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LONGITUDINAL SCHOTTKY SIGNALS OF COLD SYSTEMS WITH LOW NUMBER OF PARTICLES

Rainer.W. Hasse, GSI Darmstadt, Darmstadt, Germany, email: r.hasse@gsi.de

Very cold systems of ions with sufficiently low number of particles arrange in an ordered string-like fashion. The determination of the longitudinal momentum spread and of the transverse temperature then is no longer possible by normal Schottky diagnosis. In this paper we simulate such systems in an infinitely long beam pipe with periodic boundary conditions under the influence of all long-range Coulomb interactions by Ewald summation. Then we derive the behaviour of the longitudinal Schottky signals for cold string-like systems as well as for the transition to warmer systems when the strings break, up to hot gas-like systems. Here effects from the finite number of particles, of higher harmonics and of temperature agree with those derived analytically in the limits of very low (ordered strings) and very high (gas) temperatures.

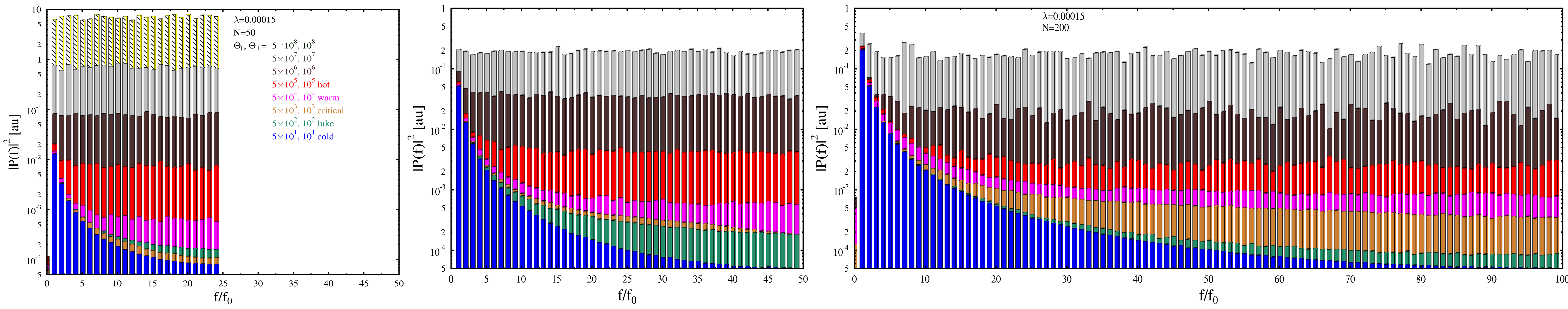

Figure 1: Calculated integrated Schottky spectra for the linar density $\lambda = 0.00015$ at various longitudinal and transverse temperatures and various number of particles N=50, 100, 200 .

In short:

- Schottky analysis is well established for many hot ions
- Theory known for cold or hot, but not in between
- ⇒ Uncertainties for few ordered cold ions
- ⇒ Molecular dynamics for few cold and hot ions
- ⇒ Transition from cold to hot
- ⇒ Few particle effects

