BUNCHED BEAM STOCHASTIC COOLING SIMULATIONS AND COMPARISON WITH DATA*

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Abstract

With the experimental success of longitudinal, bunched beam stochastic cooling in RHIC [1] it is natural to ask whether the system works as well as it might and whether upgrades or new systems are warranted. A computer code, very similar to those used for multi-particle coherent instability simulations, has been written and is being used to address these questions.

INTRODUCTION

A stochastic cooling system is a wide band feedback loop[2, 3]. A pickup signal is processed, amplified and used to drive a kicker. The difference between coasting and bunch beam stochastic cooling theory is similar to the difference between coasting and bunched beam instability theory. While the former is quite simple, the latter is still evolving.

A theory of bunched beam cooling was developed in the early eighties [4, 5, 6]. As with bunched beam stability theory, there are parameter regimes in which accurate, closed form results can be obtained. In other regimes the bunched beams act like coasting beams [7, 8]. These sort of considerations were used in the design of the RHIC longitudinal cooling system, which is now operational. Uncooled and cooled bunches are shown in Figures 1 and 2, respectively. While the general beam parameters are in line with expectations, we know of no theory capable of explaining the detailed evolution of the cooled beam. Simulations of proton test bunch cooling were fairly successful [9]. We have generalized to code to include intrabeam scattering (IBS) and transverse cooling. This note gives a detailed account of the algorithms and compares data with simulation.

Table 1: Machine and Beam Parameters for Gold

parameter	value
h=360 voltage	300 kV
h=2520 voltage	3 MV
initial FWHM bunch length	3 ns
particles/bunch	10^9
initial emittance	$15\pi\mu$m
betatron tunes	$Q_x = 28.2, Q_y = 27.2$
Lorentz factor	107
circumference	3834 m
transition gamma	22.89

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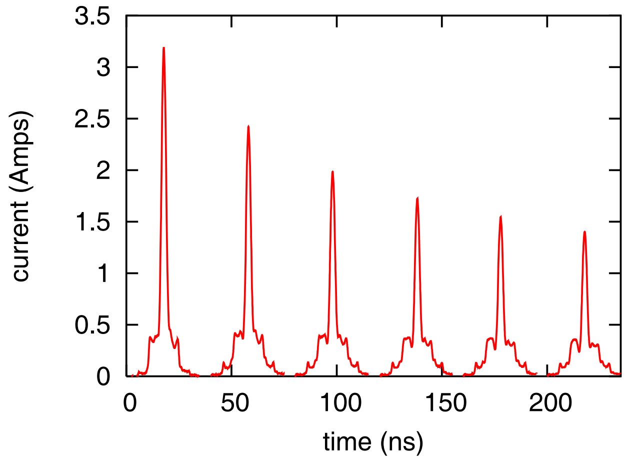

Figure 1: Evolution of the average bunch profile over a five hour RHIC store with gold beam and no cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

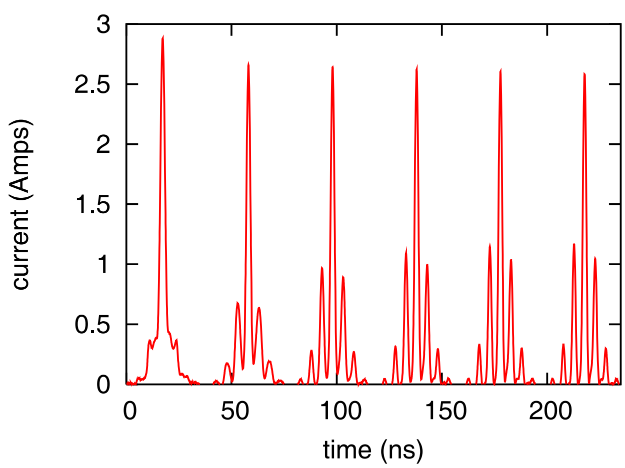

Figure 2: Evolution of the average bunch profile over a five hour RHIC store with gold beam and good longitudinal cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

SIMULATIONS

The code involves single particle dynamics and multiparticle kicks. First consider the single particle motion. The longitudinal update for a fraction of a turn χ is

$$\bar{\epsilon} = \epsilon + \chi \frac{q}{mc^2} V_{rf}(\tau) \qquad (1)$$

$$\bar{\tau} = \tau + \chi \frac{T_0 \eta}{\beta^2 \gamma_0} \bar{\epsilon} \qquad (2)$$

where τ is the arrival time of the particle with respect to the synchronous phase, $\epsilon = \gamma - \gamma_0$ is proportional to the energy deviation, γ_0 is the reference Lorentz factor for a particle of mass m and charge q, $V_{rf}(\tau)$ is the RF voltage, $\beta = v/c$, $T_0 = 1/f_0$ is the revolution period, and η is the frequency slip factor. Since the RHIC synchrotron tune is $O(10^{-3})$, the distribution of the rf voltage is not important.

