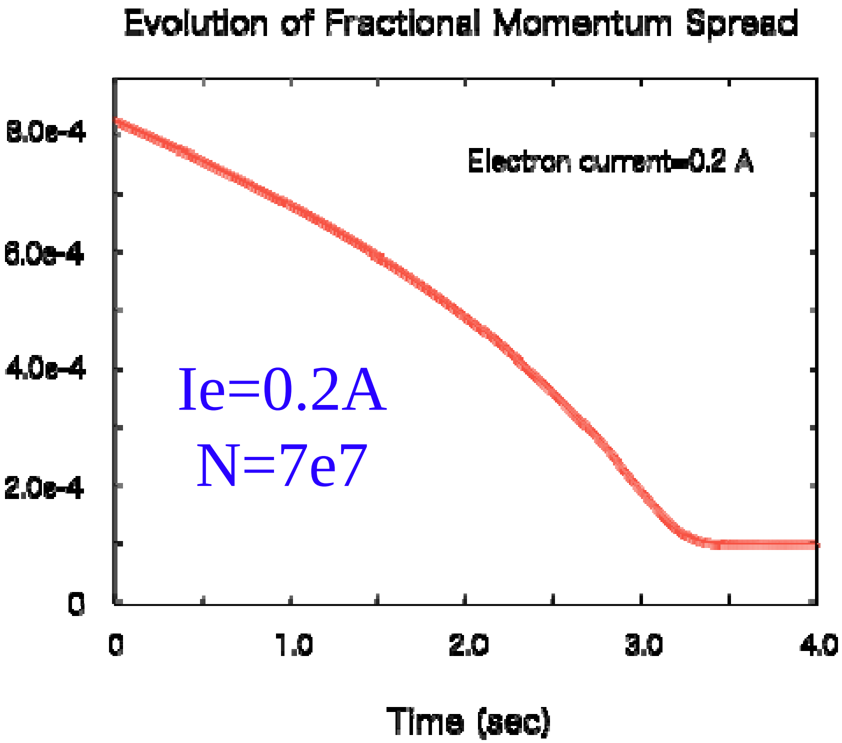
Beam parameters
40Ar18+, 65.3 MeV/u, N=7e7
Transverse emittances are fixed, and fractional momentum spread is varied.

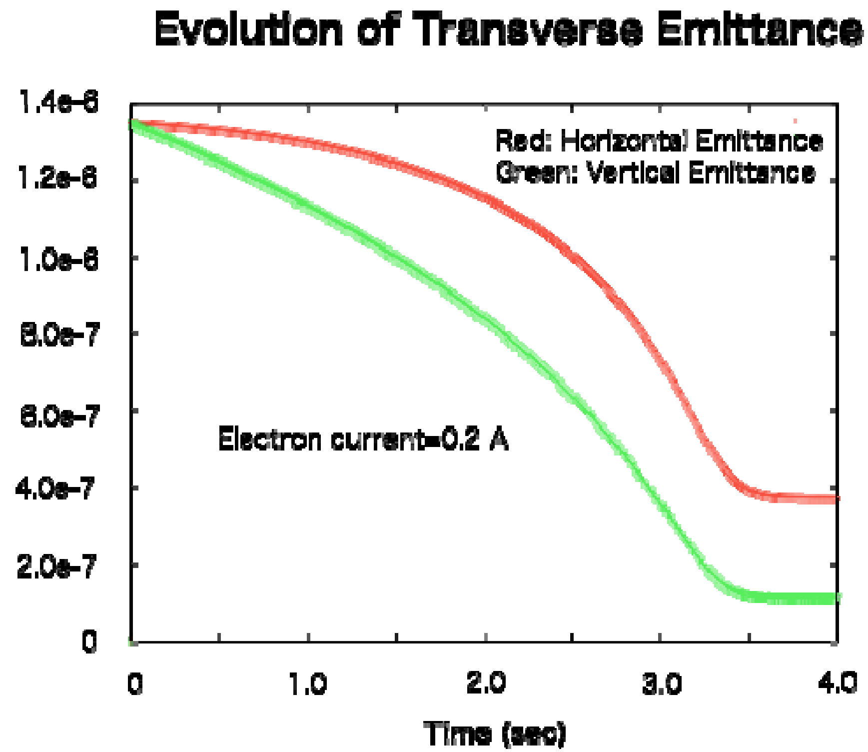


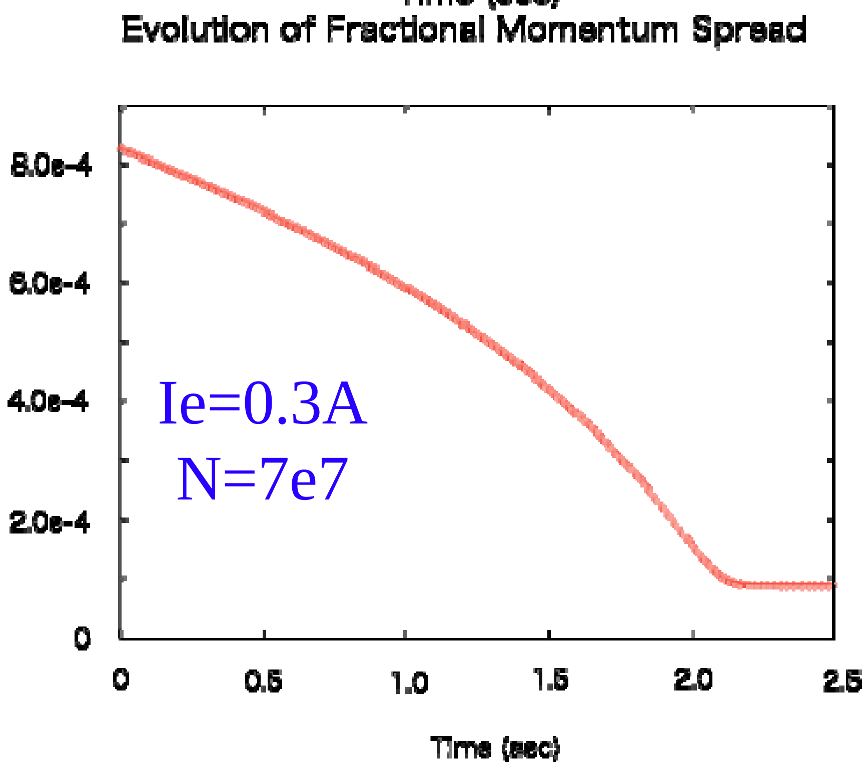


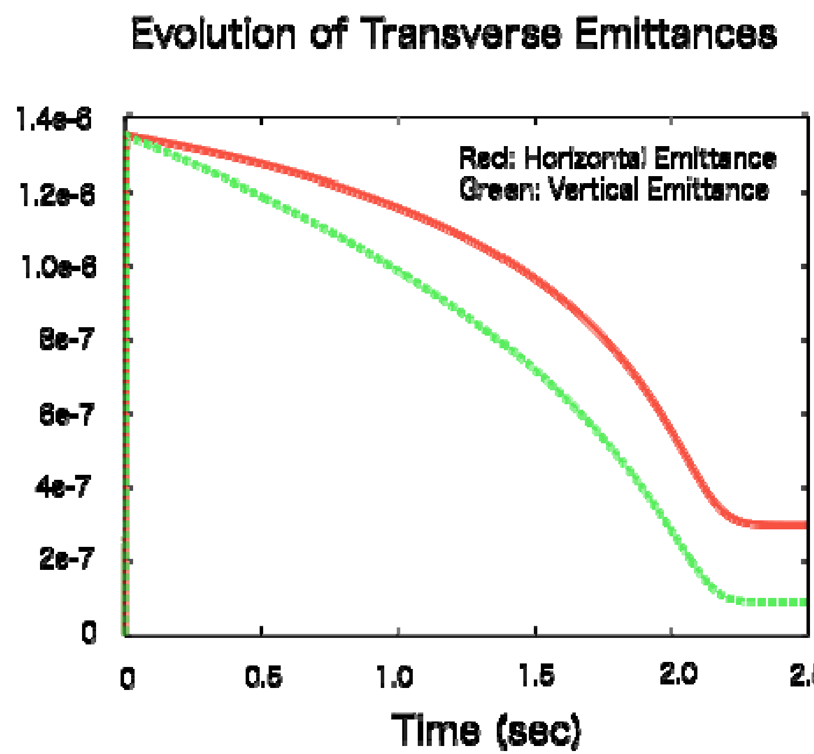


Evolution of Fractional Momentum Spread
Electron current=0.1A
IBS limit
Ie=0.1A
N=7e7
1e-3
8e-4
6e-4
4e-4
2e-4
0
0
2
4
6
8
10
(m.rad)
Evolution of Transverse Emittance
1.6e-6
1.2e-6
8e-7
4e-7
0
Electron current=0.1A
Red: Horizontal emittance
Green: Vertical emittance
0
2.0
4.0
6.0
8.0
10.0
Single Particle Calculation
"ECool.dat" u 1:4
Ie=0.1A
N=1e9
0.00085
0.0008
0.00075
0.0007
0.00065
0.0006
0.00055
0.0005
0.00045
0.0004
0.00035
0.0003
0
5
10
15
20
25
"ECool.dat" u 1:2
"ECool.dat" u 1:3
3.4e-06
3.2e-06
3e-06
2.8e-06
2.6e-06
2.4e-06
2.2e-06
2e-06
1.8e-06
1.6e-06
1.4e-06
1.2e-06
0
5
10
15
20
25


Evolution of Fractional Momentum Spread
Electron current=0.2 A
Ie=0.2A
N=7e7
8.0e-4
6.0e-4
4.0e-4
2.0e-4
0
0
1.0
2.0
3.0
4.0
Time (sec)

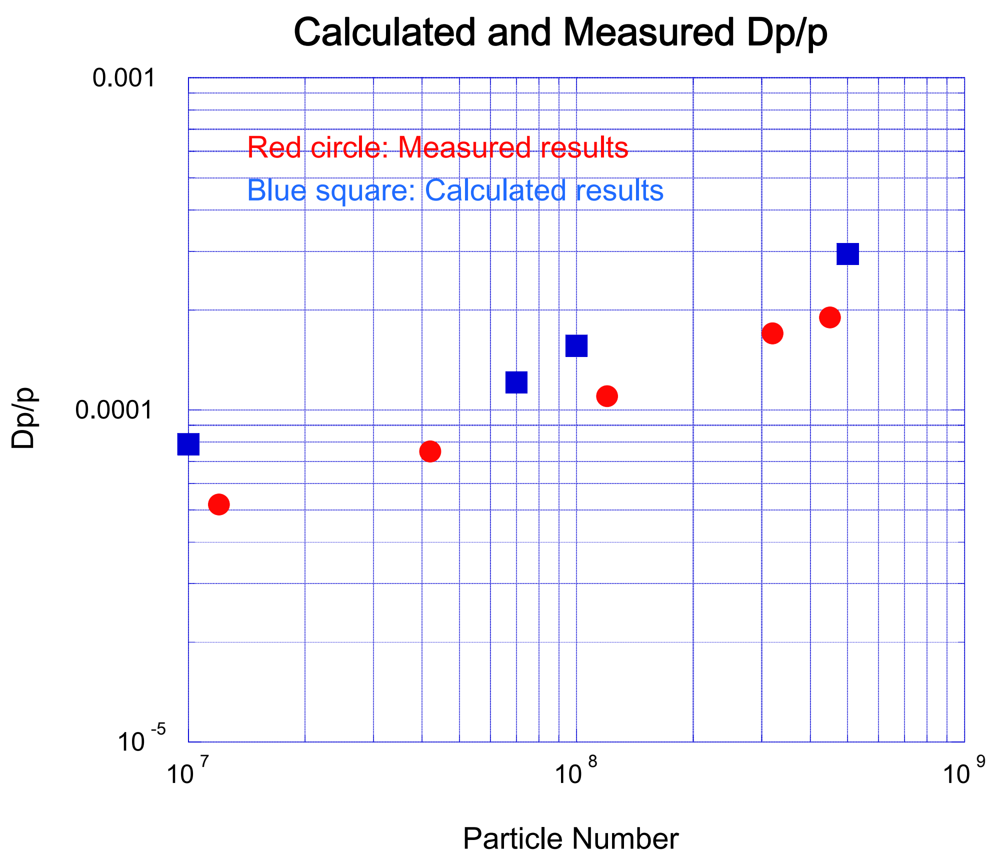

Evolution of Transverse Emittance
Red: Horizontal Emittance
Green: Vertical Emittance
Electron current=0.2 A
1.4e-6
1.2e-6
1.0e-6
8.0e-7
6.0e-7
4.0e-7
2.0e-7
0
0
1.0
2.0
3.0
4.0
Time (sec)

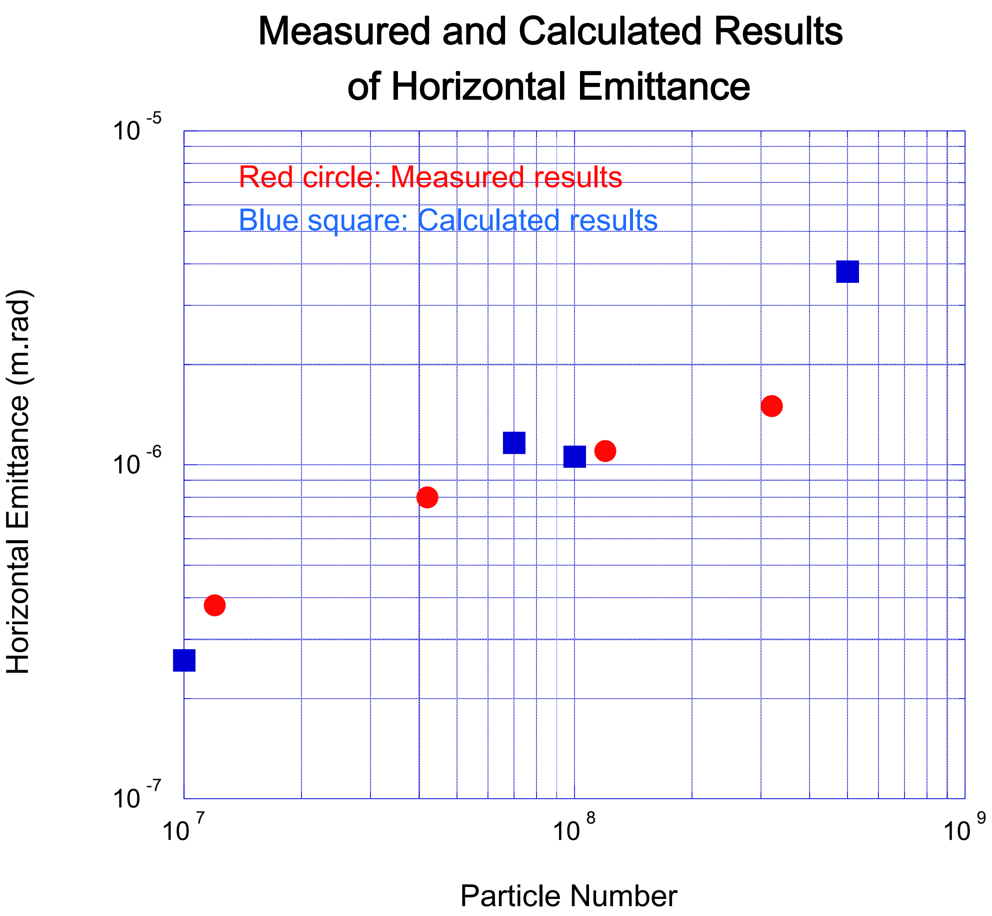

Evolution of Fractional Momentum Spread
Ie=0.3A
N=7e7
8.0e-4
6.0e-4
4.0e-4
2.0e-4
0
0
0.5
1.0
1.5
2.0
2.5
Time (sec)


Evolution of Transverse Emittances
Red: Horizontal Emittance
Green: Vertical Emittance
1.4e-6
1.2e-6
1.0e-6
8e-7
6e-7
4e-7
2e-7
0
0
0.5
1.0
1.5
2.0
2.5
Time (sec)

Comparison of Measured and Calculated Equilibrium Values of 65 MeV/u Ar Beam at ESR

Ie=100mA, Coasting beam



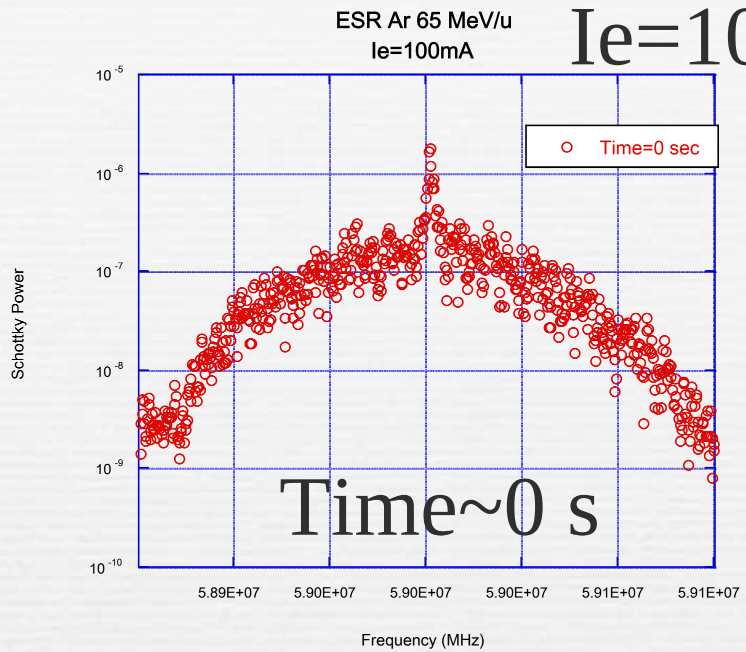


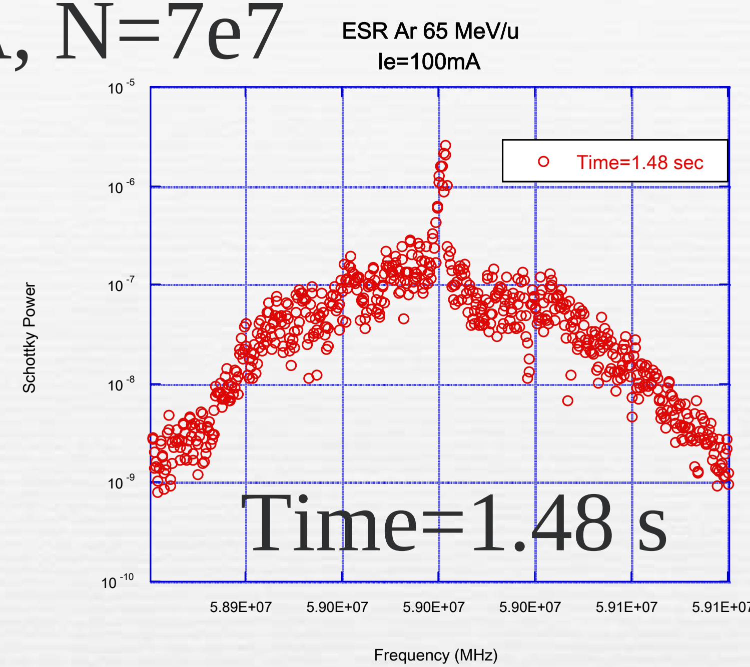
$$\frac{\Delta p}{p}(2\sigma) = 1.0 * 10^{-6.64} \times (N/B)^{0.33} \times (Ie(A)/0.1)^{-0.3}$$

