ANALYSIS OF RESONANCES INDUCED BY THE SIS-18 ELECTRON COOLER*

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Abstract

Besides the beam cooling effect, an electron cooler also acts as a non-linear optical element. This may lead to the excitation of resonances possibly resulting in an increase of the beam emittance. The aim of this work is the calculation of resonances driven by the electron space charge field in the cooler installed in the SIS heavy ion synchrotron at GSI Darmstadt. For our calculations, we used a numerical model consisting of a rotation matrix representing the ideal lattice together with a non-linear transverse kick element representing the electron cooler. Within this model, we studied the non-linear tune shift and the dominant resonance lines resulting from the interaction with the cooler.

INTRODUCTION

The space charge field in an electron cooler acts as a non-linear optical element in the lattice of a storage ring. This may lead to the excitation of additional ring resonances. Depending on the machine working point these resonances cause emittance growth and an effective heating of the beam, as it was observed e.g. in the CELSIUS cooler storage ring [1].

Electron cooling at medium energies will play an essential role in the proposed FAIR storage rings [2]. Electron cooling is already available to improve the beam quality of the intense ion beams at low energy in the existing SIS synchrotron. At low or medium beam energies, the transverse tune shift due to the direct space charge force plays an important role. The resonances excited by the non-linear space charge field of the cooler electron can potentially limit the reachable beam intensity and quality.

In this work, the excitation of resonances driven by an electron cooler is calculated within a simplified numerical model. The electron cooler is represented through a non-linear kick element in an otherwise ideal lattice. This enabled us to study only the resonances driven by the electron cooler. The MAD-X code [3] was used to perform resonance scans over a large working point area. The study is performed for parameters relevant to the electron cooler in the the SIS heavy ion synchrotron at GSI Darmstadt. This theoretical study provides the necessary information for dedicated measurements of cooler induced resonances and effects in SIS.

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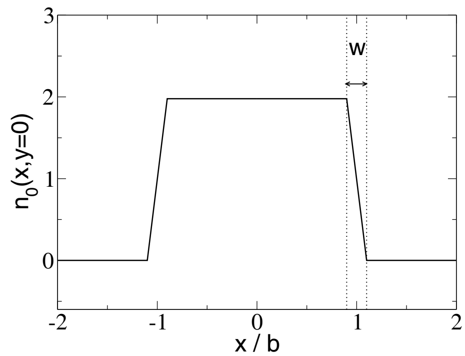

Figure 1: Normalised charge density profile used for the electron beam as provided by the beambeam element of MAD-X [3] as a function of x for $y = 0$. An edge layer with a width $w = 0.01\,b$ was used in the calculations.

PARTICLE TRACKING MODEL

In our calculations we used a simple model consisting of a rotation matrix providing the phase advance of the lattice of SIS–18 and a non-linear transverse kick introducing the force of the electron cooler in the thin lens approximation. The coordinates of a particle after the $(n+1)$-st revolution are calculated from those of the n-th revolution by

$$\begin{pmatrix} z_{n+1} \\ z'_{n+1} \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_z & \hat{\beta}_z \sin 2\pi\nu_z \\ -\frac{1}{\hat{\beta}_z}\sin 2\pi\nu_z & \cos 2\pi\nu_z \end{pmatrix} \times \begin{pmatrix} z_n \\ z'_n + \Delta z'(x_n, y_n) \end{pmatrix} \tag{1}$$

with $z = x, y$. Here, ν_z is the bare tune of the lattice, $\hat{\beta}_z$ is the unperturbed beta function in z direction at the location of the electron cooler, and

$$\Delta z'(x,y) = \frac{qq'N'}{2\pi\varepsilon_0 m_0 c^2 \beta_0^2 \gamma_0^3}\frac{z}{R^2}\int_0^R \mathrm{d}r\, r\, n_0(r) \tag{2}$$

with $R = \sqrt{x^2+y^2}$ is the transverse momentum kick depending on both spatial direction x, y. Here,

$$N' = \left|\frac{I_e L_{\mathrm{cool}}}{q'\beta_0 c}\right| \tag{3}$$

is the number of electrons in the electron cooler. q, q' are the charges of the particles in the beam considered and in the electron beam, i.e. it is $q' = -e$. n_0 is the normalised radial current distribution in the electron beam.