Only one transverse variable is considered and it will be referred to as x. The single particle transverse update for a fraction of a turn χ is

$$\bar{x} = x\cos\psi + p\sin\psi \quad (3)$$
$$\bar{p} = -x\sin\psi + p\cos\psi \quad (4)$$
$$\psi = \chi\psi_0 + \chi\frac{2\pi\xi}{\beta^2\gamma_0}\epsilon \quad (5)$$

where p is the transverse momentum variable, $\psi_0 = 2\pi Q_x$ is the on-momentum phase advance per turn, and ξ is the chromaticity. It is assumed that the rms emittance of the neglected transverse dimension is the same as the rms emittance of the dimension tracked. For no transverse cooling this is a fairly good approximation under normal RHIC conditions. With transverse cooling we invoke sufficient coupling, or cooling in both transverse dimensions.

The effect of IBS was included by first calculating the rms growth rates for the beam being simulated. This was done using Piwinski's formulae [10] with the smooth lattice approximation. The emittance growth rates are

$$\frac{1}{\sigma_j^2}\frac{d\sigma_j^2}{dt} = \alpha_{j0}, \quad (6)$$

where $j = x, y, p$. The growth rates in the handbook are for amplitudes, with *eg* $1/T_p = \alpha_{p0}/2$. For the actual RHIC beam one finds comparable growth in the two transverse directions, $\alpha_x \approx \alpha_y$, so the next step is to define an average transverse growth rate for the physical beam $\alpha_{\perp 0} = (\alpha_{x0} + \alpha_{y0})/2$. Typical rms growth times are of order an hour, but there is no need to directly simulate such a large number of turns. Instead, one can simply choose the number of simulation turns one wishes to calculate in order to model a given number of turns in the actual machine. Suppose we wish to model N_m turns in the real machine with N_c turns in a computer simulation. Let $R = N_m/N_c > 1$ be the number of machine turns divided by the number of simulation turns. By using the rms growth rates $\alpha_{p1} = R\alpha_{p0}$ and $\alpha_{\perp 1} = R\alpha_{\perp 0}$, the simulation will show the same growth with R fewer computations than a direct simulation. The final modification is due to the fact that the line densities in Figure 1 are not close to gaussian, while equation (6) is defined for gaussian bunches. The IBS rates are proportional to the beam density and, correspondingly, the local value of beam current. Define a form factor $F(t) = I(t)\sigma_t 2\sqrt{\pi}/Q$ where $I(t)$ is the instantaneous beam current, σ_t is the rms bunch length, and Q is the total bunch charge. The IBS momentum kick given to a particle on a given turn is $\Delta p = \sigma_p\sqrt{\alpha_{p1}T_0F(t)}rand$, where $rand$ is a gaussian random deviate with zero mean and unit standard deviation. The rms value of Δp for gaussian $I(t)$ equals Piwinski's value, and the same form factor is used for transverse kicks. This is equivalent to applying coasting beam formulas to longitudinal slices within the beam, with the caveat that the rms momentum spread and rms transverse emittance are calculated for the beam as a whole.

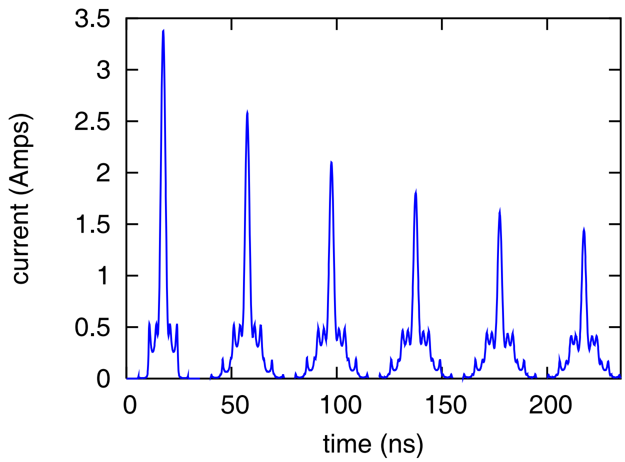

Figure 3: Simulation of the average bunch profile over a five hour RHIC store with gold beam and no cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

The cooling algorithm exploits the fact that, for fixed gain and bandwidth, the cooling time is proportional to the number of particles [2, 11, 12, 4, 5, 6, 3]. While this is a well known result we will present an alternate derivation.