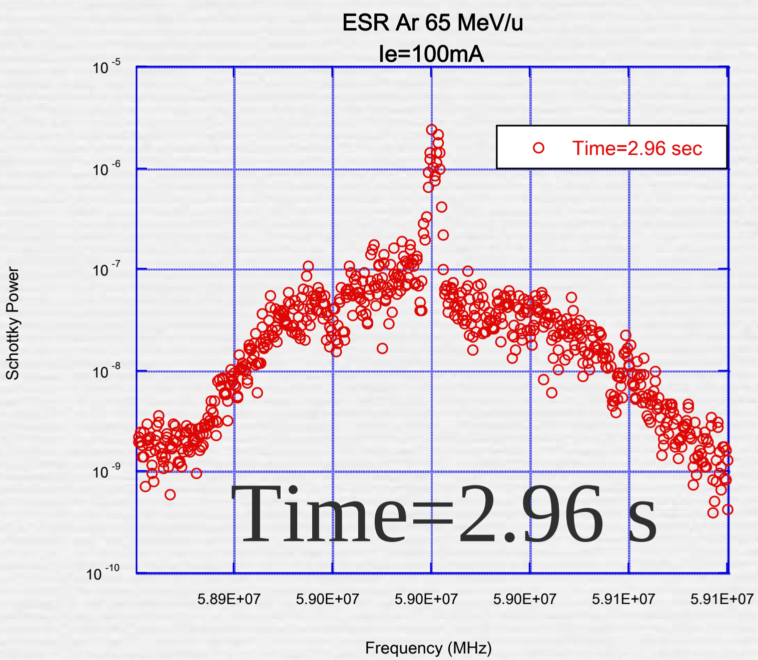
Scaling laws

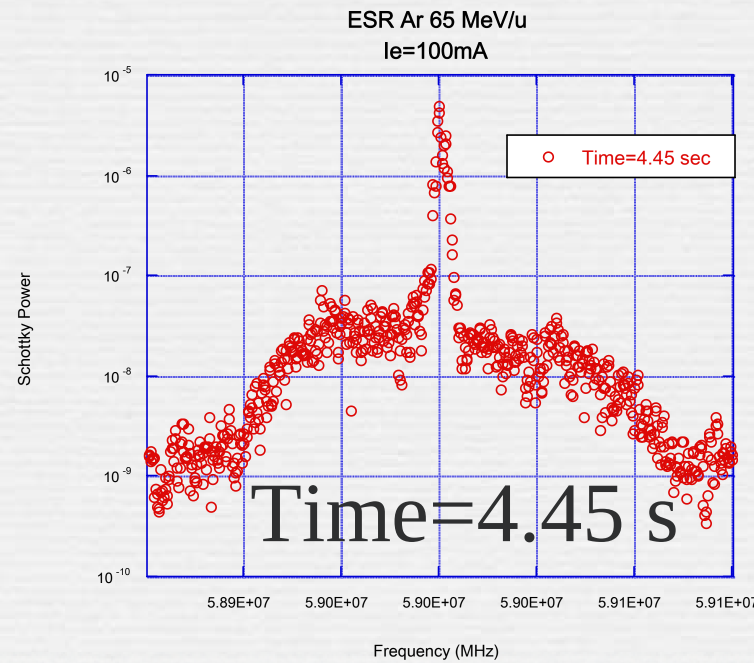
Measured Evolution of Ar Beam at ESR

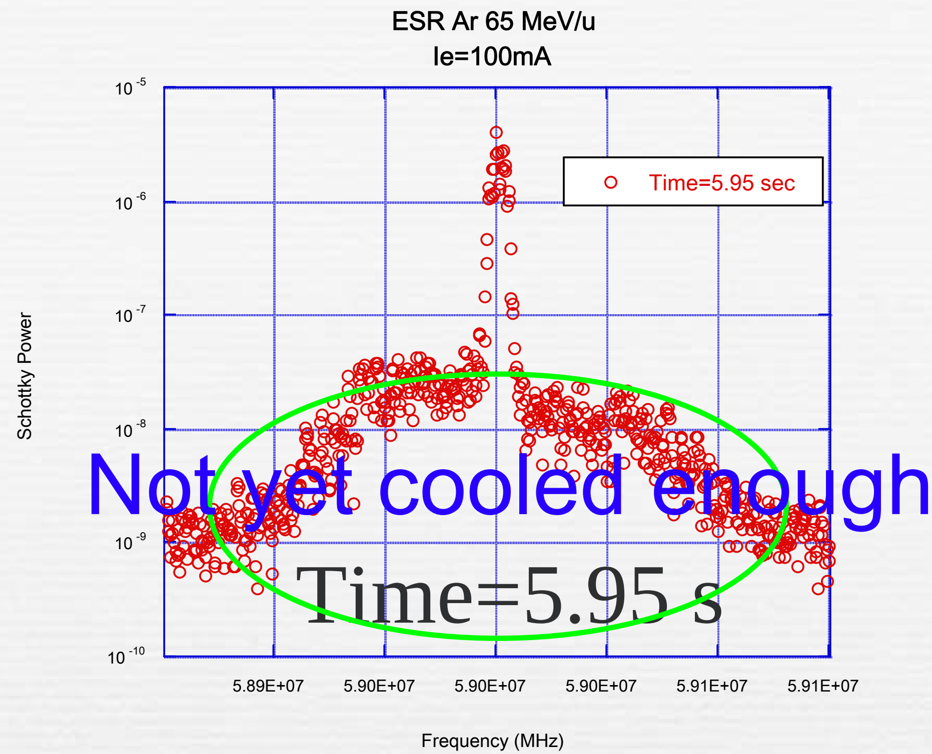
Ie=100 mA, N=7e7

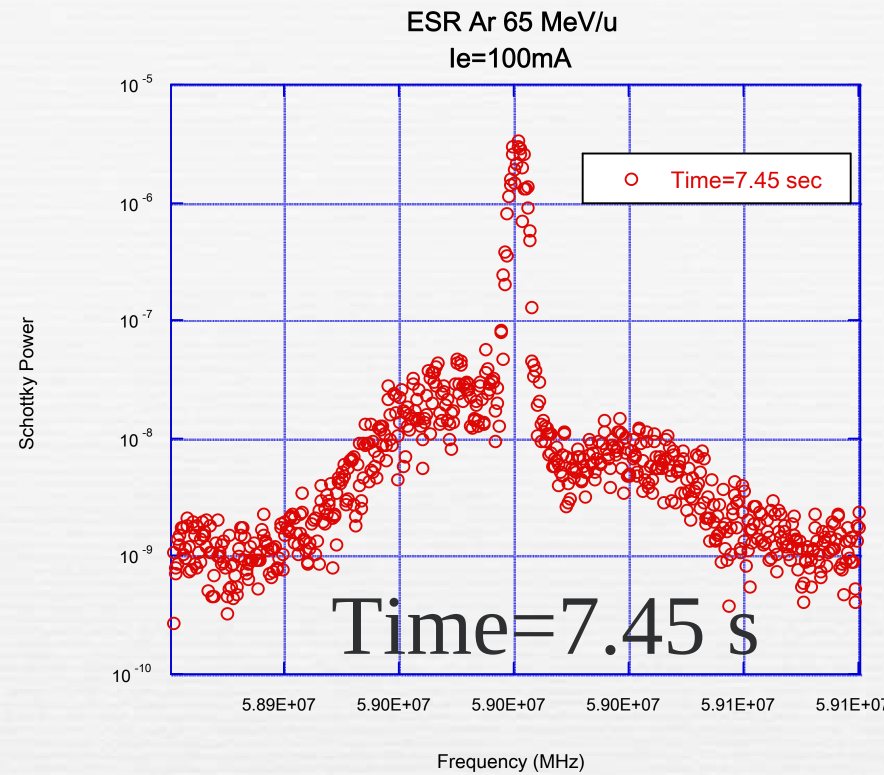


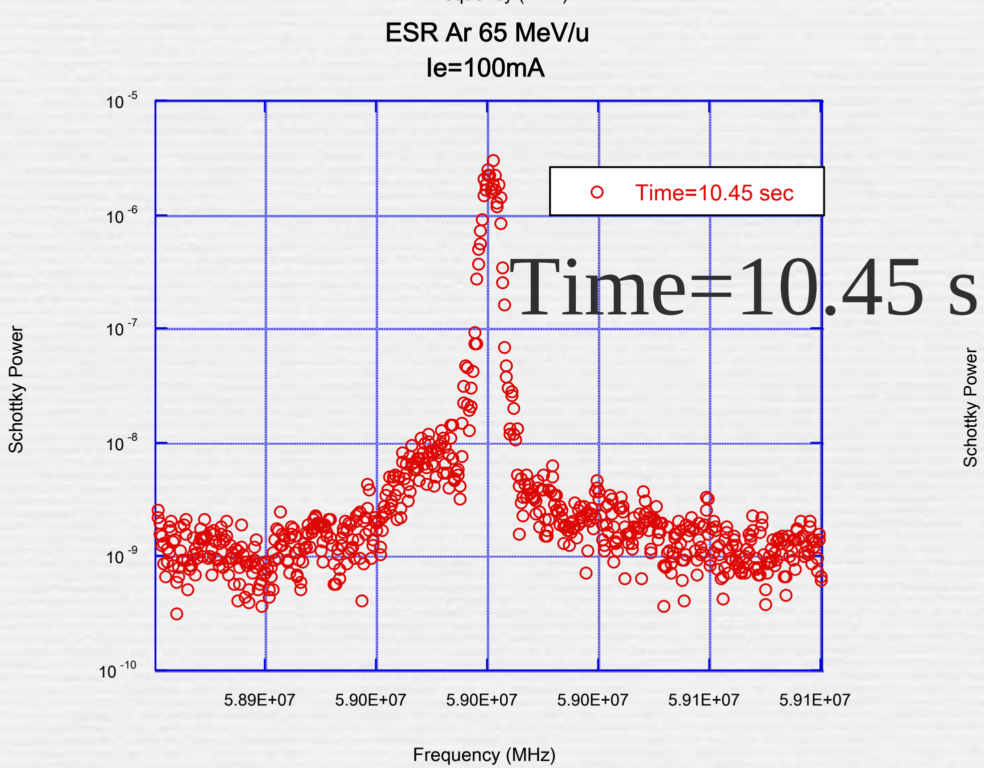


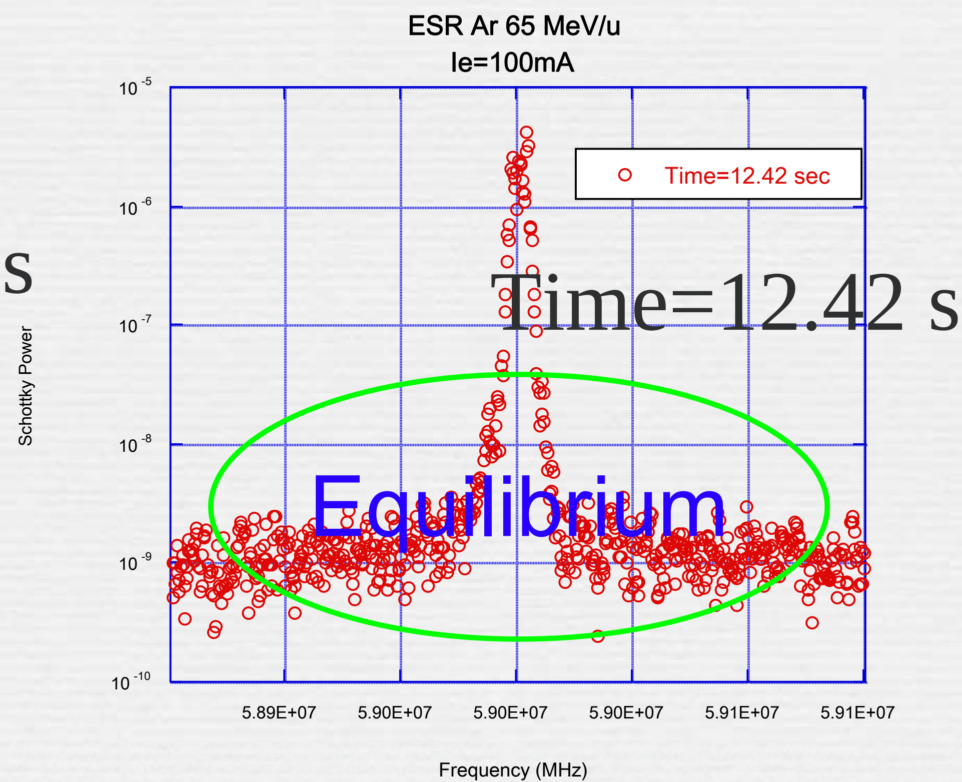




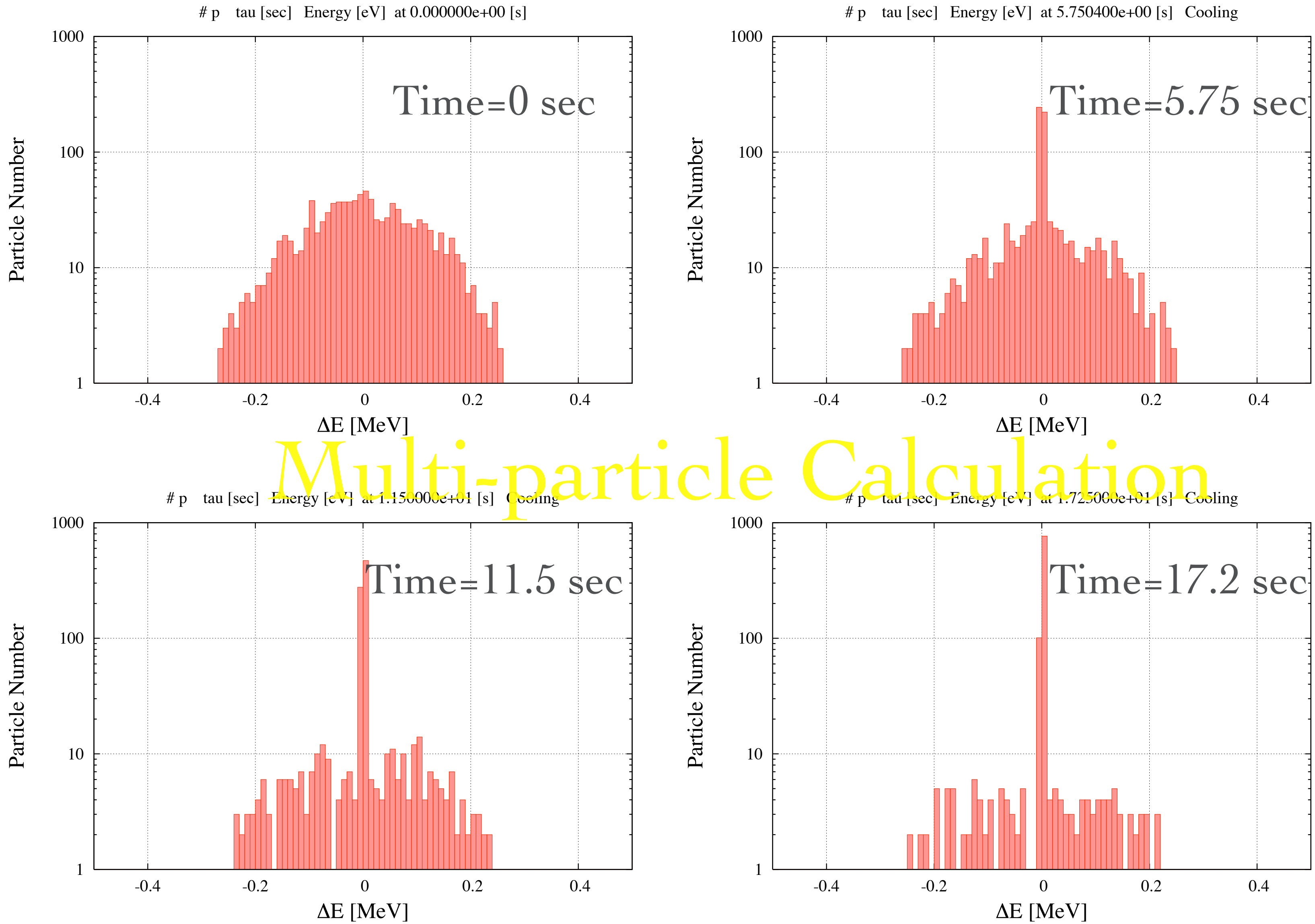

ESR Ar 65 MeV/u
Ie=100mA
Time=5.95 sec
Schottky Power
Frequency (MHz)
Not yet cooled enough
Time=5.95 s


ESR Ar 65 MeV/u
Ie=100mA
Time=7.45 sec
Schottky Power
Frequency (MHz)
Time=7.45 s


ESR Ar 65 MeV/u
Ie=100mA
Time=10.45 sec
Schottky Power
Frequency (MHz)
Time=10.45 s


ESR Ar 65 MeV/u
Ie=100mA
Time=12.42 sec
Schottky Power
Frequency (MHz)
Time=12.42 s
Equilibrium