The electron beam of an electron cooler usually has a radial shape with a constant current density in the centre

Table 1: Parameter of SIS–18 used in the calculations and taken from [4] and [5], and quantities calculated with them. K_1 corresponds to equation (4) and $\Delta\nu_x, \Delta\nu_y$ to equation (5).

Particle	U^{73+}
Injection energy E	$11.4\,\mathrm{MeV/u}$
Relativistic factors $\beta_0,\ \gamma_0$	$0.15,\ 1.01$
Cooling length L_{cool}	$3\ \mathrm{m}$
Electron current I_e	$0.3\ \mathrm{A}$
Cathode radius r_{cath}	$12.7\ \mathrm{mm}$
Adiab. expansion factor f_E: used, (range)	$3,\ (1\ ...\ 8)$
Electron beam radius ($b = r_{\mathrm{cath}}\sqrt{f_E}$)	$22\ \mathrm{mm}$
Beta function in the cooler ($\hat\beta_x,\ \hat\beta_y$)	$8\ \mathrm{m},\ 15\ \mathrm{m}$
Eff. focal strength K_1	$-0.010\ \mathrm{m}^{-1}$
Resulting tune shift $\Delta\nu_x, \Delta\nu_y$	$0.0066, 0.012$
Eigen space charge tune shift $\Delta\nu_{sc}$	up to -0.25

and a thin edge layer. We could use the beambeam element of the MAD-X code for the representation of the electron cooler. The profile is shown in figure 1.

In the region of constant current density, a momentum kick of this shape acts as an element with a field gradient having an effective focal strength K_1. For a sufficiently small width of the edge layer w, the focal strength is given by

$$K_1 := k_1 L_{\mathrm{cool}} = \frac{\Delta z'}{z} \approx \frac{qeN'}{2\pi\varepsilon_0 m_0 c^2 \beta_0^2 \gamma_0^3 b^2}. \quad (4)$$

For the resulting linear tune shift, one can write

$$\Delta\nu_z = -\frac{\hat\beta_x K_1}{4\pi}. \quad (5)$$

Our calculations were performed for U^{73+} ions using the parameters given in table 1.

RESULTS

To make the resonances visible, the relative rms beam radius variation

$$w_{z,\mathrm{rel}} = \frac{\sigma_{f,z}}{\sigma_{i,z}}, \quad z = x, y, \quad (6)$$

with

$$\sigma_{(i,f),z} = \sqrt{\bar{z^2}} = \frac{1}{N}\sqrt{\sum_{k=1}^{N} z^2_{k,(i,f)}} \quad (7)$$

was calculated as a function of the tune values ν_x, ν_y of the rotation matrix. N is the number of the test particles tracked. We used a particle beam with a Gaussian initial profile.

To have a realistic tune range, we searched for resonances in a tune window defined by $\nu_x \in [4.05, 4.3]$ and $\nu_y \in [3.2, 3.45]$, which is near the working point $(\nu_x, \nu_y) = (4.2, 3.4)$ given in [4], and which does not contain a half integer resonance. On the other hand, it was

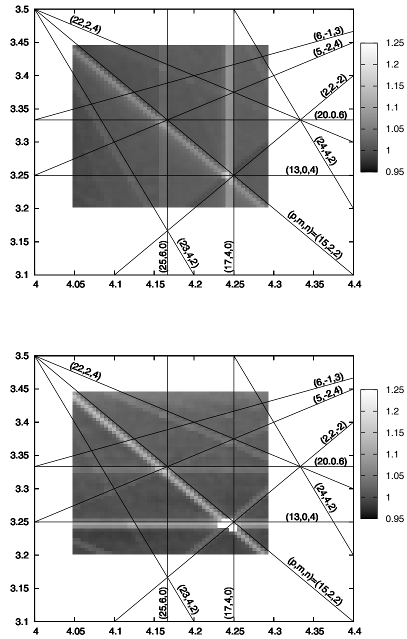

Figure 2: Relative rms beam width in x and y direction, upper and lower picture, respectively. The colour scale is within $w_{\mathrm{rel}} \leq 1$ (dark grey) and $w_{\mathrm{rel}} \geq 2$ (white).