Consider $N \gg 1$ harmonic oscillators with frequencies $\Omega_j = \Omega_0 + \omega_j$, with $|\omega_j| \ll \Omega_0$. The equation of motion for oscillator j is

$$\ddot{x}_j + \Omega_j^2 x_j = -\frac{2g\Omega_0}{N}\sum_{m=1}^{N}\dot{x}_m, \quad (7)$$

where $\dot{x} = dx/dt$ and g is the cooling gain. In this model N represents the number of particles per sample in an actual cooling system, and we consider a large mixing factor. Set $x_j = a_j\exp(-\lambda t - i\Omega_0 t)$ and keep leading order terms to yield

$$(\lambda - i\omega_j)a_j = \frac{g\Omega_0}{N}\sum_{m=1}^{N}a_m. \quad (8)$$

dividing through by $\lambda - i\omega_j$ and summing over j yields the dispersion relation

$$1 = \frac{g\Omega_0}{N}\sum_{m=1}^{N}\frac{1}{\lambda - i\omega_m}. \quad (9)$$

For almost all values of g equation (9) has N distinct solutions, so no information has been lost. Let the coarse grained, normalized distribution for the frequencies be $f(\omega)$ and limit the discussion to the case were

$$\int_{-\infty}^{\omega_j} f(\omega)d\omega = \frac{j-1/2}{N}, \tag{10}$$

so that the frequencies are nearly evenly spaced when viewed over short ranges of ω. In the vicinity of frequency ω the spacing is $\Delta\omega = 1/(Nf(\omega))$. Assume the existence of an inertial range M with $1 \ll M \ll N$. Consider a solution to equation (9) with $|Im(\lambda) - \omega_K| \lesssim \Delta\omega_K = 1/Nf(\omega_K)$. For frequencies near ω_K the sum in (9) resembles a "picket fence" , while for frequencies far from ω_K the sum is well approximated by an integral. Then

$$\begin{aligned}
&\sum_{m=1}^{N} \frac{1}{\lambda - i\omega_m} \\
= &\sum_{|m-K|<M} \frac{1}{\lambda - i\omega_m} + \sum_{|m-K|\geq M} \frac{1}{\lambda - i\omega_m} \\
\approx &\sum_{|m|<M} \frac{1}{\lambda - i\omega_K - im\Delta\omega_K} + \sum_{|m-K|>M} \frac{i}{\omega_m - \omega_K} \\
\approx &\sum_{k=-\infty}^{\infty} \frac{1}{\lambda - i\omega_K - ik\Delta\omega_K} \\
+ &\; iN \int_{-\infty}^{\infty} \frac{\omega - \omega_K}{0^+ + (\omega-\omega_K)^2} f(\omega)d\omega. \qquad (11)
\end{aligned}$$

Use the identity [13]

$$\lim_{M\to\infty} \sum_{k=-M}^{M} \frac{1}{z - ik} = \pi \frac{\exp(2\pi z) + 1}{\exp(2\pi z) - 1},$$

set

$$X(\omega_K) = \Omega_0 \int_{-\infty}^{\infty} \frac{\omega - \omega_K}{0^+ + (\omega - \omega_K)^2} f(\omega)d\omega,$$

and set $R(\omega_K) = \pi\Omega_0 f(\omega_K)$ to obtain

$$\exp[2\pi Nf(\omega_K)(\lambda - i\omega_K)] = \frac{1 + gR - igX}{1 - gR - igX}. \tag{12}$$

The right hand side of (12) is independent of N so $Re(\lambda) \propto 1/N$. Equation (12) with $X = 0$ is compared with the eigenvalues obtained from exact, numerical solution of equation (8) in Figure 4. Figure 5 shows a close in view comparing the exact and approximate eigenvalues as a parametric function of g. The excellent agreement suggests that the $1/N$ scaling is robust for $N \gtrsim 50$.