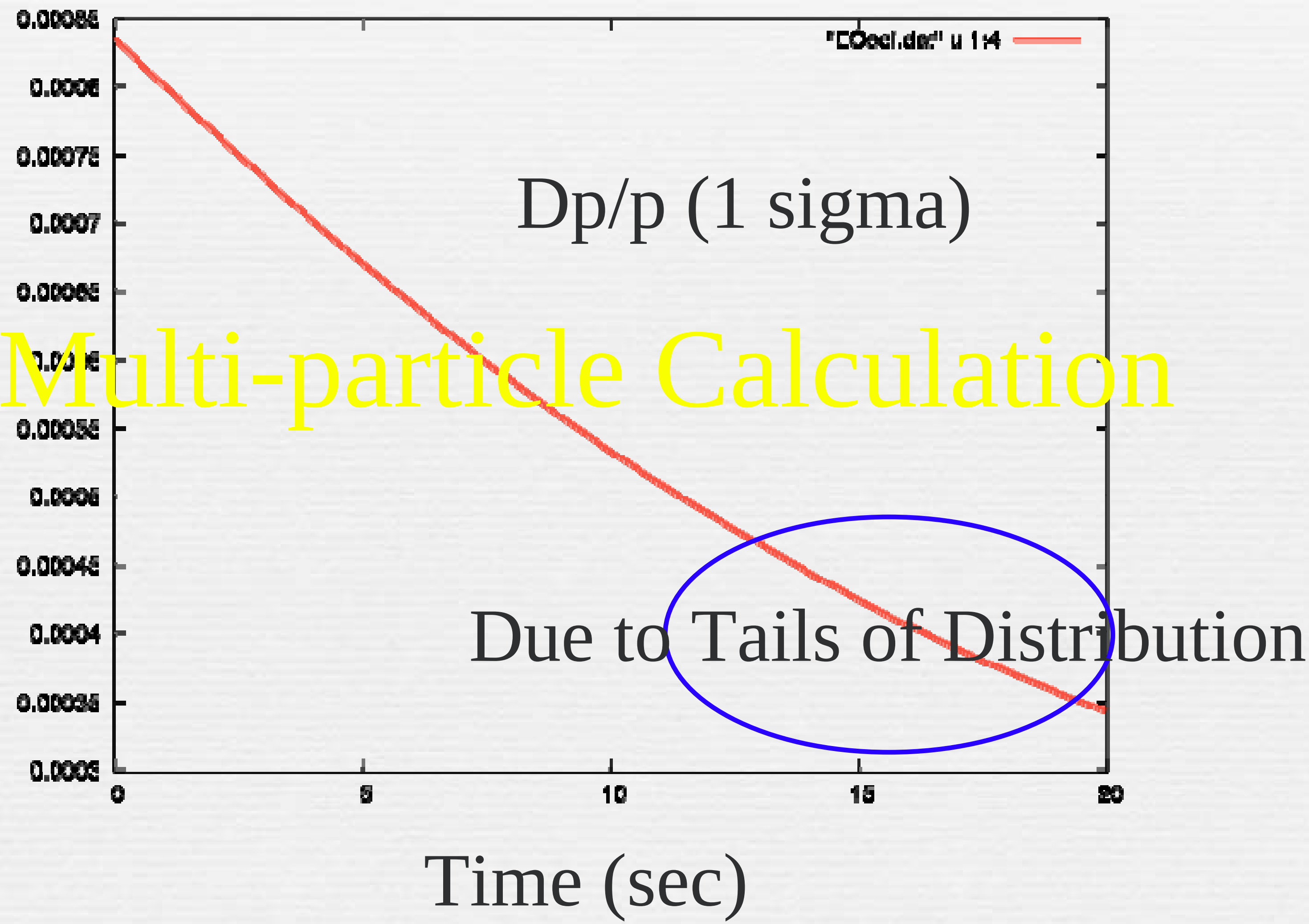
Calculated Energy Spectrum (Ie=0.1A)



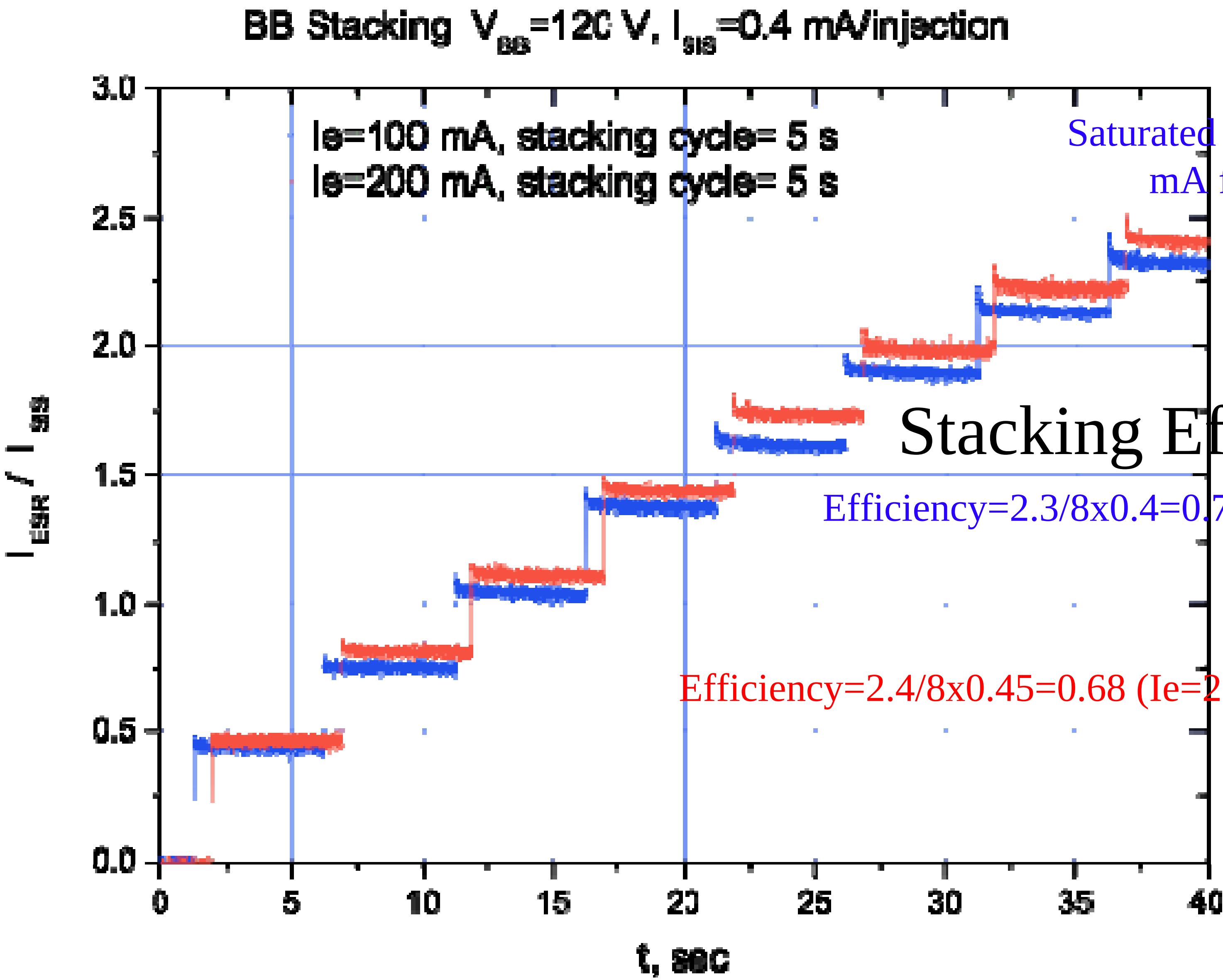
Electron Cooling, Ie=0.1A

(Multi particle calculation)

Initial distribution: Ar 65.3 MeV/u
Gaussian truncated at +/- 3 sigma,
DeltaP/P(1 sigma)=8.25e-4
Transverse emittance(1 sigma)=1.35e-6 m.rad



Measured stacking process



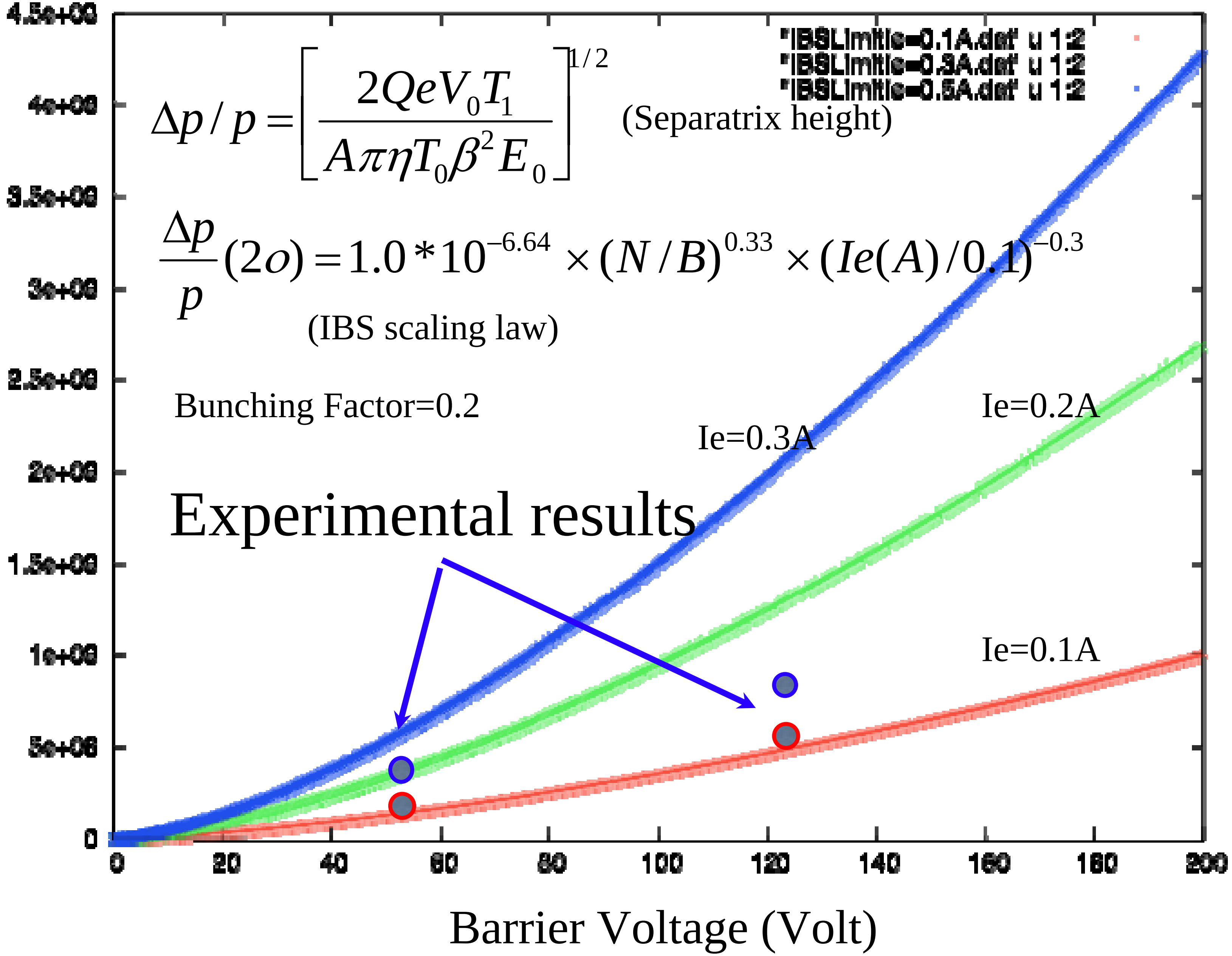
Saturated stacked current=1.1~1.2 mA for Ie=100~200mA

Stacking Efficiency~70 %

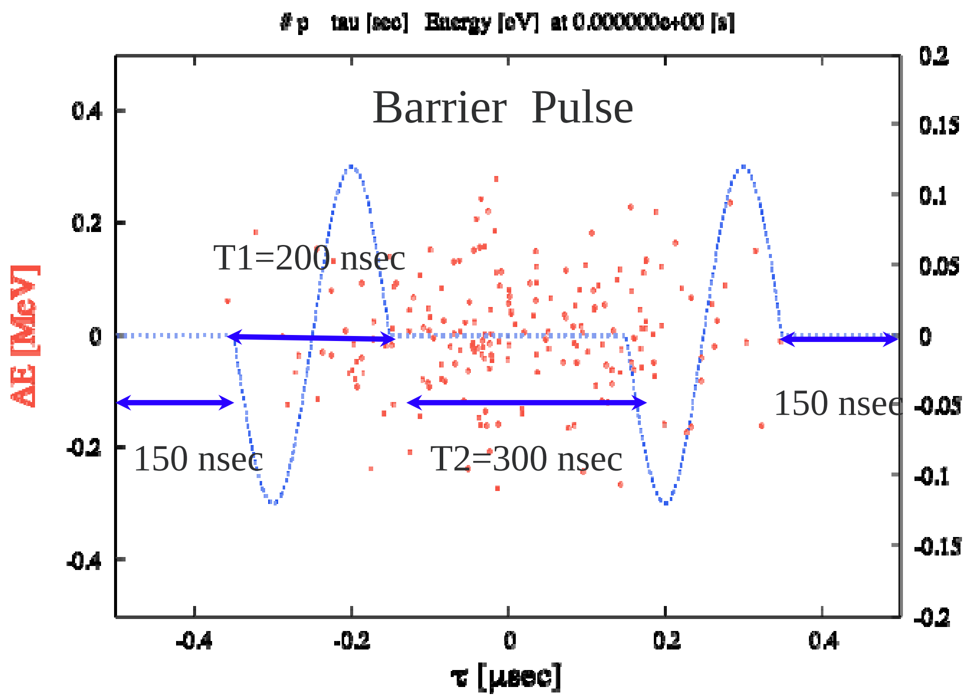
Efficiency=2.3/8x0.4=0.72 (Ie=100mA)

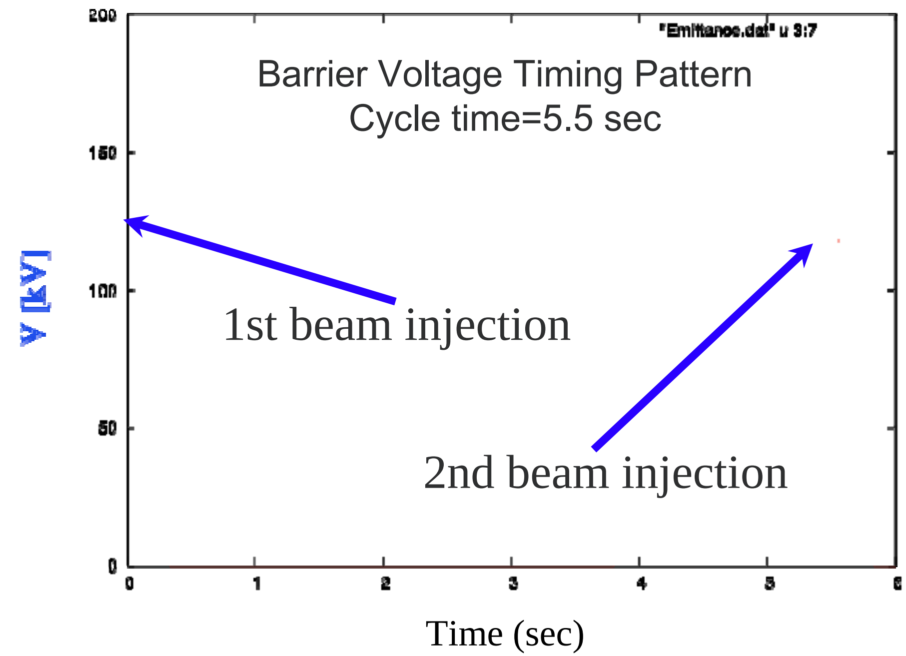
Efficiency=2.4/8x0.45=0.68 (Ie=200mA)

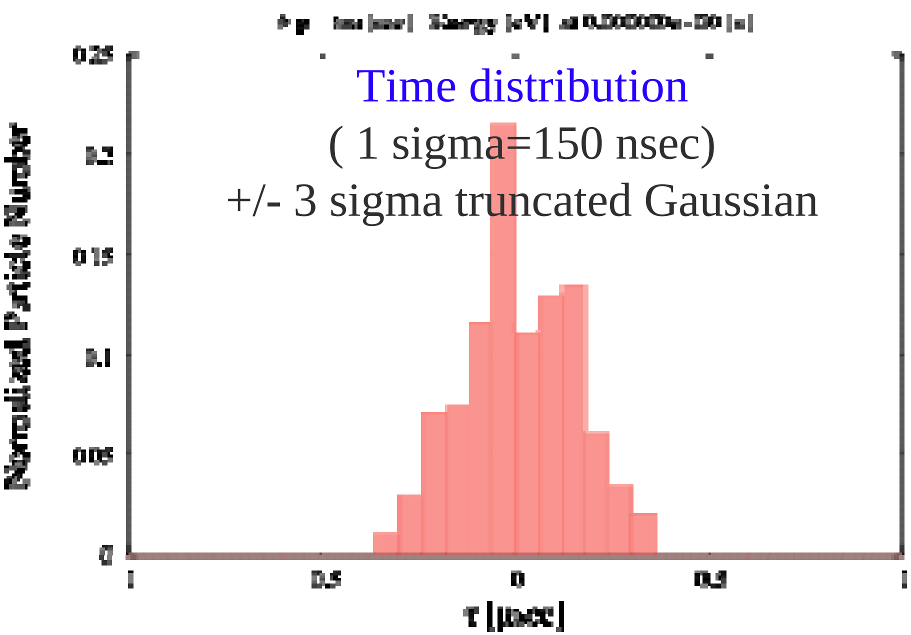
Calculation of Accumulated Ion Numbers limited by IBS with ESR Empirical IBS Formula

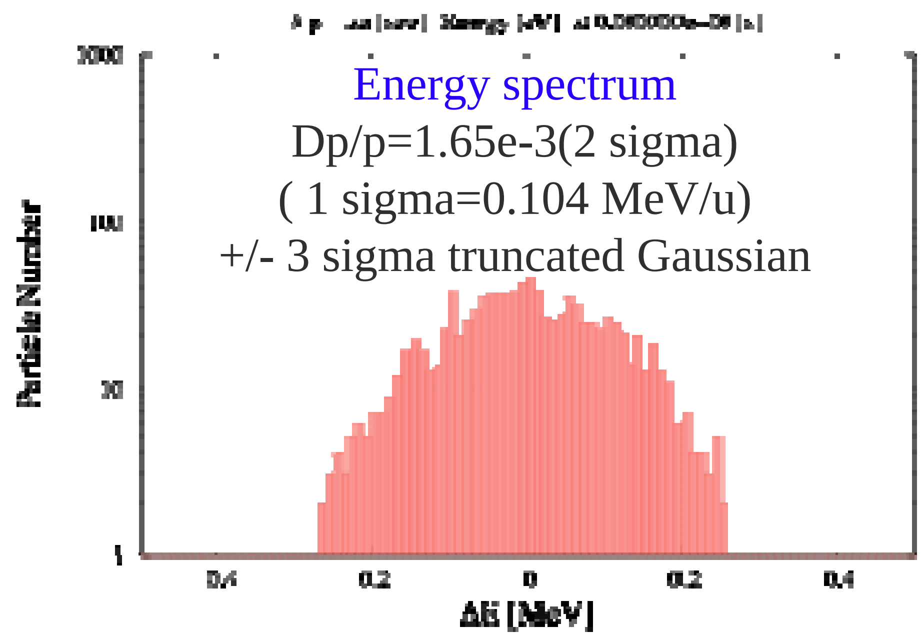


Initial condition of simulation for ESR experiments









Particle tracking method (1)