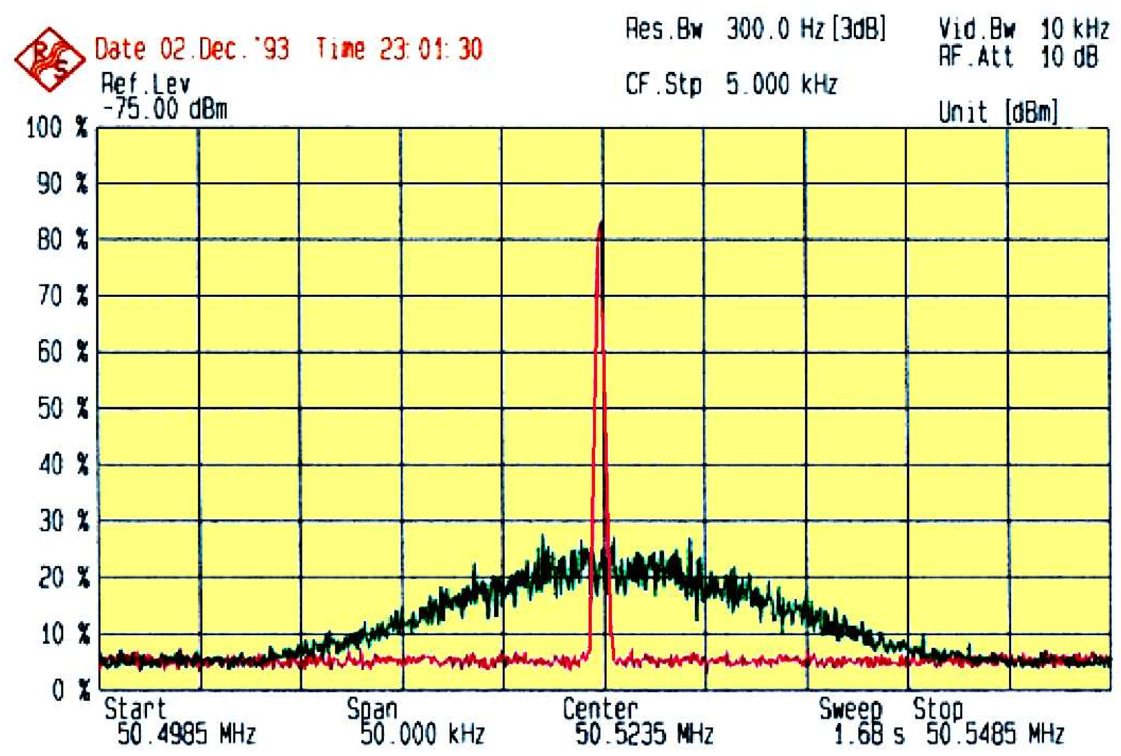

Figure 2: Early (1993) Schottky spectra from the ESR before (green) and after (red with $\delta p/p = 2 \times 10^{-5}$) electron cooling.

Theory: [1, 2]

$$HOT: \quad |P_{\text{gas}}|^2 \propto T_{||} \qquad (1)$$

$$COLD: \quad |P_{\text{string}}|^2 \propto \frac{N^2}{n^2} \qquad (2)$$

Procedure:

- Chose density λ or average distance d
- Cylindrical tube with periodic boundary conditions
- Randomly distributed according to temperatures Θ = (kinetic energy)/(q^2/d)
- Molecular dynamics
- Reflect or pass each other?
- Good statistics ⇒ Red curve of Fig.3, ref. [5]
- Good statistics ⇒ Necessary conditions for ordered strings ref. [7]
- Record signals at pickup, Fourier transform and integrate the Schottky signals over one harmonic
- ⇒ Spectra of Fig.1
- ⇒ Widths of Fig.4 → thermal
- ⇒ Hot systems of Fig.5 linear with Θ, eq.[1]
- ⇒ Hot systems of Fig.6 I independent of particle number for $N > 100$, new effects for for $N < 100$
- ⇒ Cold systems for Fig.7 obey $\propto 1/n^2$ and $\propto N^2$, eq.[2]