By exploiting the scaling with N a comparatively small number of macroparticles can be tracked over a reasonable number of turns and the results scaled to the real beam being modeled [9]. It is then possible to simulate an accurate model of the cooling system. Consider the longitudinal cooling system in RHIC. Let $I_0(t)$ be the beam current at the pickup as a function of time. This is sampled on a fine grid with $t_k = k\Delta t$. For N_m macroparticles the beam current at the pickup on turn n is

$$I_0(t_k, n) = \frac{q_m}{\Delta t} \sum_{m=1}^{N_m} \hat{\delta}(\tau_m^p(n) - t_k), \tag{13}$$

where $\tau_m^p(n)$ is the arrival time of macroparticle m at the pickup on turn n, q_m is the charge of a macroparticle, and $\hat{\delta}(t)$ is a triangle function of full width $2\Delta t$ and height one. The macroparticle charge is $q_m = Q/N_m$, with Q the total charge on the real beam being modeled. We use two, cascaded one turn delay filters so the effective current driving the kicker on turn n is

$$I_1(t_k, n) = I_0(t_k, n) - 2I_0(t_k, n-1) + I_0(t_k, n-2). \tag{14}$$

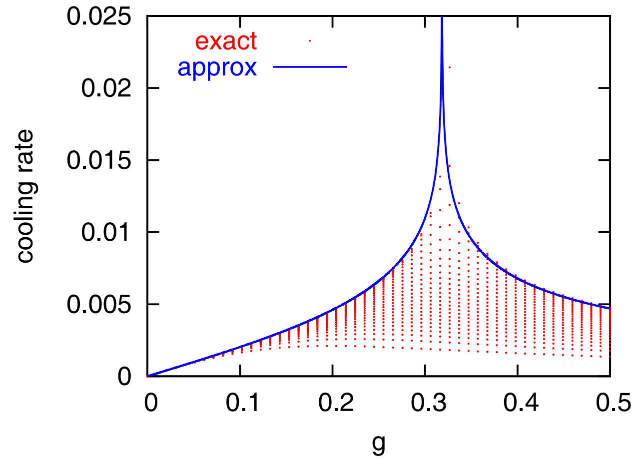

Figure 4: Comparison of actual values of $Re(\lambda)$ versus gain with those obtained from equation (12) with $X = 0$ for a rectangular frequency distribution with $N = 51$. The numerical solution had one eigenmode with a monotonically growing eigenvalue, which is not fully shown.

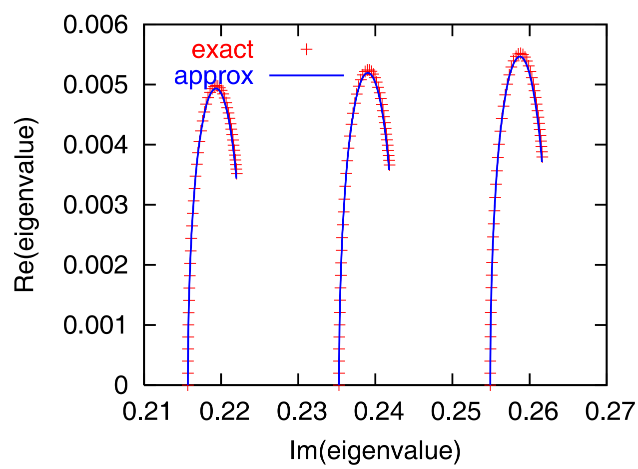

Figure 5: Evolution of λ as a function of gain for the exact, numerical solution and equation (12). The oscillator frequencies were uniformly spaced with $\omega_j = j/N$ and $N = 51$. Comparable agreement is obtained with a gaussian distribution.

The RHIC system uses a bank of cavities with frequencies spaced by 200 MHz, and a traversal filter drive. The cavity bandwidths of 10 MHz are sufficient so that the kick decays between bunches, but within a single bunch on a single turn the kick is nearly periodic with period $\tau_0 = 5$ ns. The simulation takes this periodicity to be perfect and uses the kicker drive current,

$$I_2(t_k, n) = \sum_m I_1(t_k - m\tau_0, n), \tag{15}$$

where the limits on m are chosen so that I_2 is correct for $0 \le t_k \le \tau_0$. The current I_2 drives the effective wakefield. The wakefield is defined by a lower frequency f_1, an upper frequency f_2, and the effective longitudinal resistance at these two frequencies, R_1 and R_2, respectively. The needed phase shift is incorporated yielding a longitudinal wakefield