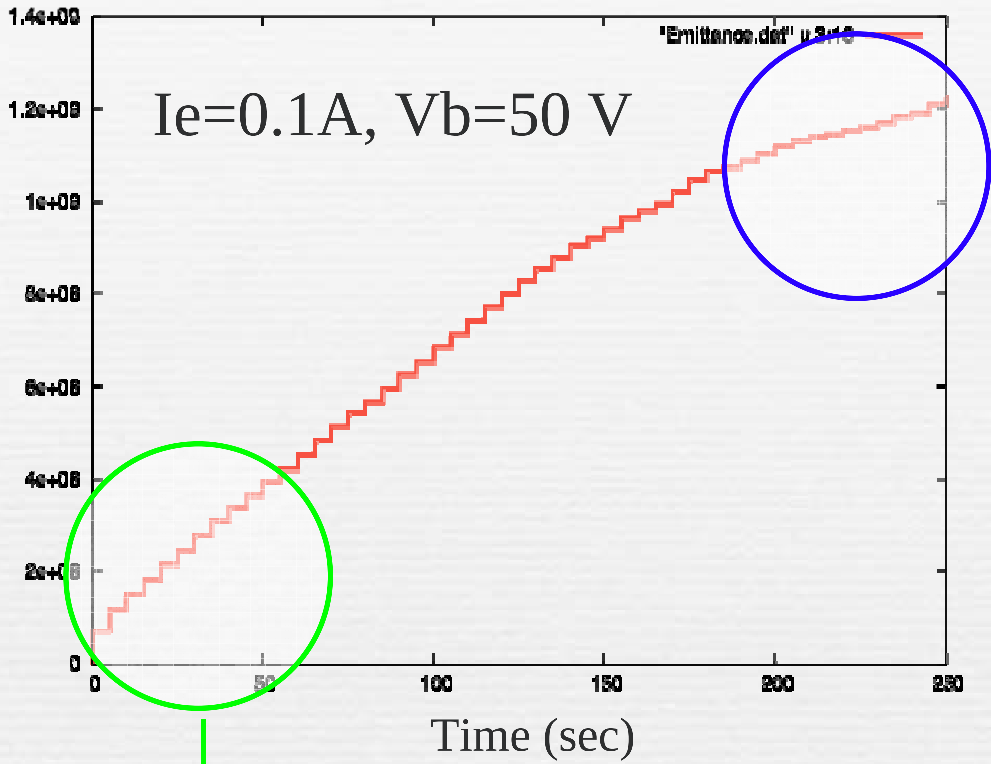
IBS effects are taken account as the cooling electron current control with following formula.

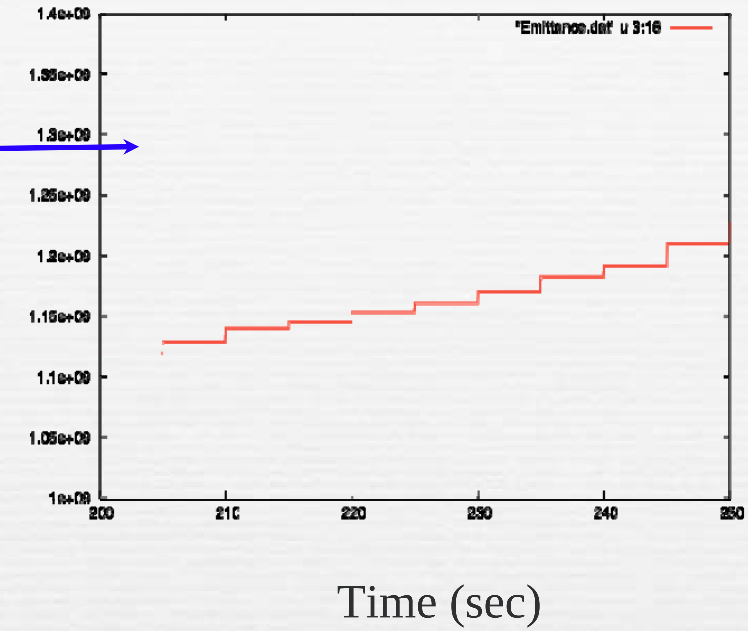
$$\frac{1}{\sigma_p}\frac{d\sigma_p}{dt} = \left(\frac{1}{\tau_{c,p}} + \frac{1}{\tau_{IBS,p}}\right)$$

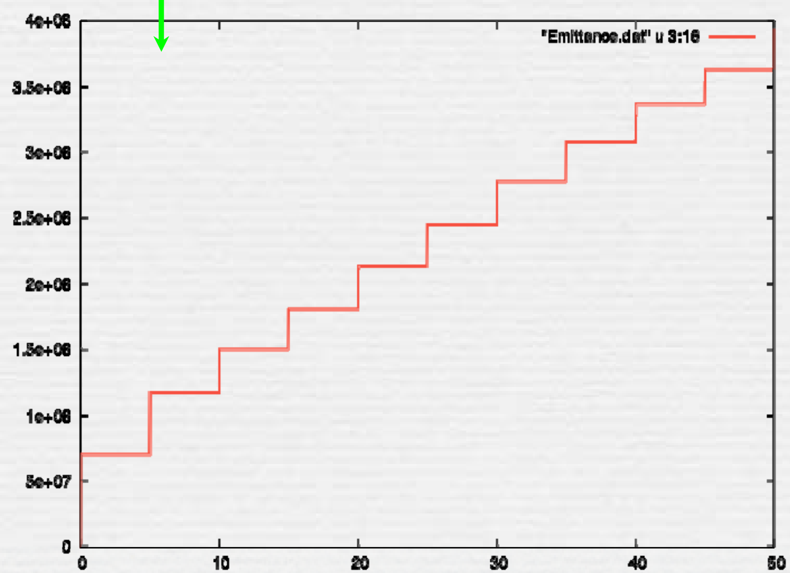
$$\frac{1}{\tau_{c,p}} = gI_0, \quad \frac{1}{\tau_{c,p}} + \frac{1}{\tau_{IBS,p}} = gI_{eff}$$

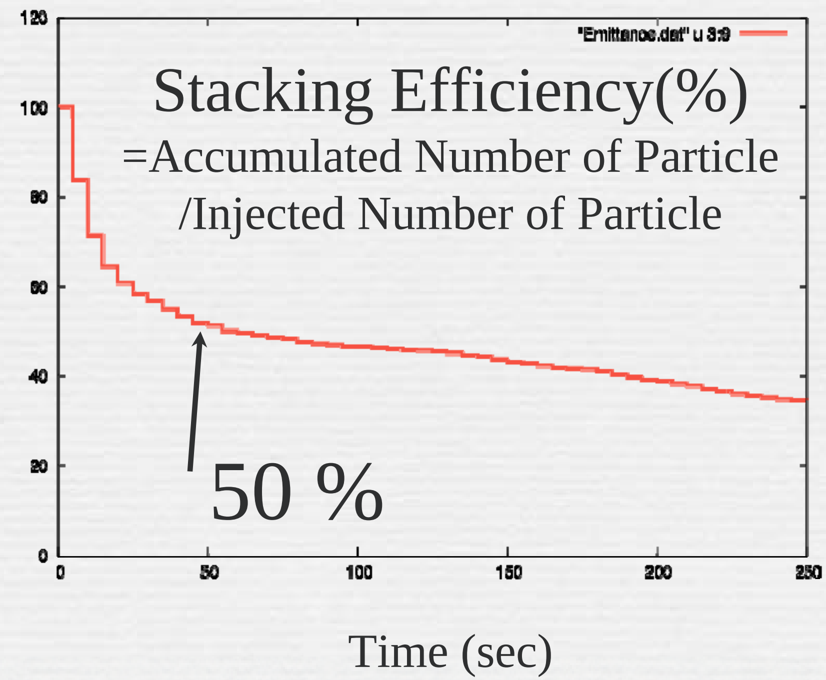
$$I_{eff} = I_0 * \left(1 + \frac{\tau_{c,p}}{\tau_{IBS,p}}\right)$$

Accumulated Particle Number & Stacking Efficiency



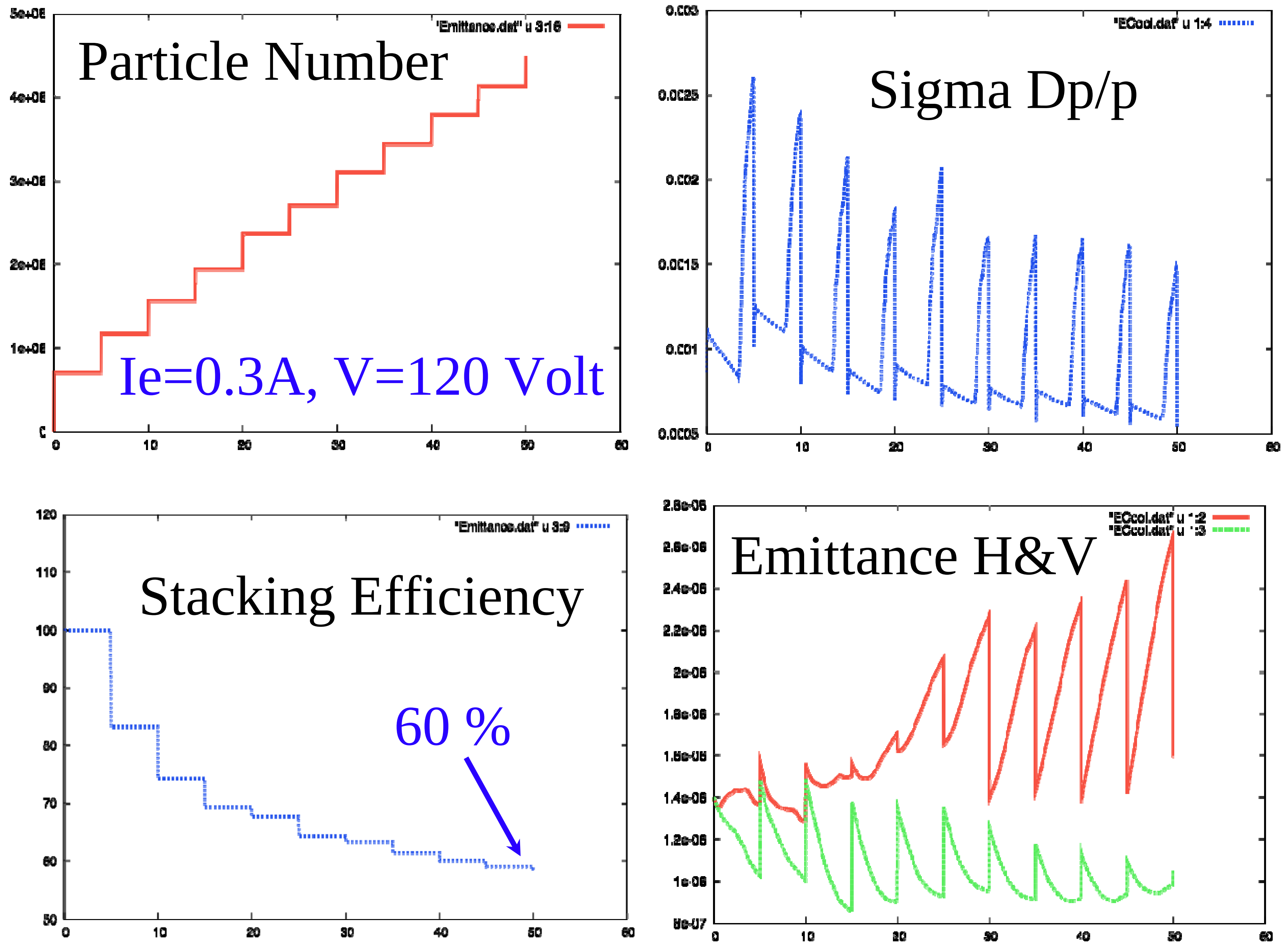




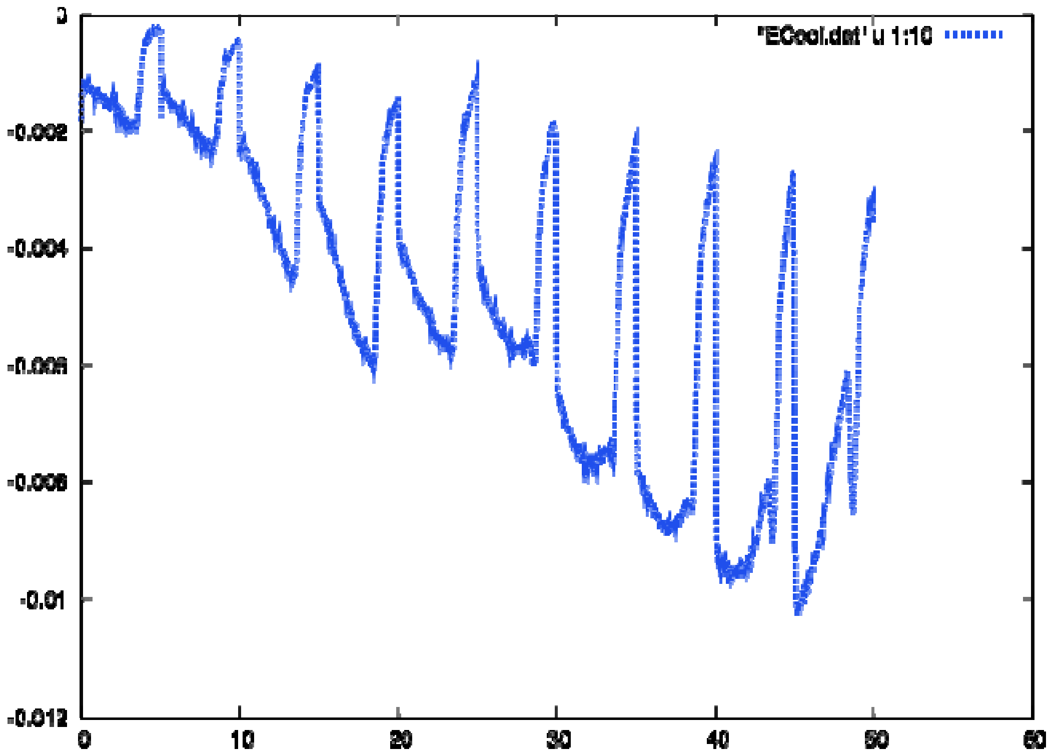


Particle Tracking (2)

IBS effects are taken into account so as to heat the emittance and momentum spread of each particle with use of analytically calculated IBS heating rates and random number.

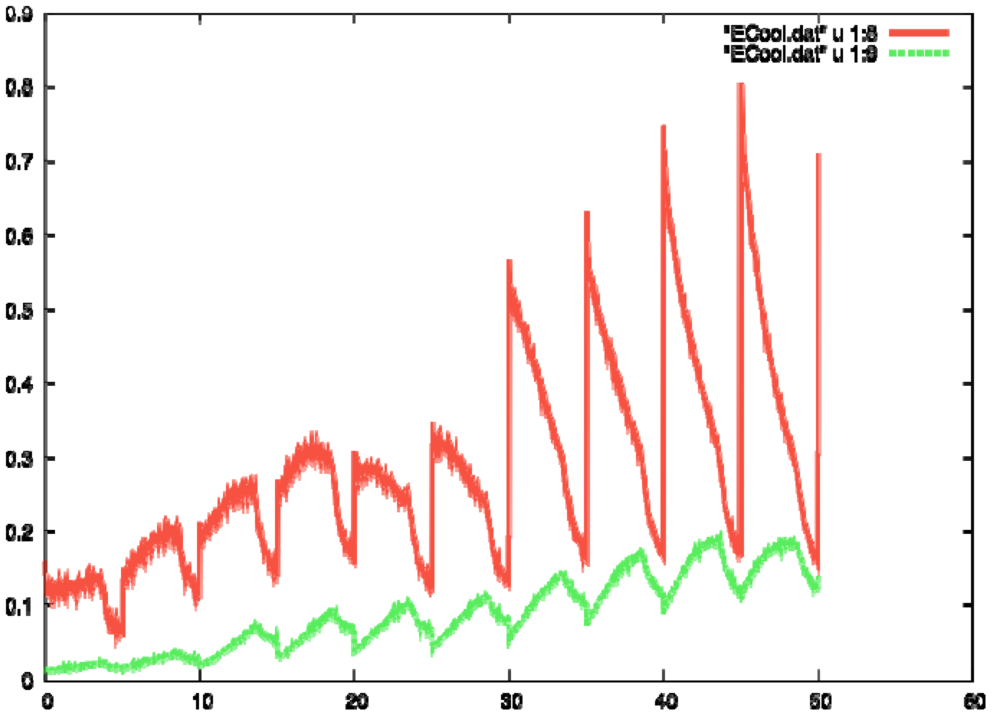


Particle Number
'Emittance.dat' u 3:16
Ie=0.3A, V=120 Volt
Sigma Dp/p
'ECool.dat' u 1:4
Stacking Efficiency
'Emittance.dat' u 3:9
60 %
Emittance H&V
'ECool.dat' u 1:2
'ECool.dat' u 1:3



Dp/p IBS Heating Rate (1/sec)

Transverse Emittance Heating Rate (1/sec)