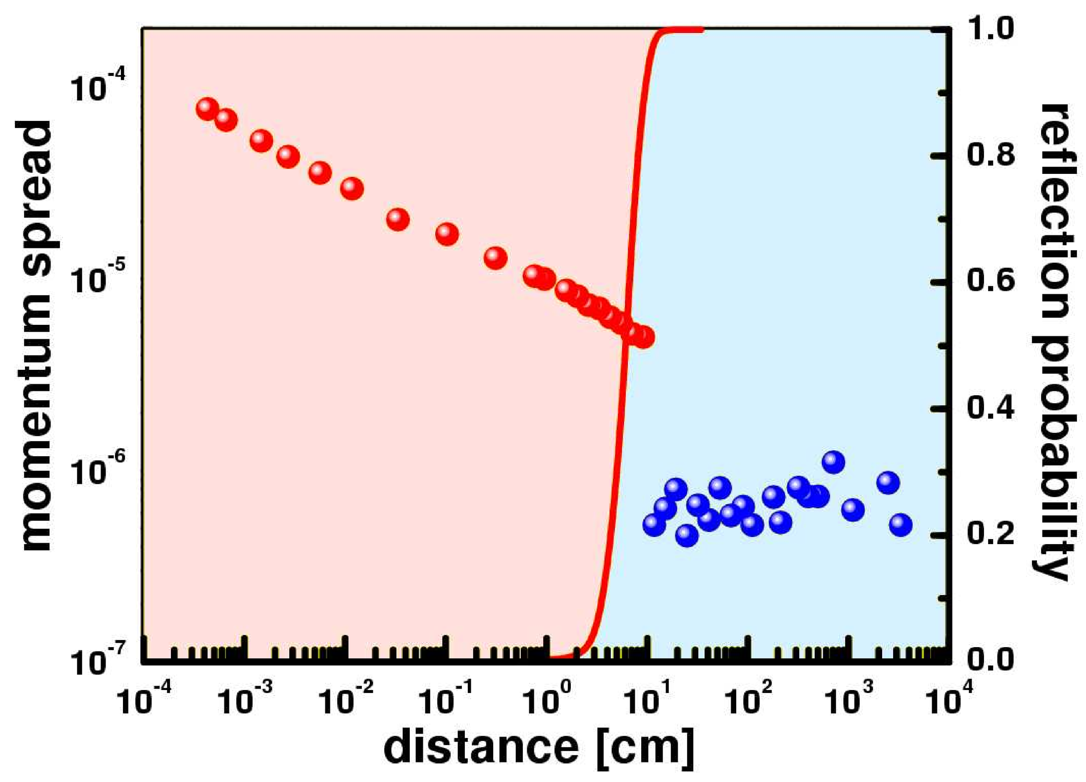

Figure 3: Momentum spreads of an U^{92+} beam at 360 MeV/u. The red line is the calculated reflection probability. After Steck [4] and Hasse [5].

Results:

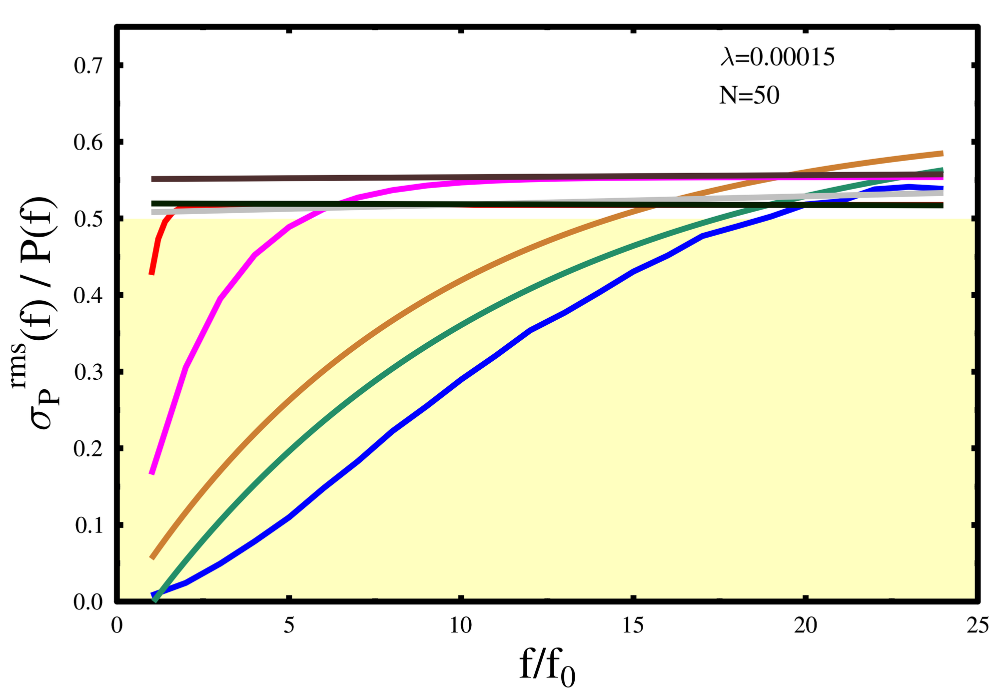

Figure 4: Rms width over average value of the integrated Schottky spectra of Fig.1 for various temperatures (same color code).