$$W(\tau) = 2 \int_{f_1}^{f_2} df R(f) \sin(2\pi f \tau), \tag{16}$$

where $R(f)$ is linear between f_1 and f_2. The voltage is obtained by convolving I_2 with $W(\tau)$ using a fast Fourier transform with an interval τ_0. This defines the voltage on $[0, \tau_0]$. The particles are then tracked from the pickup to the kicker and the kick is applied. For particles that arrive outside $[0, \tau_0]$, the kick is taken as periodic with period τ_0. Figure 6 shows a simulation of longitudinal cooling for the data in Figure 2. The simulations are in fair agreement with the data. The simulations do not include the few percent burn-off losses, which are of order the difference between the simulation and the data. We will treat this as an estimation error and go on to develop the algorithms for transverse cooling.

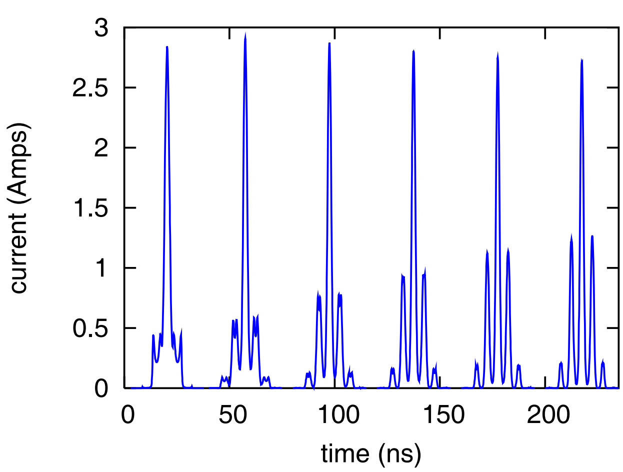

Figure 6: Simulation of the average bunch profile over a five hour RHIC store with gold beam and good cooling. Initial conditions are shown on the left and each trace to the right is one hour later.

For N_m macroparticles, the dipole weighted beam current at the pickup on turn n is

$$D_0(t_k, n) = \frac{q_m}{\Delta t} \sum_{m=1}^{N_m} x_m^p(n) \hat{\delta}(\tau_m^p(n) - t_k), \tag{17}$$

where $x_m^p(n)$ is the transverse offset for particle m at the pickup on turn n and all other symbols are the same as in equation (13). A lower frequency ($f_{1\perp}$), upper frequency ($f_{2\perp}$), and transverse impedances $R_{\perp 1}$ and $R_{\perp 2}$ are defined. The transverse wakefield is

$$W_\perp(\tau) = 2 \int_{f_{1\perp}}^{f_{2\perp}} R_\perp(f) df \cos(2\pi f \tau). \tag{18}$$

As of now there is no filtering on D_0 and the kick is obtained by convolving D_0 and $W_\perp$. We assume cavity kickers with same $1/\tau_0$ frequency spacing.

As a starting point we simulated transverse cooling without longitudinal cooling or intrabeam scattering. This parameter regime allows for a particularly clean test of the scaling law for cooling rate as a function of macroparticle number, as shown in Figure 7. The horizontal scale is the normalized longitudinal energy,

$$H_s(\epsilon, \tau) = \frac{T_0 \eta m c^2}{2\beta^2 \gamma_0} \epsilon^2 - \int_0^\tau dt q V_{rf}(t). \tag{19}$$

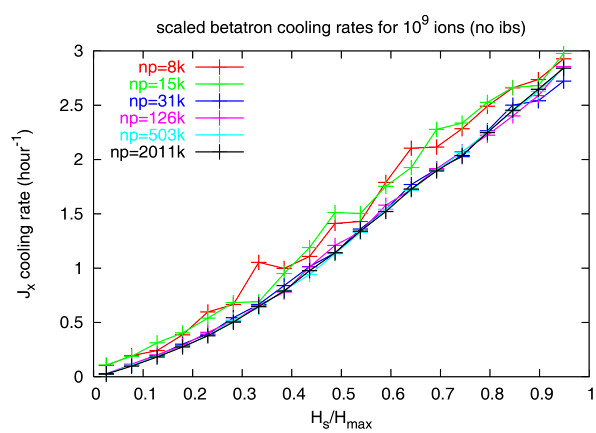

Figure 7: Transverse cooling rate versus the value of the longitudinal hamiltonian. Similar results are shown in [4, 5]

The strong dependence of transverse cooling rate on longitudinal energy was predicted by Chattopadhyay [4, 5], and design options for transverse cooling in the SPS included a higher harmonic RF cavity in an attempt to fix the problem [14]. In RHIC this problem is solved by longitudinal diffusion, from both IBS and the longitudinal stochastic cooling system. Diffusion causes the longitudinal energy of individual particles to migrate. For RHIC parameters the cooled beam shows almost no correlation of transverse action with longitudinal energy.