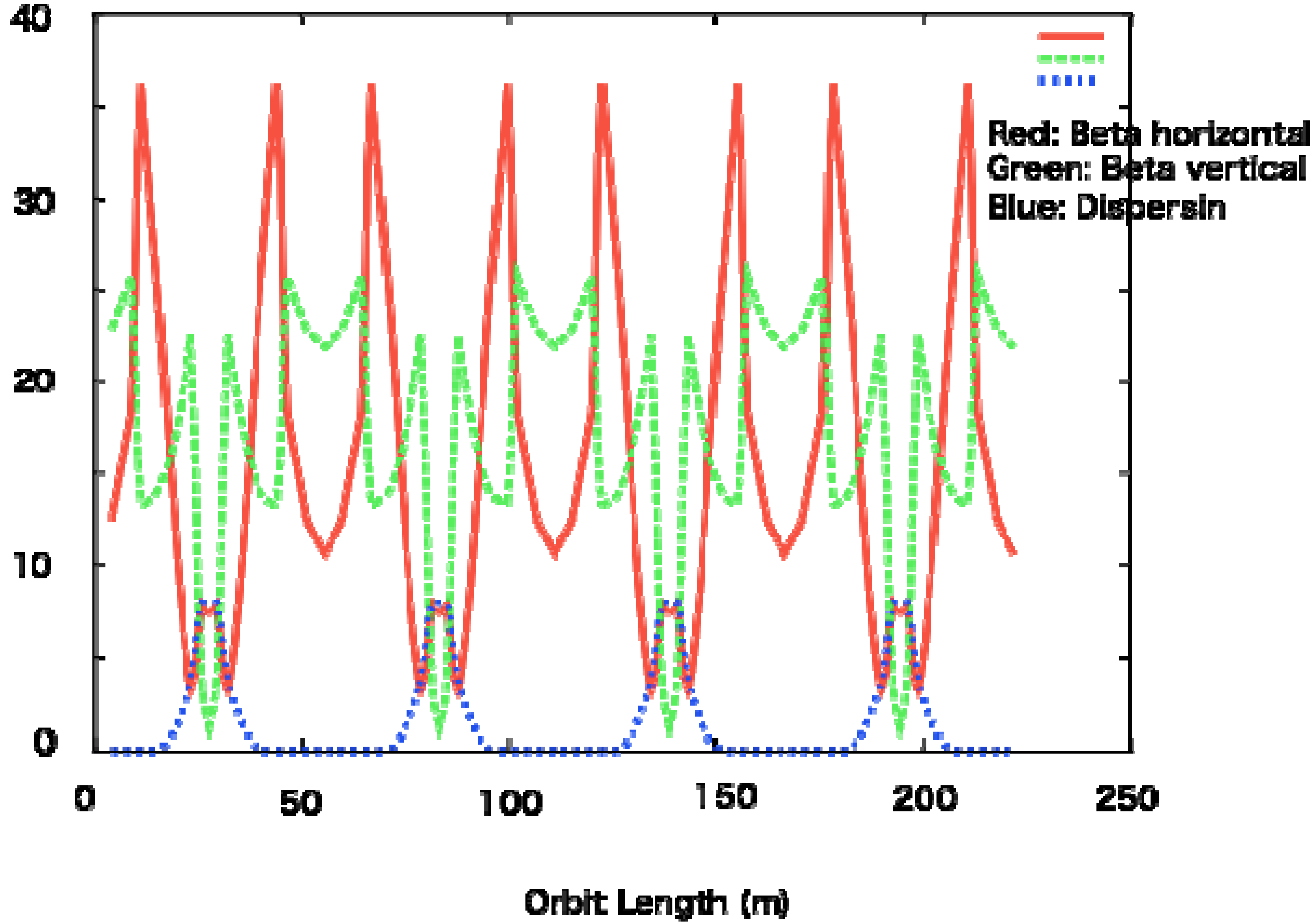


NESR Parameters

132Sn50+, 740 MeV
Injection Particle Number N=1e8
Initial Horizontal emittance=0.5 pai (mm.mrad)
Vertical emittance=0.5 pai (mm.mrad)
DeltaP/P=2.5e-4
(1 sigma)

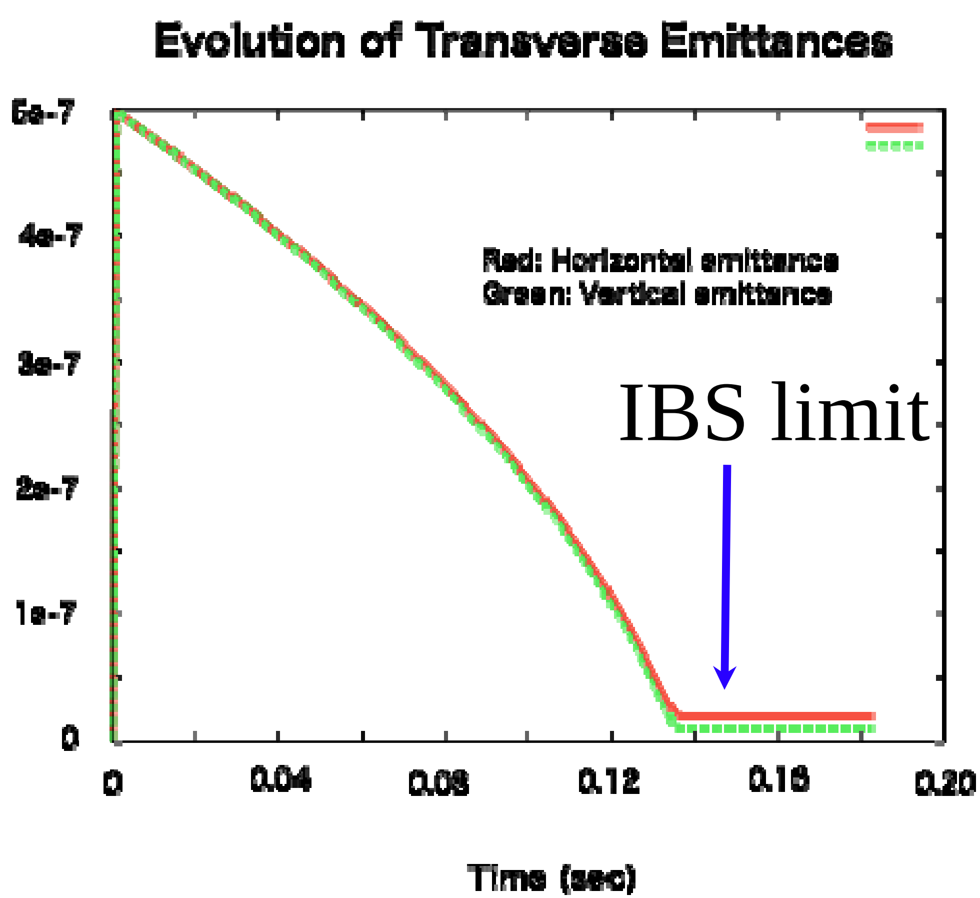
Electron cooler parameters
Cooler length=5m,
Horizontal Beta function=10m,
Vertical Beta function=22m
Dispersion=0 m,
Current=1.0 A
Electron Diameter=1.0e-2 m

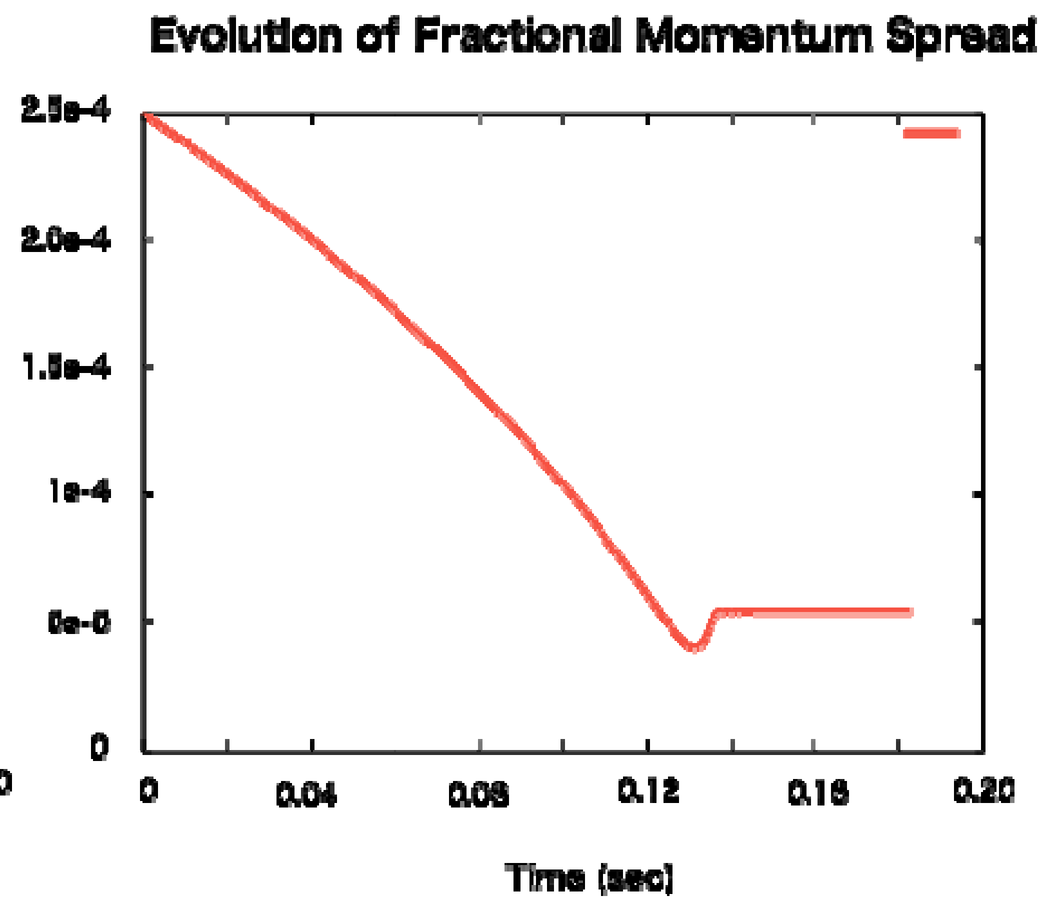
Twiss Functions of NESR



Evolution of DeltaP/P and Emittances

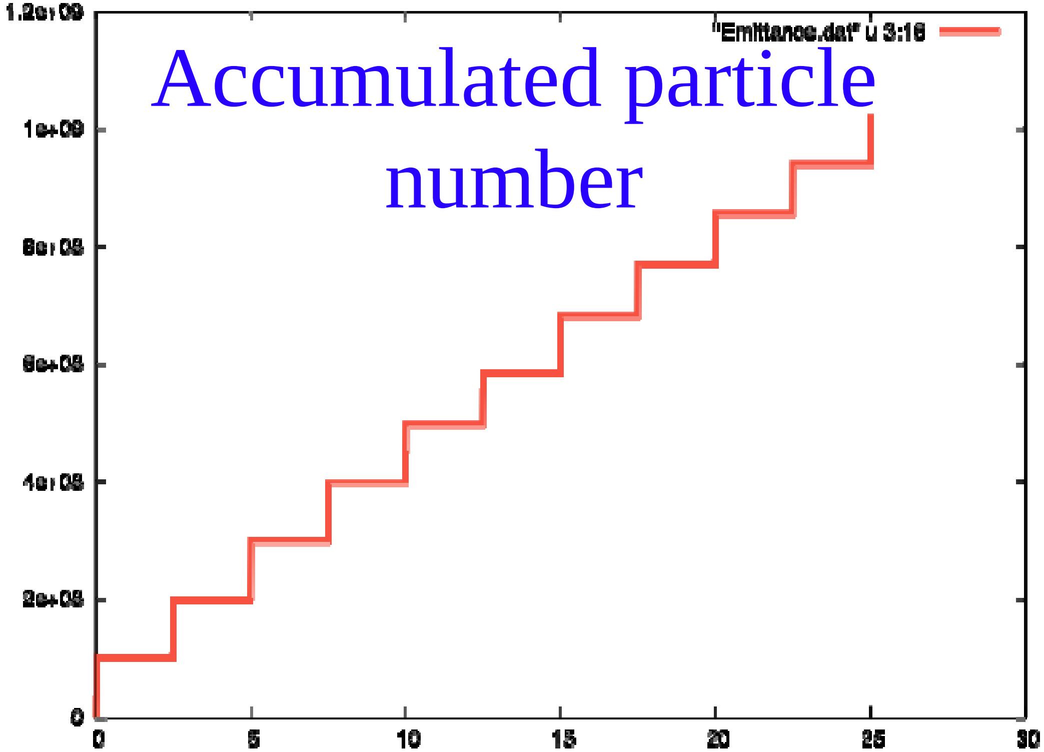
Initial beam parameters
Beam energy=740 MeV/u
DeltaP/P=2.5e-4
Horizontal emittance=0.5 pai (mm.mrad)
Vertical emittance=0.5 pai (mm.mrad)
Number of particles=1e8

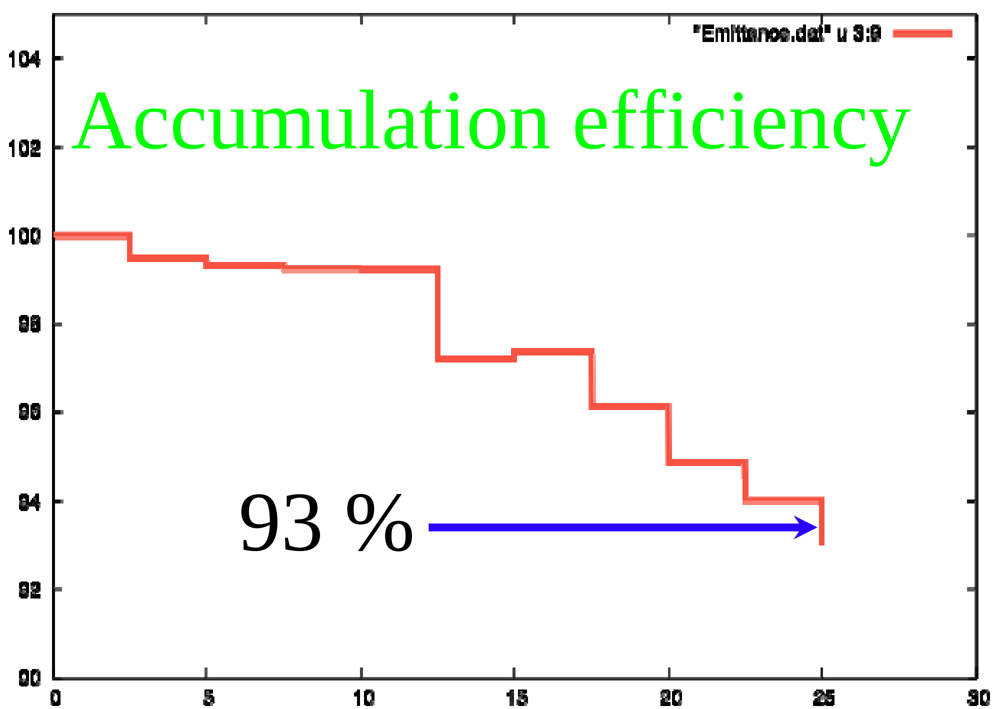


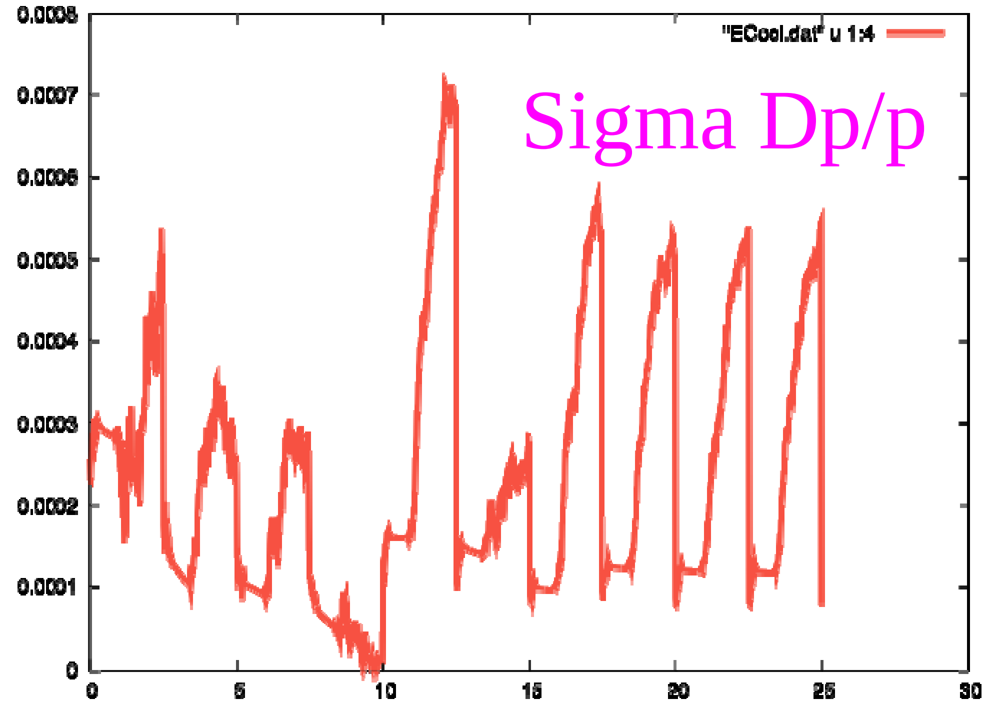


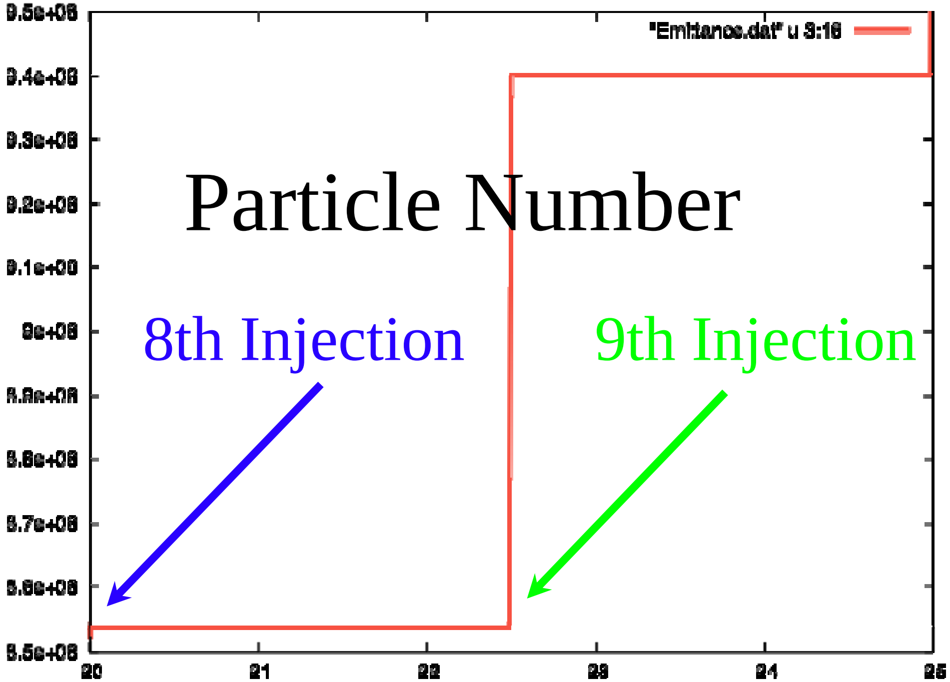
Particle Tracking Method (1)