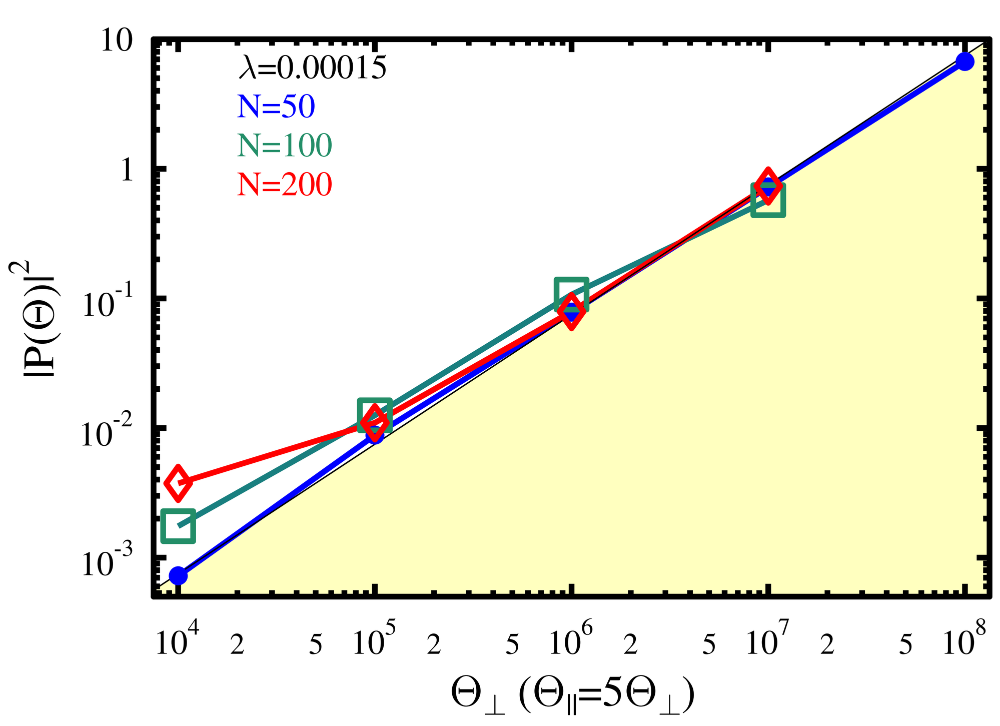

Figure 5: Temperature dependence of hot integrated Schottky signals for various particle numbers.