pointed out in [1] within an analytic model, that an electron cooler, with a round electron beam, excites only resonances of even order, where, additionally, the resonances strength decreases with increasing order. Hence, we searched only for resonances of order 4 and 6.

The initial rms beam width in both spatial directions were chosen to be equal to the radius of the electron beam, i.e. $\sigma_{i,x} = \sigma_{i,y} = b = 22$ mm, see table 1. The resulting initial rms emittances are different due to the different emittances, it is $\epsilon_x = 61.0\,\mathrm{mm\ mrad}$ and $\epsilon_y = 32.3\,\mathrm{mm\ mrad}$.

The resonances found can be seen in figure 2. The straight black lines denoting the resonances satisfy the relation

$$p = m\nu_x + n\nu_y. \quad (8)$$

The positions of the resonances found in our tune scan by detecting the enhancement of the beam width are near these lines, which denote the positions of the resonances with respect to the unperturbed tune ν_x, ν_y of the rotation matrix. So, we could identify every resonance line found. The resonances found in the tune scan are slightly shifted to smaller tunes compared to the resonance lines of the rotation matrix. The reason for that is, that an electron cooler contrary to higher order multipoles yields also a linear tune shift $\Delta\nu_z(x, y)$. On the other hand, the position of the resonances depend on the total tune $\nu_{z,\mathrm{tot}} = \nu_z + \Delta\nu_z(x, y)$. So, the corresponding tune of the rotation matrix ν_z is smaller than the total tune $\nu_{z,\mathrm{tot}}$.

As expected, we found resonances of order 4 generally being the strongest resonances followed by resonances of

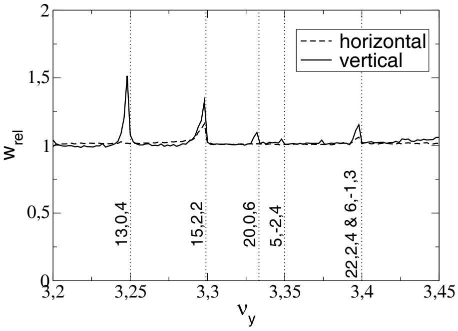

Figure 3: Relative beam width as a function of the vertical tune within the range $\nu_y \in [3.1, 3.45]$ for one horizontal tune $\nu_x = 4.2$. So, this figure is an extract of figure 2.

the order 6. Additionally, only so called sum resonances and resonances depending only on the tune of one direction lead to a significant beam blow up, what seems to be reasonable, see e.g. [6]. Here, latter lead to an enhancement of the beam size only in one direction. So, the resonances $(p, m, n) = (17, 4, 0)$ and $(25, 6, 0)$ are visible only in the upper picture of figure 2 showing the relative beam width in horizontal direction, whereas the resonances with $(p, m, n) = (13, 0, 4)$ and $(20, 0, 6)$ appear only in the lower picture of that figure and in the solid line in figure 3 showing the relative extension of the beam width in vertical direction. Figure 3 also shows, that the widths of resonances of different order do not significantly differ from each other. For that reason, a quantitative verification of the widths using the analytic model in [1] is not possible.