NESR, 740 MeV/u, 132Sn50+
Ie=1A, Vb=2kV
Cycle Time =2.5 sec

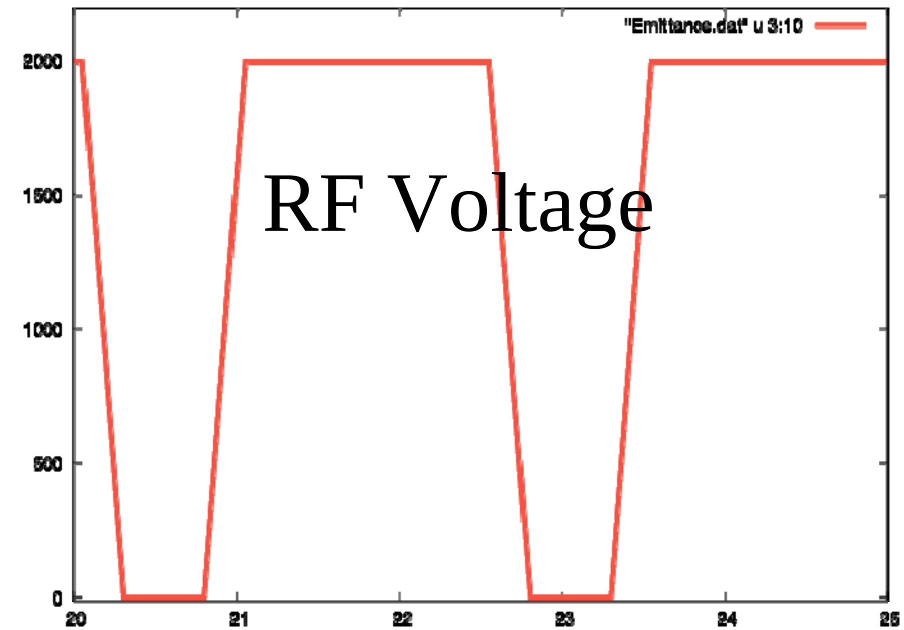




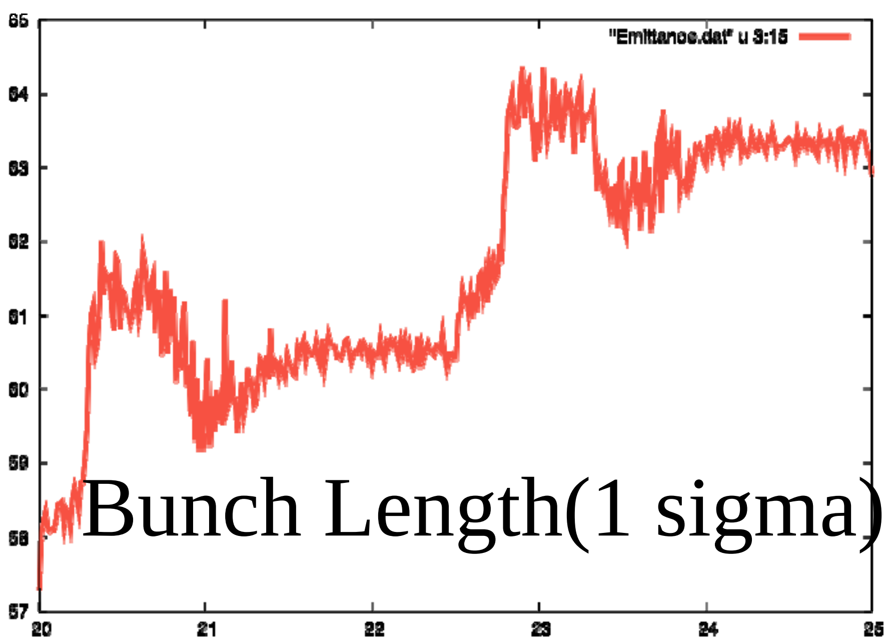




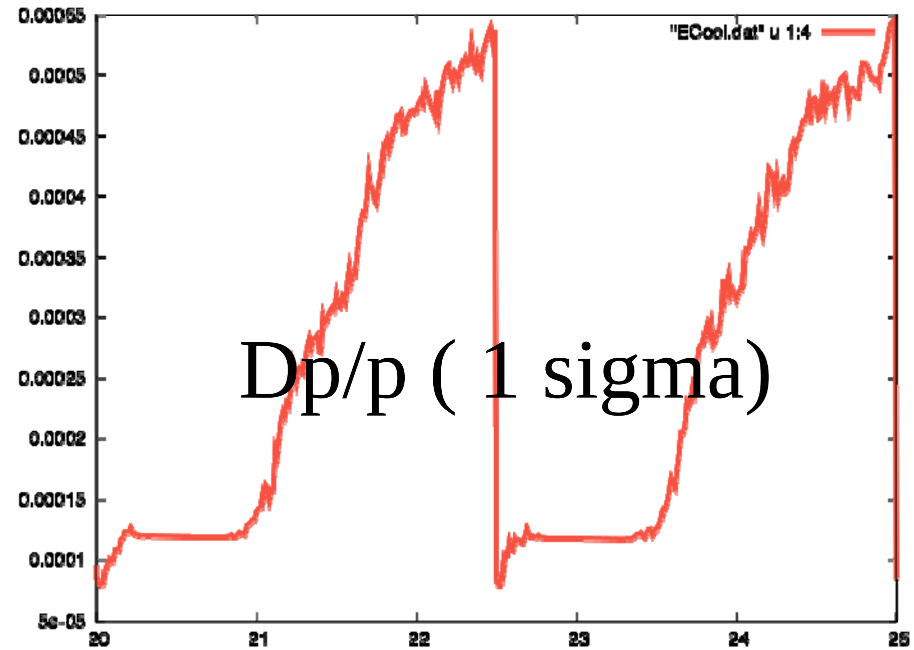
"Emittance.dat" u 3:16
Particle Number
8th Injection
9th Injection
20
21
22
23
24
25



"Emittance.dat" u 3:10
RF Voltage
2000
1500
1000
500
0
20
21
22
23
24
25

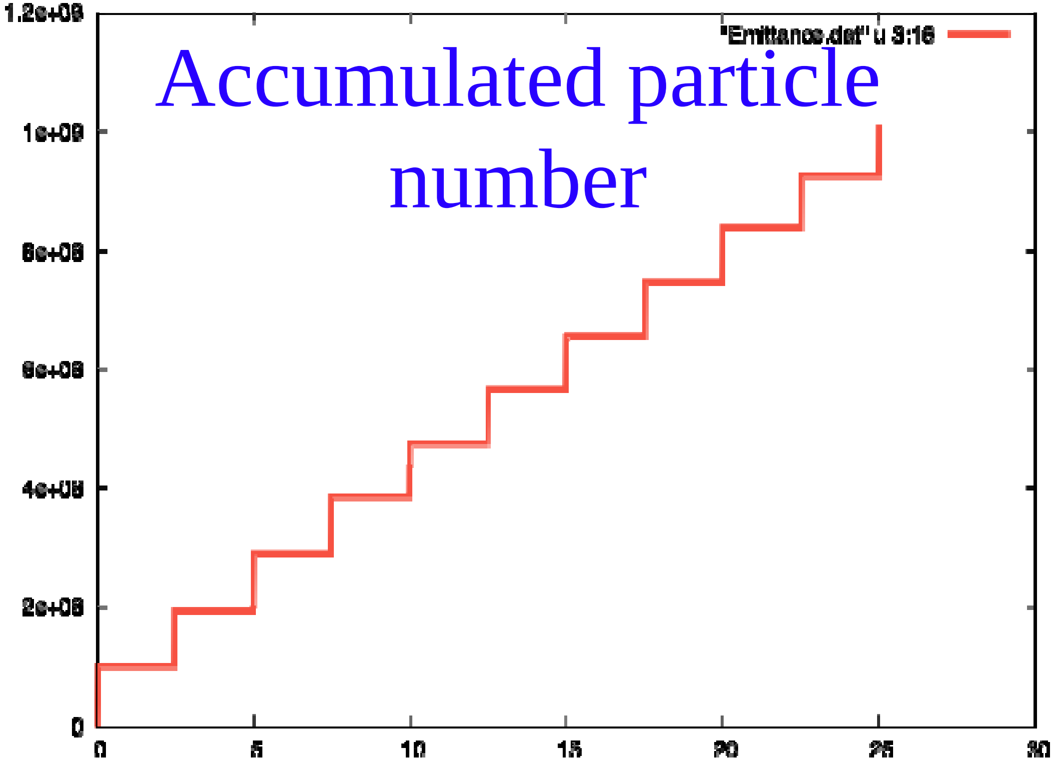


"Emittance.dat" u 3:15
Bunch Length(1 sigma)
65
64
63
62
61
60
59
58
57
20
21
22
23
24
25

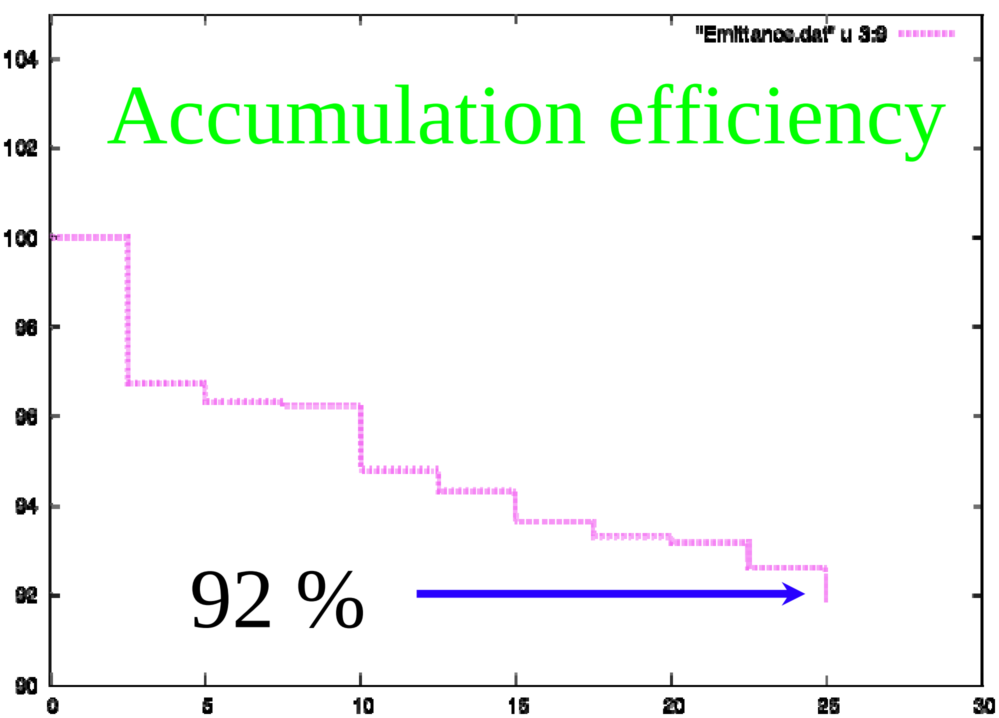


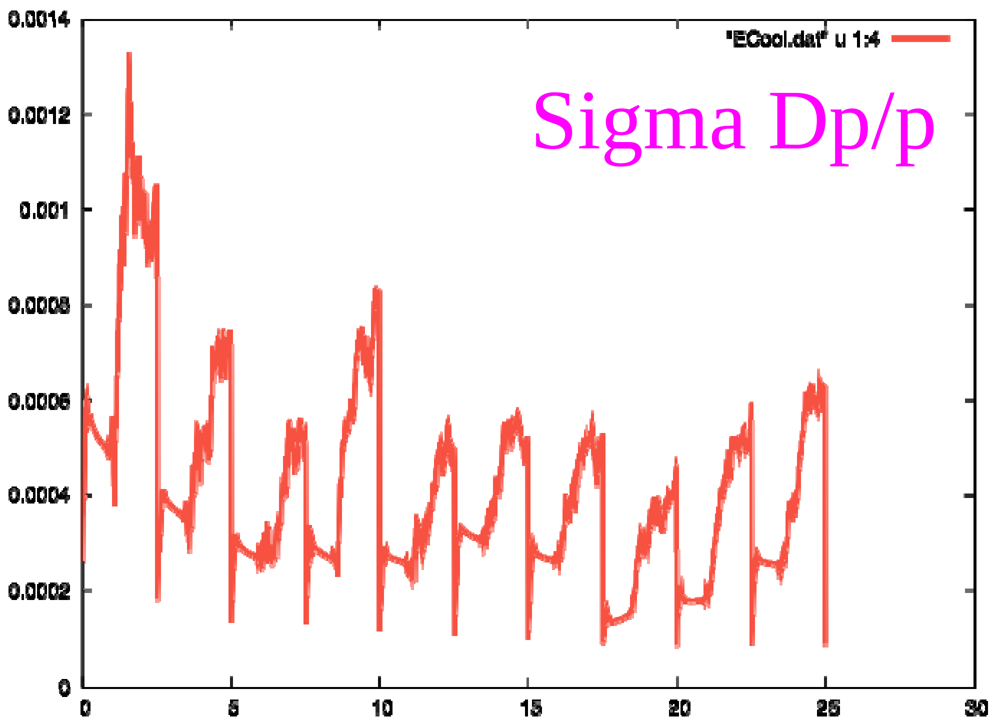
"ECool.dat" u 1:4
Dp/p (1 sigma)
0.00055
0.0005
0.00045
0.0004
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
20
21
22
23
24
25

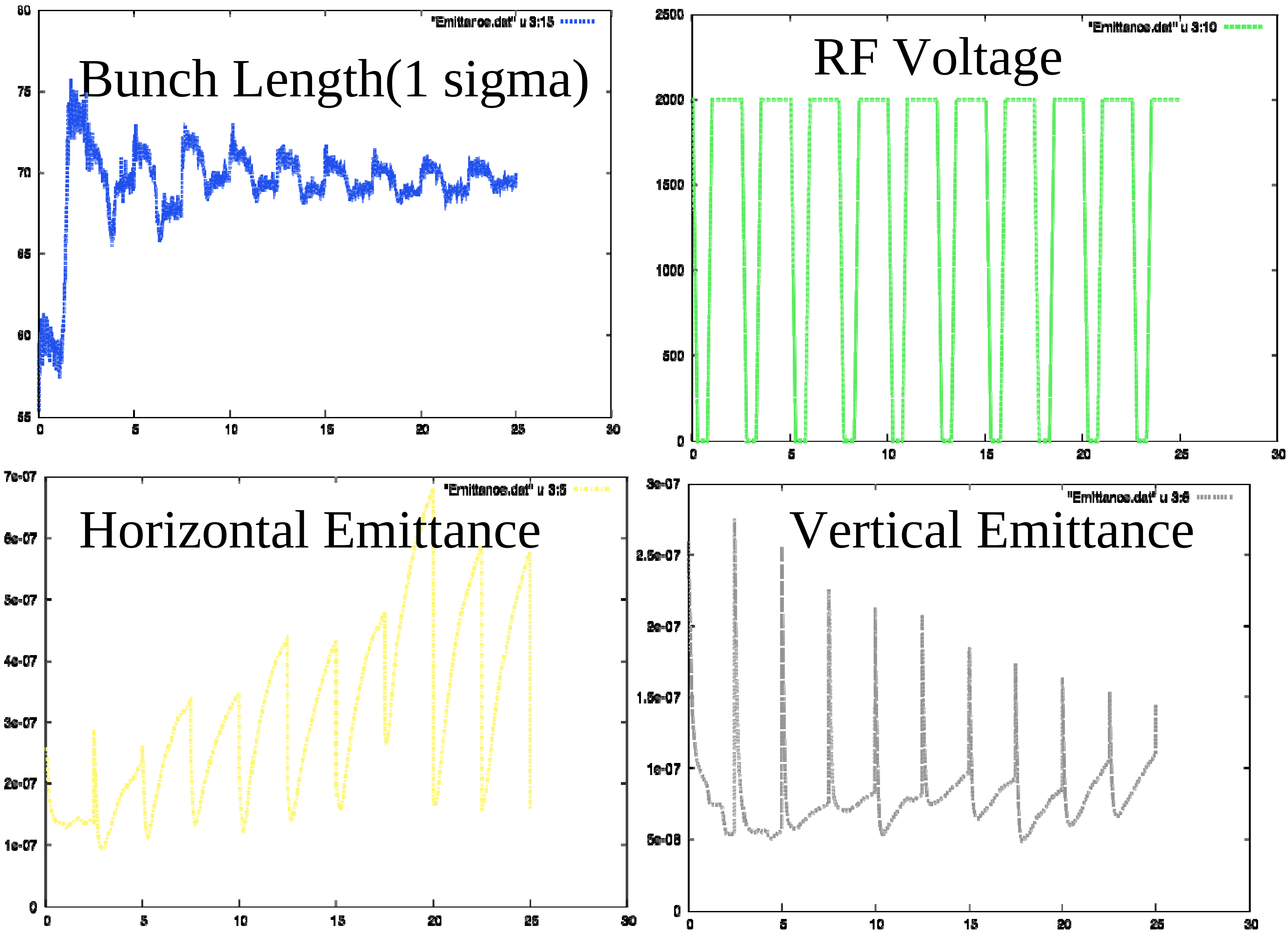
Particle Tracking Method (2)