In addition to the large scale migration of the particles in H_s, the inclusion of IBS in the simulation can enhance the short term mixing [15]. Simulations with too few macroparticles would overestimate the effectiveness of the cooling system. Figure 8 shows that our simulations with $50,000$ macroparticles should be fine.

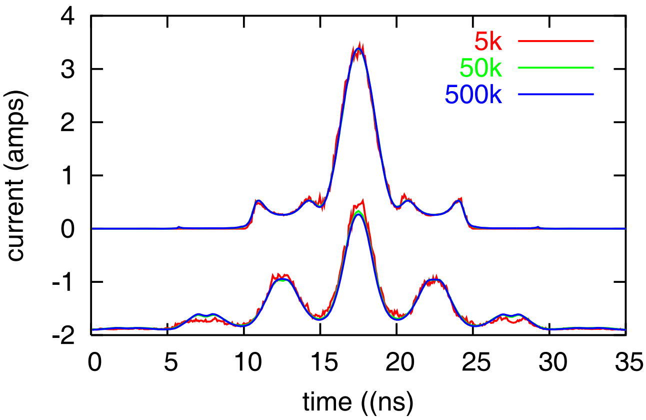

Figure 8: Test of convergence with both cooling and IBS. The initial profiles for 5000, $50,000$, and $500,000$ macroparticles are shown in the upper traces. The lower traces show the profiles at 2000, $20,000$, and $200,000$ turns, respectively. This corresponds to 10^9 gold ions evolving over 85 minutes.

Simulations for 10^9 gold ions per bunch, with both longitudinal and transverse cooling are shown in Figures 9 and 10. We assumed 5 MV on the $h = 2520$ RF system and clean rebucketing. The $1/6$th turn delay for the longitudinal cooling system will utilize the 70 GHz microwave link we are currently developing.

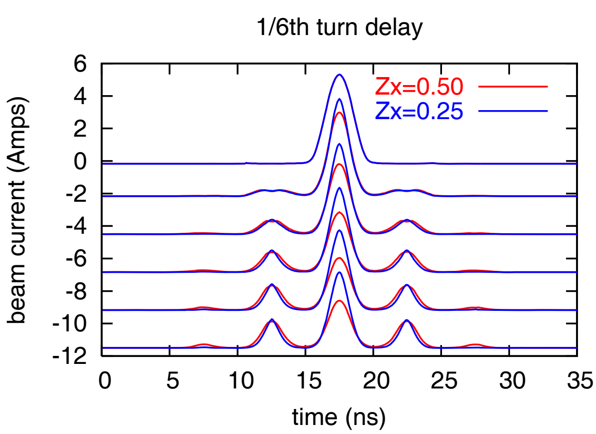

Figure 9: Simulated longitudinal profiles over 5 hours with two different transverse cooling gains and $1/6$th turn delay. The transverse gain of 0.25 utilized only a single one turn delay in the longitudinal cooling system, while the gain of 0.5 used the same cascaded delays we use now.

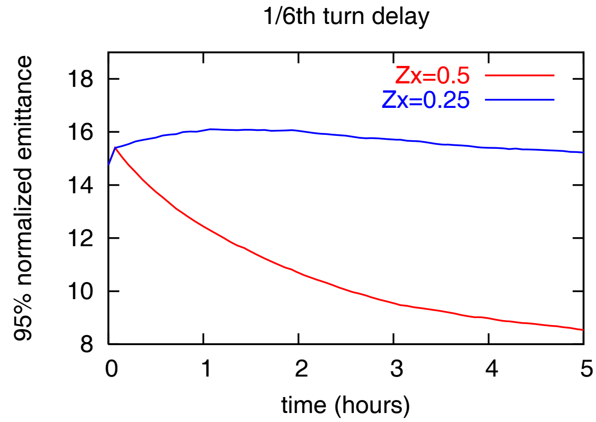

Figure 10: Simulated transverse emmittance over 5 hours with two different transverse cooling gains. The parameters are the same as those in Fig 9.

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