Under the conditions defined in table 1, we obtained a tune shift due to the electron cooler of $\Delta\nu_x = 0.0066, \Delta\nu_y = 0.012$, what is so small, that the according tune spread does not cross any resonance found in our calculation. On the other hand, the tune shift due to the eigen space charge of the beam has an size up to $\Delta\nu_{sc} \approx -0.25$. Therefore, the according tune spread will cross some of the strong resonances shown in figure 2. For that reason, we observed the beam behaviour at a working point near a strong resonance in more detail. In particular, we investigated the growth of the beam width depending on the initial beam width and the number of revolutions. One can see in figure 4 the increase of the beam width occurring only in vertical direction because of the working point $\nu_x = 4.1$, $\nu_y = 3.245$ close to the resonance given by $(p, m, n) = (13, 0, 4)$. We found, that only particles being initially at the edge of the beam increase their distance from the beam centre, whereas particles in the core stay there. So, the number of test particles having a betatron amplitude larger than the radius of the electron beam changed from 2049 at the beginning to 2056 after 100000 revolutions, what is an increase by $0.3\ \%$. Here, the total number of test particles was 5000, and it was $\sigma_{i,x} = \sigma_{i.y} = 0.5\ b$. The according vertical rms width increased from $\sigma_{i,y} = 22$ mm to $\sigma_{f,y} = 26.4$ mm and so, by about 20 %. Note, that we have a 4-dimensional Gaussian distribution in a round

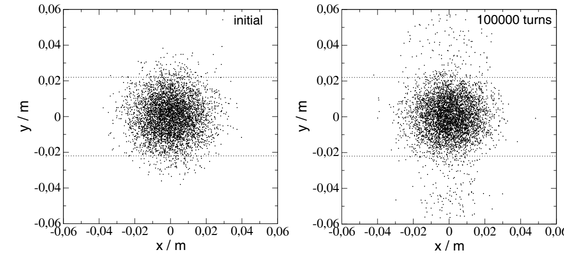

Figure 4: Spatial beam profile for $\sigma_{i,x} = \sigma_{i,y} = 0.5b$ at the beginning of a run and after 100000 revolutions. The working point is $\nu_x = 4.1$, $\nu_y = 3.245$.

beam, where the number of particles N_{in} being always closer to the beam centre than a certain distance R is given by

$$N_{in} \propto \int_0^{R/\sigma} x^3 \mathrm{e}^{-\frac{x^2}{2}} \mathrm{d}x. \tag{9}$$

The observation coincides with the statement, that resonances are driven only by the non-linear part of the electric field of the electron beam.

One exception from that general statement is the growth of the beam width due to an half-integer resonance. A resonance of this kind is driven by its nature by a quadrupole error in the lattice, i.e. by a purely linear perturbation. Such a perturbation can not cause an emittance growth. The beam blow up is caused by the growth of the beta function, see e.g. [6]. Hence, the condition $\sigma_z \propto \sqrt{\beta_z}$ is valid. Here, the full width of the tune range with a beta function enhanced by a factor 2 or more is given by the half-integer stopband integral

$$J_p = \frac{1}{2\pi} \oint \hat{\beta} k(s) \mathrm{e}^{-\mathrm{i}p\phi} \mathrm{d}s. \tag{10}$$

This provides the possibility to compare tracking results to an analytic expression. So, we performed calculations with a particle beam having an initial extension $\sigma_{i,x} = \sigma_{i,y} =$

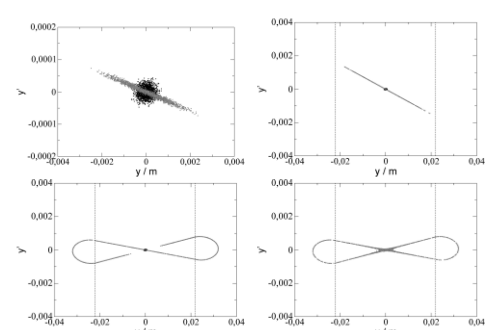

Figure 5: Vertical phase space plot for an initial beam width $\sigma_{i,y} = 0.01\ b \ll b$ at $\nu_x = 4.2,\ \nu_y = 3.45$ after several number of revolutions: $N_{\mathrm{rev}} = 20,\ 50,\ 100,\ 10000$. The vertical straight lines in pictures 2 – 4 denote the radius of the electron beam b. The black points in the centre denote the initial positions of the particles in phase space.