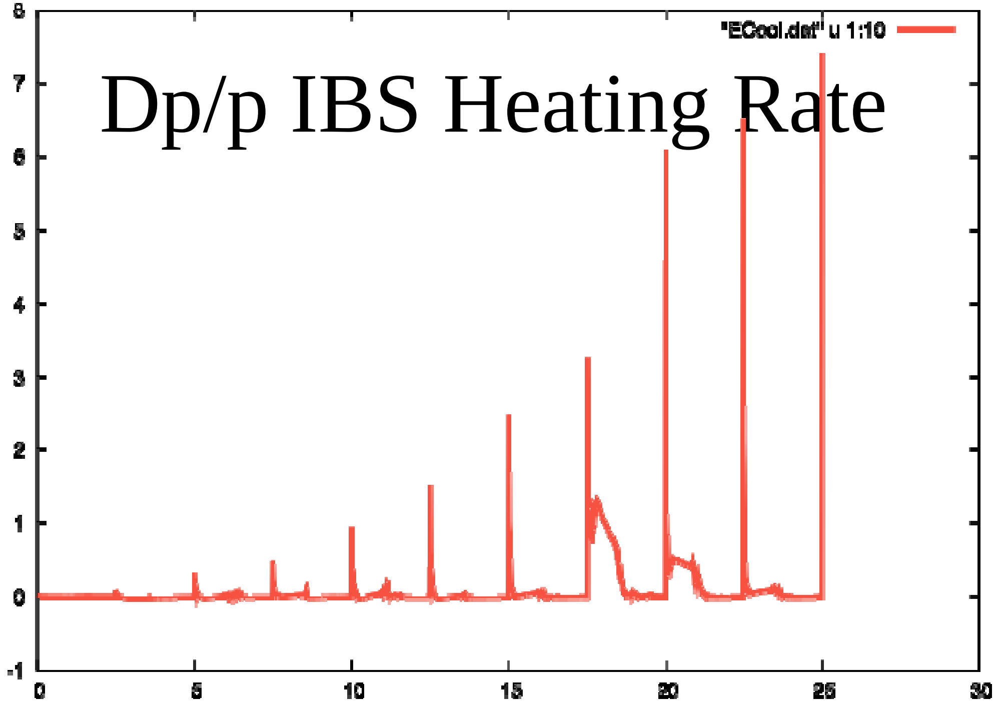
NESR, 740 MeV/u, 132Sn50+
Ie=1A, Vb=2kV
Cycle Time =2.5 sec

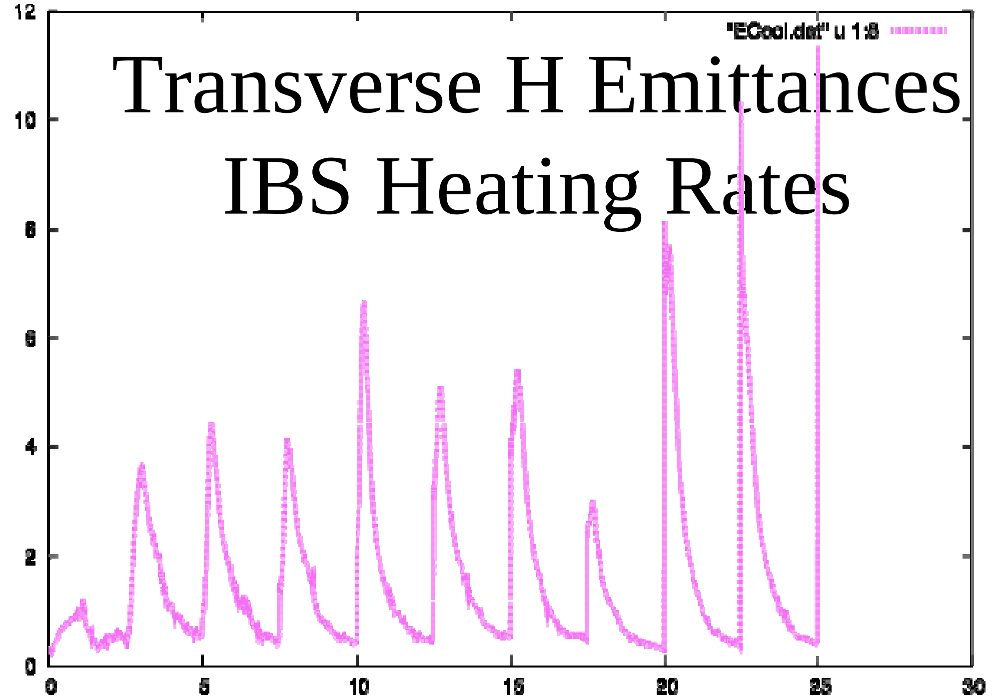


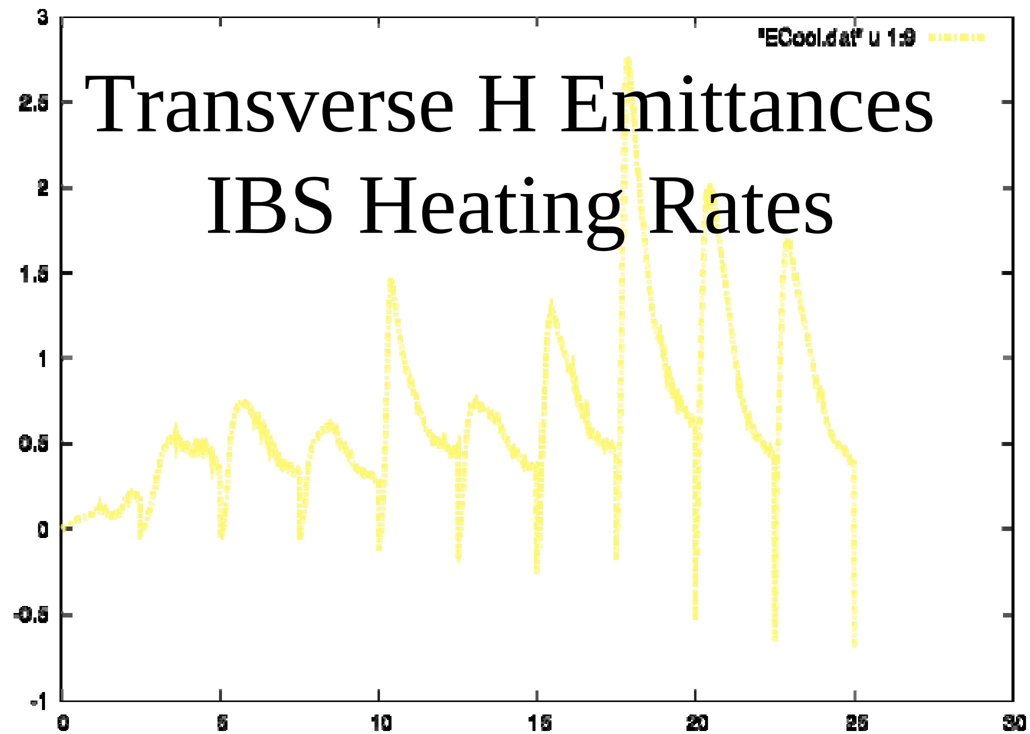


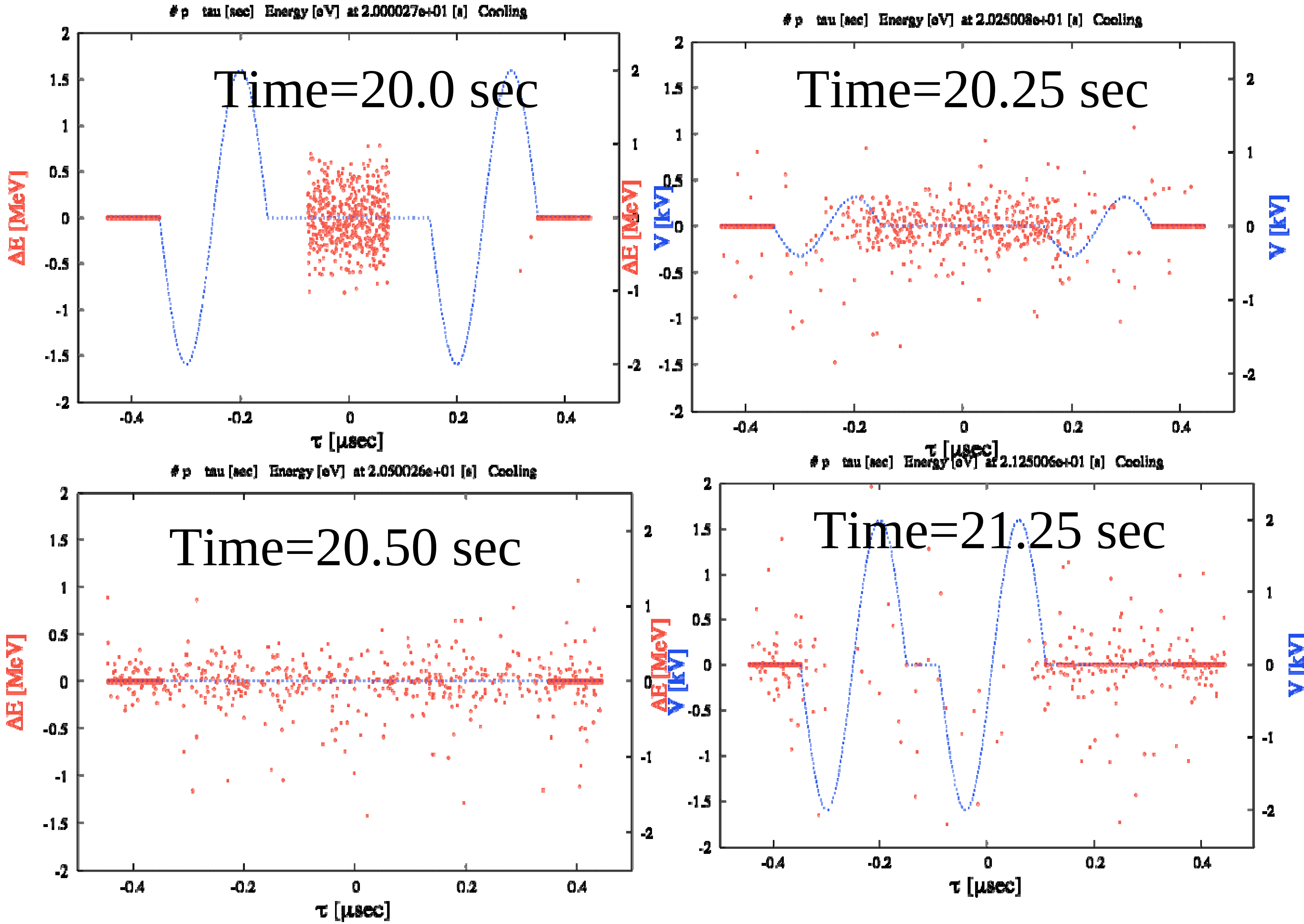


Bunch Length(1 sigma)
"Emittance.dat" u 3:15
RF Voltage
"Emittance.dat" u 3:10
Horizontal Emittance
"Emittance.dat" u 3:5
Vertical Emittance
"Emittance.dat" u 3:6

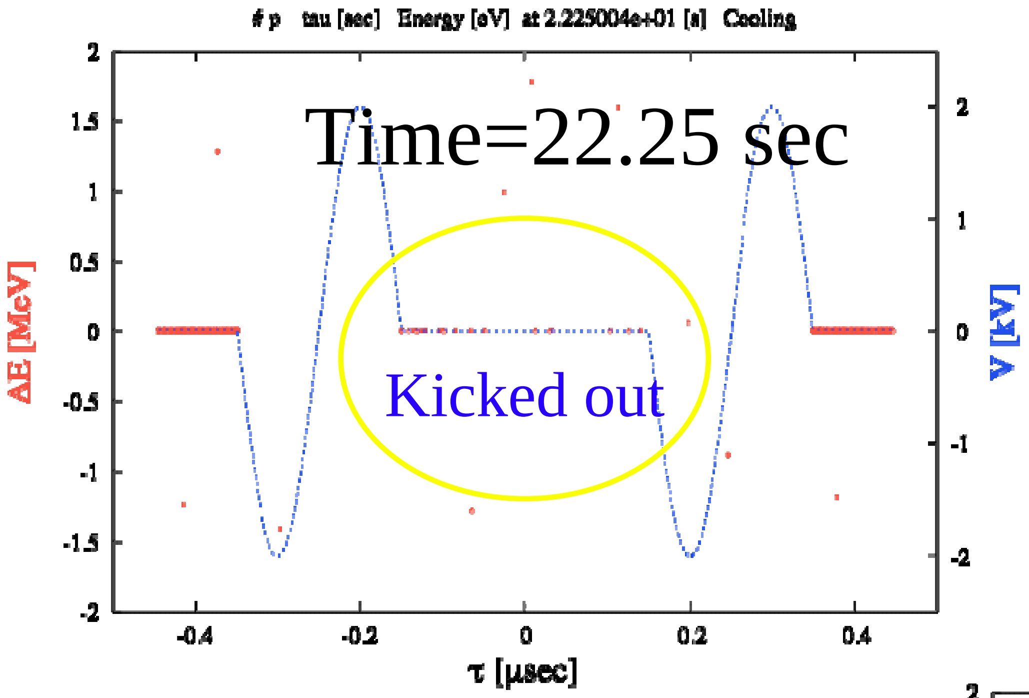

Dp/p IBS Heating Rate
"ECool.dat" u 1:10
-1 0 1 2 3 4 5 6 7 8
0 5 10 15 20 25 30


Transverse H Emittances
IBS Heating Rates
"ECool.dat" u 1:8
0 2 4 6 8 10 12
0 5 10 15 20 25 30

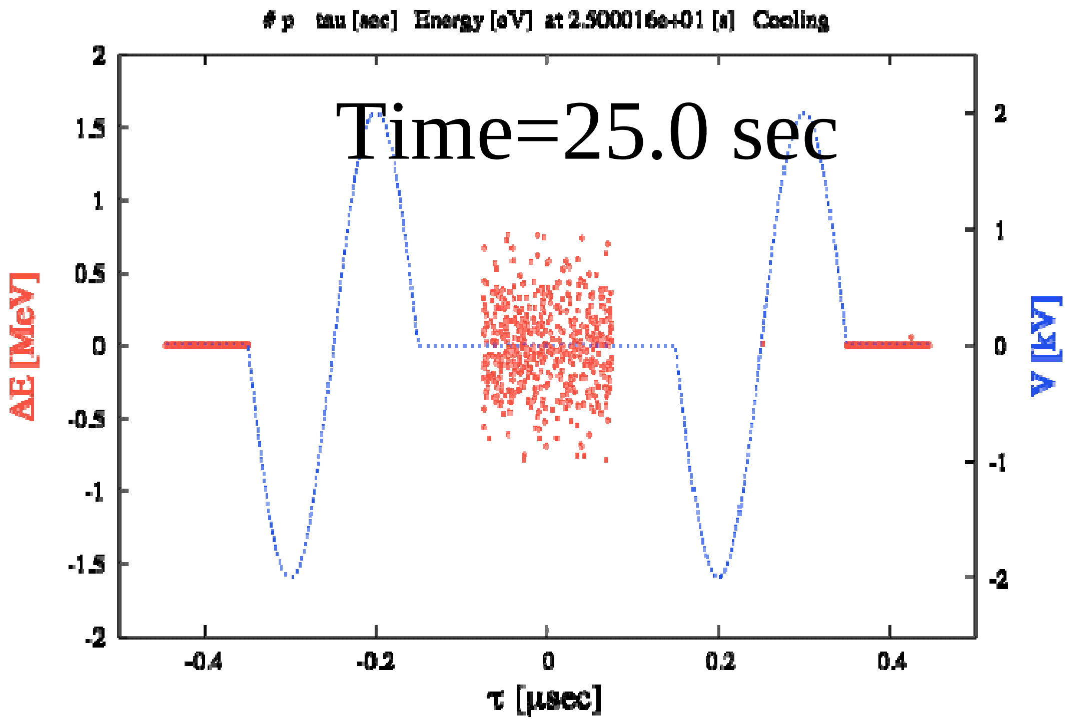

Transverse H Emittances
IBS Heating Rates
"ECool.dat" u 1:9
-1 -0.5 0 0.5 1 1.5 2 2.5 3
0 5 10 15 20 25 30



# p tau [sec] Energy [eV] at 2.000027e+01 [s] Cooling
Time=20.0 sec
# p tau [sec] Energy [eV] at 2.025008e+01 [s] Cooling
Time=20.25 sec
# p tau [sec] Energy [eV] at 2.050026e+01 [s] Cooling
Time=20.50 sec
# p tau [sec] Energy [eV] at 2.125006e+01 [s] Cooling
Time=21.25 sec
ΔE [MeV]
V [kV]
τ [μsec]



# p tau [sec] Energy [eV] at 2.225004e+01 [s] Cooling
Time=22.25 sec
Kicked out
ΔE [MeV]
V [kV]
τ [μsec]



# p tau [sec] Energy [eV] at 2.500016e+01 [s] Cooling
Time=25.0 sec
ΔE [MeV]
V [kV]
τ [μsec]

Governing Equations & Space Charge Calculation

- Equation of Time in Phase Space

$$\frac{d\tau}{dt} = \frac{\eta}{\beta^2}\frac{\Delta E}{E_0},$$

- Equation of Energy Difference

$$\frac{d\Delta E}{dt} = \frac{q}{m}\frac{V_{bb}}{T_0} - \beta c \frac{q}{m} K_c \Delta E - \frac{q}{m}\frac{g}{4\pi\epsilon_0\gamma^2}\frac{d\lambda}{d\tau}.$$