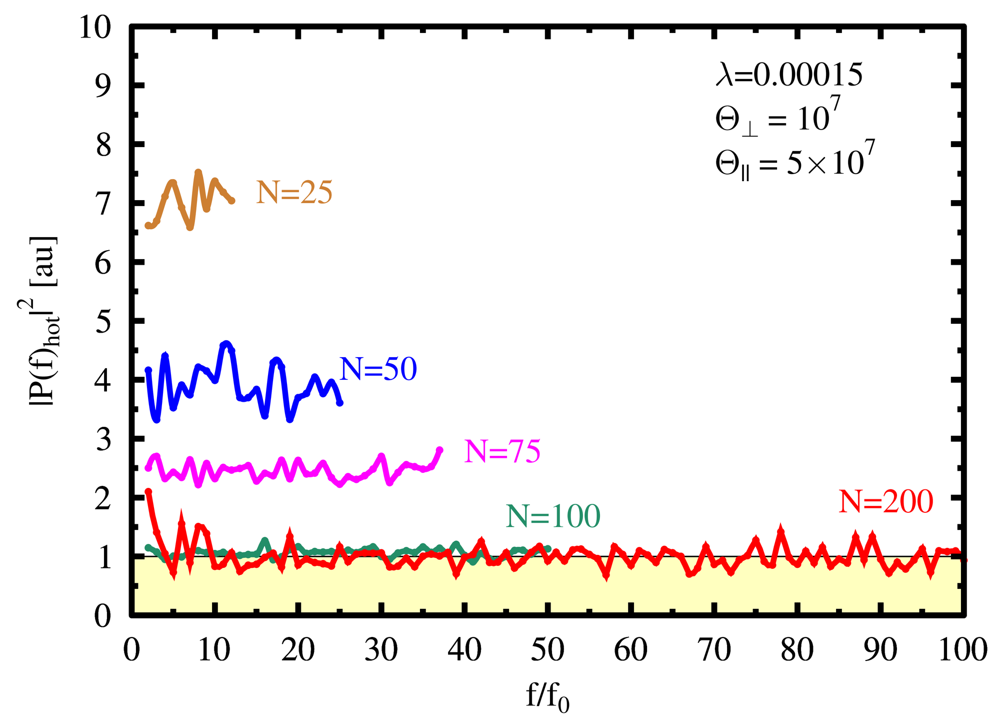

Figure 6: Particle number dependence of hot integrated Schottky signals vs. harmonic normalized to unity for $N \to \infty$.

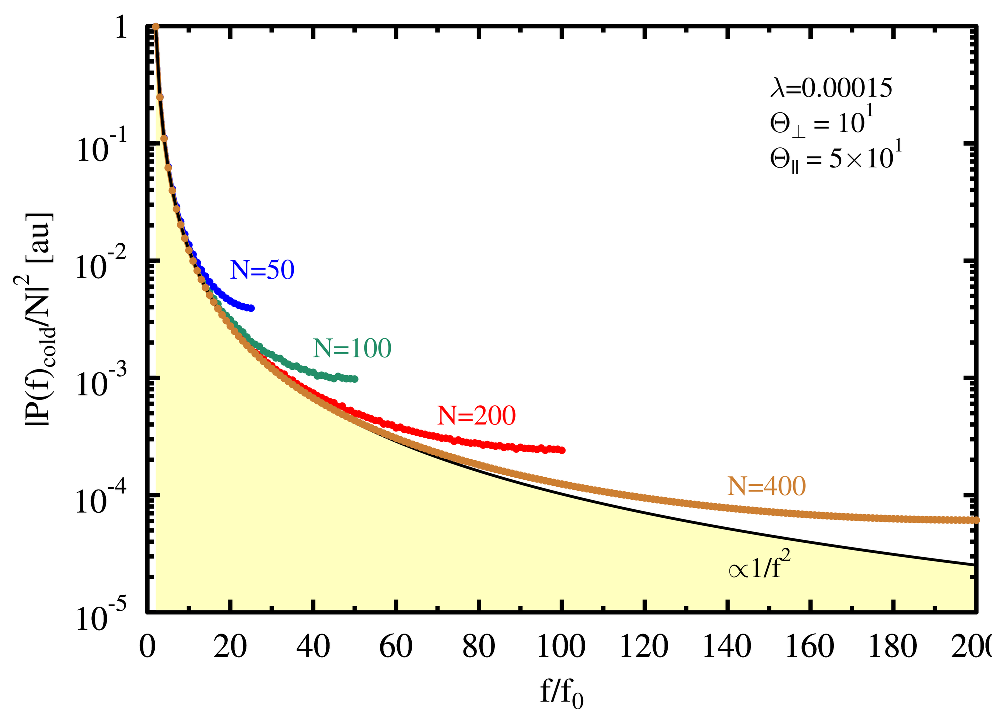

Figure 7: Particle number dependence of cold integrated Schottky signals (divided by N^2) vs. harmonic.

References

[1] S. Chattopadhyay, *Some Fundamental Aspects of Fluctuations and Coherence in Charged-particle Beams in Storage Rings*, Proc. CERN Accel. School, 1984, CERN Report 84-11 (1984).

[2] D. Boussard, *Schottky Noise and Beam Transfer Function Diagnostics*, Proc. CERN Accel. School, 5th Advanced Course, p. 749, CERN Report 95-06 (1995).

[3] R.W. Hasse, J.P. Schiffer, Ann. Phys. (NY) **203** (1990) 419.

[4] M. Steck, Hyperfine Interact. **99** (1996) 245; M. Steck, Nucl. Phys. **A626** (1997) 473c.

[5] R.W. Hasse, Phys. Rev. Lett. **83** (1999) 3430.

[6] R.W. Hasse, M. Steck, *Proc. EPAC Vienna*, 2000, p. 274 (JACoW TUBF201).

[7] R.W. Hasse, Phys. Rev. Lett. **90** (2003) 204801-1.