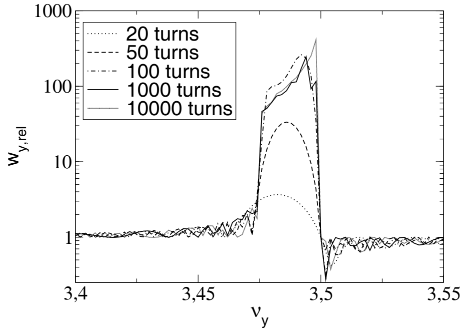

Figure 6: Relative vertical rms beam width $w_{y,\mathrm{rel}}$ after different numbers of revolutions. Note, that it is $\sigma_{i,y} = 0.01\, b$ and, so $w_{y,\mathrm{rel}} = 100$ refers to an absolute final rms beam width $\sigma_{f,y} = b$. It is $\nu_x = 4.2$.

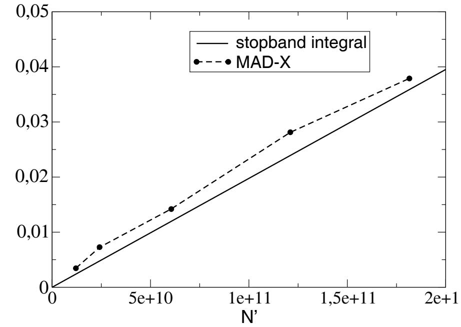

Figure 7: Full rms beam width after 1000 revolutions calculated using MAD-X compared to the half integer stopband integral J_7 from equation 10.

$0.01\, b \ll b$ to satisfy the condition of a pure linear perturbation. Due to the momentum kick, the beam extension in vertical direction started to increase. Here, the phase space ellipse became only stretched without increasing its area, as long as the vertical beam size had not exceeded the radius of the electron beam. After about 50 revolutions, the beam size exceeded the electron beam leading to the deformation of the phase space ellipse, as figure 5 shows. So, the condition for the applicability of the stopband integral was principally no longer valid. Fortunately, the width of the tune range with enhanced beam width remained almost constant also when the beam width exceeded the electron beam radius, see figure 6. So, it was possible to evaluate the full width of this range and compare it to the half-integer stopband integral. We found a very good agreement between them for several values of the electron number in the cooler, as one can see in figure 7.

SUMMARY

We studied the resonances generated by the space charge force of the electron beam in the SIS–18 cooler. The initial rms radius of the ion beam was adjusted to the radius of the electron beam. Resonances up to the 6th order could be identified. Furthermore, we could qualitatively reproduce the dependency of the resonance width on the resonance order as given by an analytic model in reference [1]. This model predicts that the resonance width decreased, when the order of a the resonances is enhanced. A quantitative reproduction of the beam width using an analytic model was possible only for the half integer resonance. Here, we found a good agreement between the resonance width and the width given by the analytic half-integer stopband integral.

Beyond that, we found, that the resonances driven by the electron cooler are of a similar width as those driven by higher order multipoles in the lattice of SIS–18 [7].

An important result is that the resonances induced by the electron cooler can lead to a strong increase of the width and the emittance of an ion beam. This can lead to a reduced cooling rate and so to an effective heating mechanism. Furthermore, the resonances are an additional possible constraint for the choice of the tune, because they could limit the extension of the space charge tune spread due to the self fields of the beam and therefore leading to the reduction of the space charge limit. On the other hand, this effect could be weakened by the fact, that an emittance growth due to the interplay of the direct space charge tune shift and spread on one hand and the resonances on the other hand arises only from particles having from the beginning a betatron amplitude larger than the radius of the electron beam. The number of these particles can be chosen by choosing a proper value for the width of the ion beam. On the other hand, it is desirable to keep this number small in any case to avoid a reduction of the electron cooling rate. So, the limiting influence of the resonance on the size of the direct space charge tune spread might possibly minimised by proper initial conditions.

Nevertheless, the interplay of the cooler induced resonances, the direct space charge tune shift, and other machine resonances requires further investigations. This will be the topic of future studies.

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