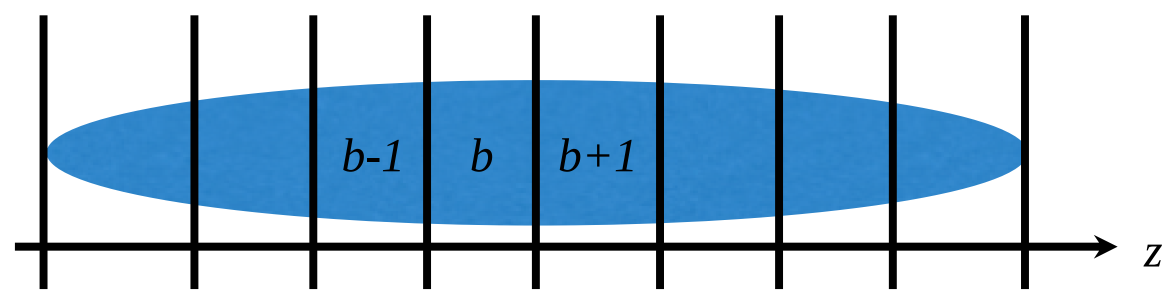
Barrier Bucket Voltage

Electron Cooling Force

Space Charge Potential

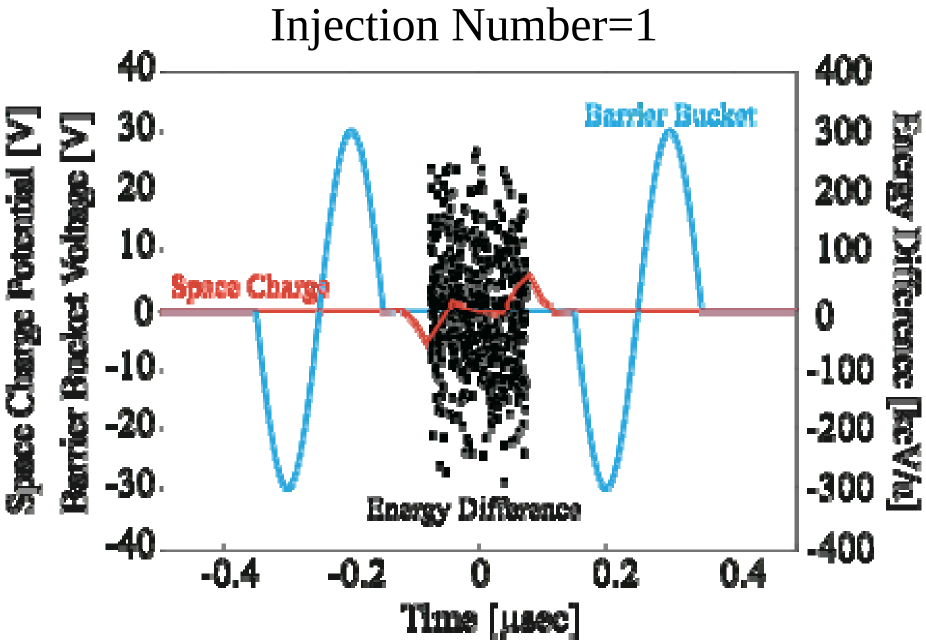
$$E_z = -\frac{g}{4\pi\epsilon_0{\gamma_0}^2}\frac{d\lambda}{dz},$$

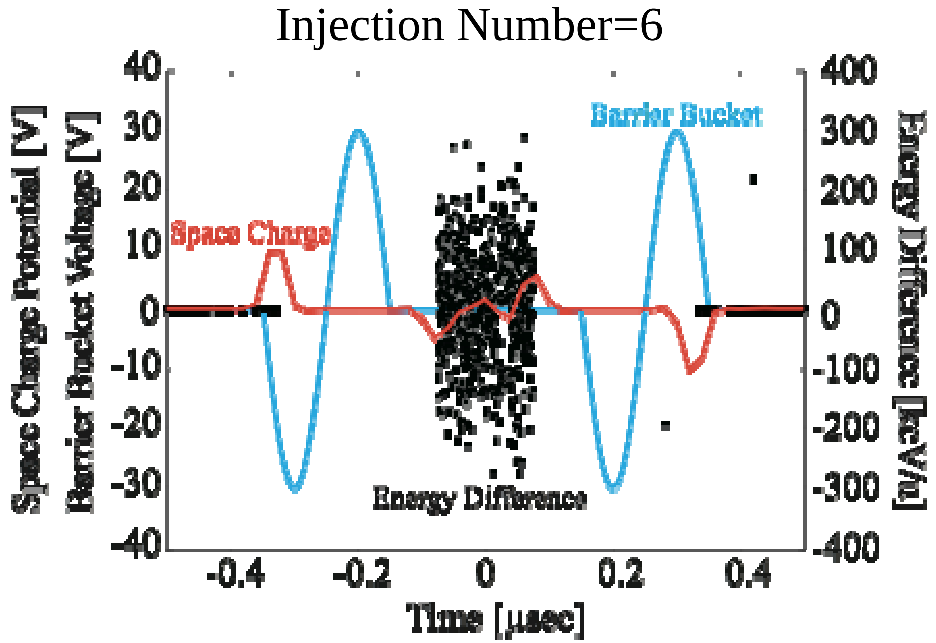
Particle In Cell along the Orbit
Cell Number=50
Particle Number=1000/shot

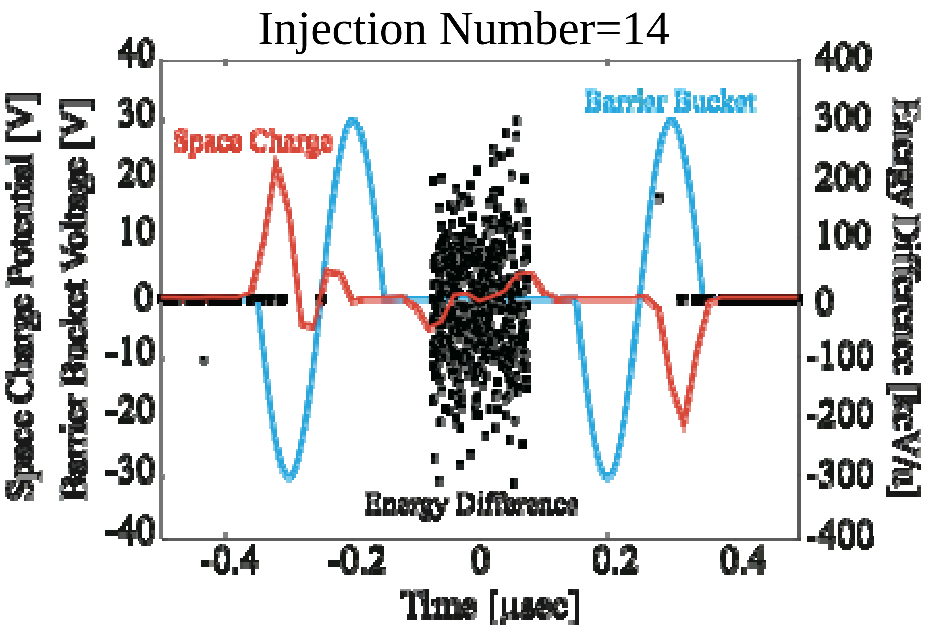


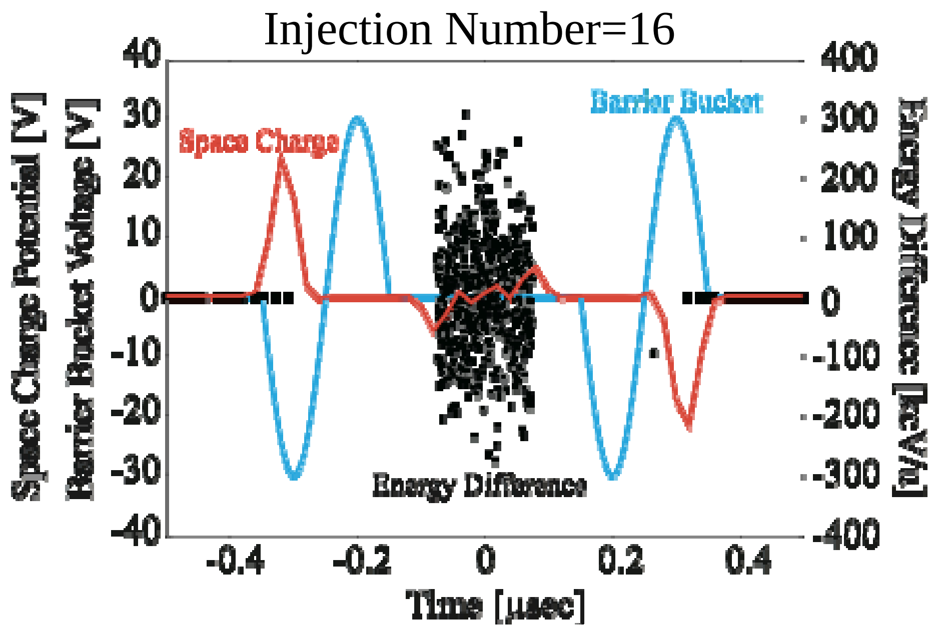
Particle Map, Space Charge Potential, Barrier Bucket Voltage

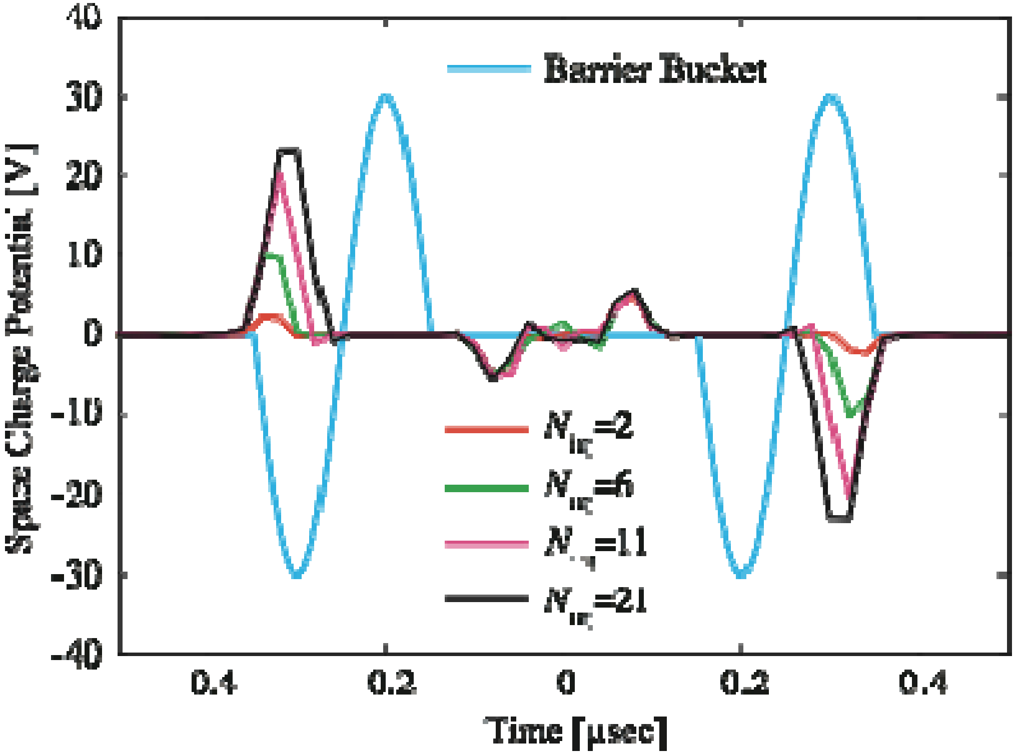
Ie=0.5A, N=7e7/injection





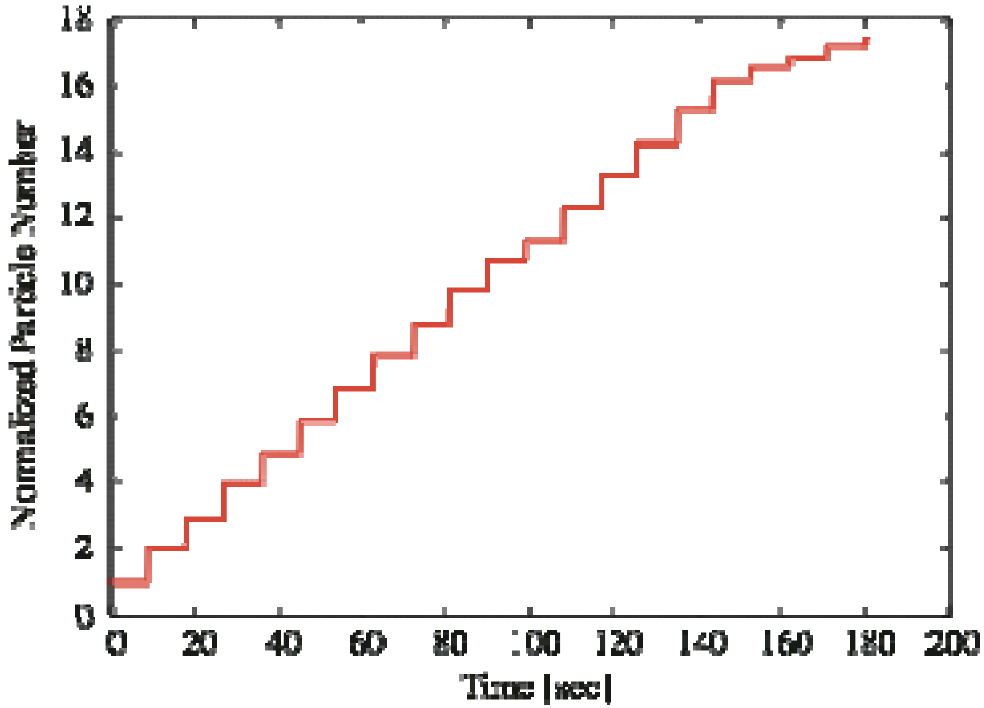






Barrier Voltages and Space Charge Potential

Accumulation of Ions

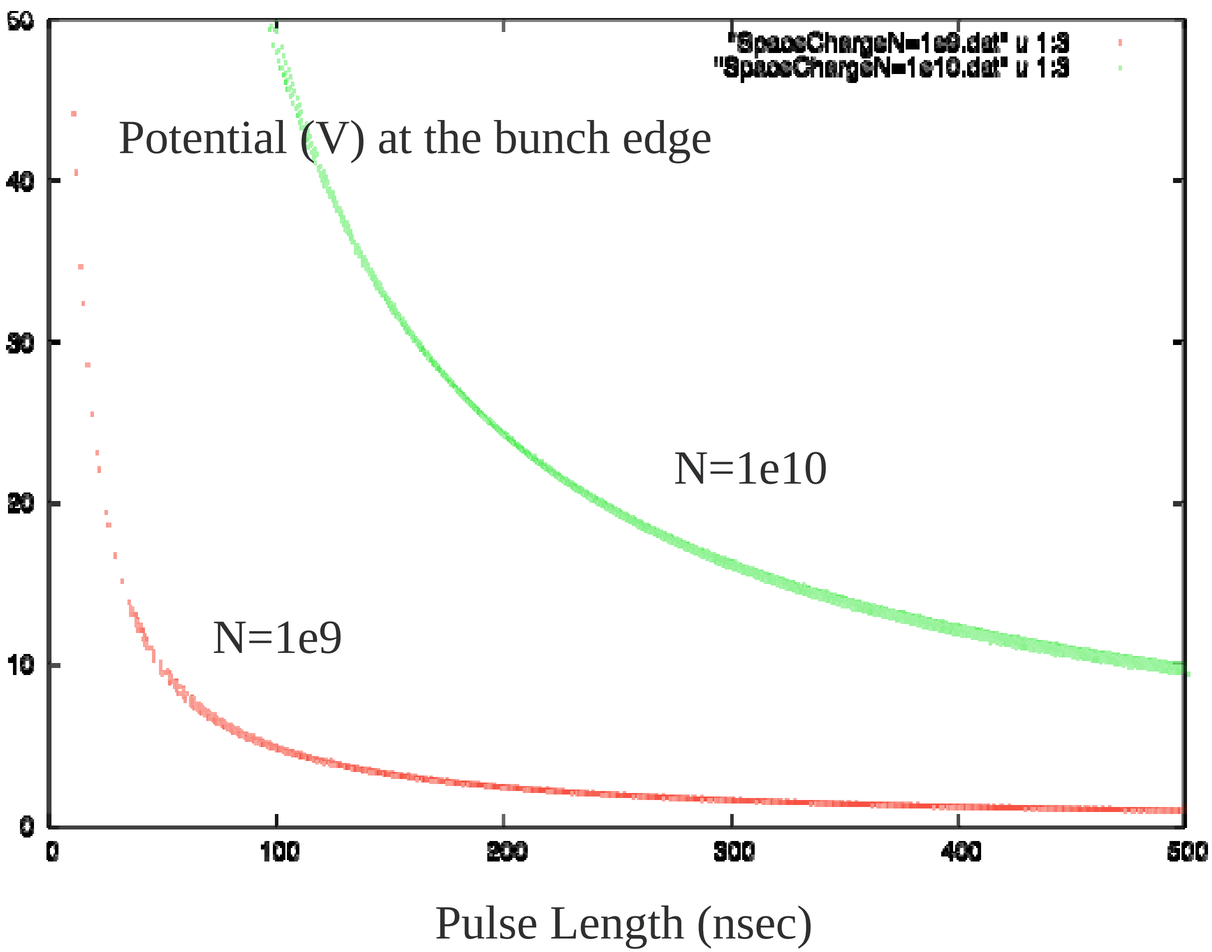


Electrical Potential in Ellipsoidal Bunch with Uniform Density

$$\phi = \frac{\rho}{2\varepsilon_0}\left(\frac{1-M_E}{2}r^2 + M_E z^2\right) \qquad M_E = \frac{1-\xi^2}{\xi^2}\left(\frac{1}{2\xi}\ln\frac{1+\xi}{1-\xi} - 1\right)$$

$$\xi = \left(1-\left(r_b/z_b\right)\right)^{1/2}$$

M. Reiser, *Theory and Design of Charged Particle Beams*, Wiley, New York, (1994).



z_b : Bunch Edge Length

r_b : Bunch Edge Radius

z : Longitudinal Length from Bunch Center

r : Radius from Bunch Center

ρ : Charge Density

Summary

1. Equilibrium with electron cooling and IBS.

Single particle calculation of electron cooling show that the equilibrium values of transverse emittances, are fairly well in agreement with the experimental results while the calculated equilibrium momentum spread give smaller values than the experimental ones by factor ~2.

2. Evolution of electron cooling.

Multi-particle cooling calculation shows that the calculated evolution of particle spectrum are in fairly well in agreement with experimental Schottky signal analysis with time.

3. Limit of stacked number of ions with use of scaling laws.

With use of scaling laws of equilibrium momentum spread with electron current and particle number, we made analytical calculation of limit of accumulated number of ions. For the case of Ie=100mA, it gives well agreement with the experimental ones while for the higher electron current the discrepancy is factor 2~3. Space charge repulsion force ?

4. IBS effects are taken into account by two methods, 1) Control the cooling electron current, 2) Random heating with IBS rates.
Numerical values of sigma for transverse emittance and momentum spread are calculated at each computing cycle, and they are used to calculate the IBS heating rates. The cooling electron current strength is controlled by taking the IBS heating rates into account (method 1), or random heating with IBS rates (method 2).

5. Stacking efficiency.
ESR case, calculated accumulation efficiency is ~60 % while the measured ones are around 70 % with 10 times injection with repetition cycle of 5 sec.
For the NESR, 132Sn50+, 740 MeV/u ions, the accumulation efficiency is calculated at 92 % when the cycle time is 2.5 sec, Ie=1A and Vb=2kV at 10 batches injection.

6. Space charge repulsion force. (T. Kikuchi et al., paper)
It reaches to +/- 25 Volt (ESR case) at the edge of accumulated area when the particle number is 1e9. It decrease the effective barrier voltage substantially. NESR case is under study.