LONGITUDINAL SCHOTTKY SIGNALS OF COLD SYSTEMS WITH LOW NUMBER OF PARTICLES

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Abstract

Very cold systems of ions with sufficiently low number of particles arrange in an ordered string-like fashion. The determination of the longitudinal momentum spread and of the transverse temperature then is no longer possible by normal Schottky diagnosis. In this paper we simulate such systems in an infinitely long beam pipe with periodic boundary conditions under the influence of all long-range Coulomb interactions by Ewald summation. Then we derive the behaviour of the longitudinal Schottky signals for cold string-like systems as well as for the transition to warmer systems when the strings break, up to hot gas-like systems. Here effects from the finite number of particles, of higher harmonics and of temperature agree with those derived analytically in the limits of very low and very high temperatures.

INTRODUCTION

Schottky analysis has been an efficient tool for the determination of the momentum spread of a heavy ion beam. After the construction of the electron cooler [1] in the ESR ring, see [2], at GSI in 1990 a pickup was installed and connected to a Schottky device. From the width of the signal the momentum spread $\delta p/p$ can be deduced, see e.g. Fig. 1.

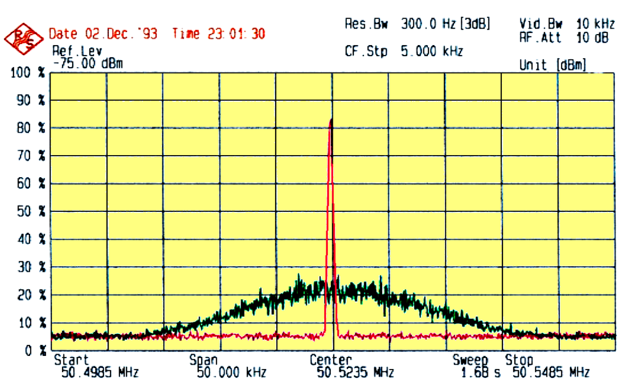

Figure 1: Early (1993) Schottky spectra from the ESR before (green) and after (red with $\delta p/p = 2 \times 10^{-5}$) electron cooling.

At high intensities of cooled systems momentum spreads below 10^{-5} could be reached. For very low densities, on the other hand, and if cooled properly, $\delta p/p$ decreases until intrabeam scattering breaks down [3]. Then the momentum spread levels off at a very low level of the order of 10^{-6} only due to ripples of the power supplies etc, see Fig. 2. These two regions are well separated by a a well defined jump in $\delta p/p$.

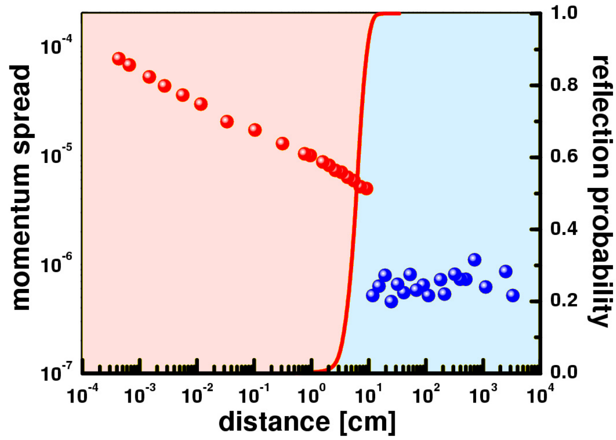

Figure 2: Momentum spreads of an U^{92+} beam at 360 MeV/u. The red line is the calculated reflection probability, see below. After Steck [3] and Hasse [4].

This effect has been detected in the ESR for various ions from protons and carbon up to uranium. Afterwards it was also confirmed in different storage rings like SIS18 at GSI [5], Cryring at Stockholm [6], and, recently, at the S-LSR at ICR, Kyoto University [7].

These results posed a challenge to theory and were soon explained in ref. [4]: If the interparticle distance becomes as small as 10 cm and if the ions are sufficiently cold in the transverse direction then the ions arrange in a string-like fashion and they repel each other rather than passing. Lateron in ref. [8] simple general criteria were derived for the existence of such Coulomb strings which turned to be out to be valid for all storage rings.

SCHOTTKY SIGNALS

A particle passing by at the Schottky pickup induces a signal called the Schottky noise. The theory of Schottky noise has first been applied to stochastic cooling at CERN. It can be found in various CERN accelerator school lectures e.g. by Chattopadhyay [10] or Boussard [11]. For an ideal (hot) gas at high density the signal is proportional to the longitudinal kinetic energy (or temperature),

$$|P_{\rm gas}|^2 \propto T_{||}\ , \tag{1}$$

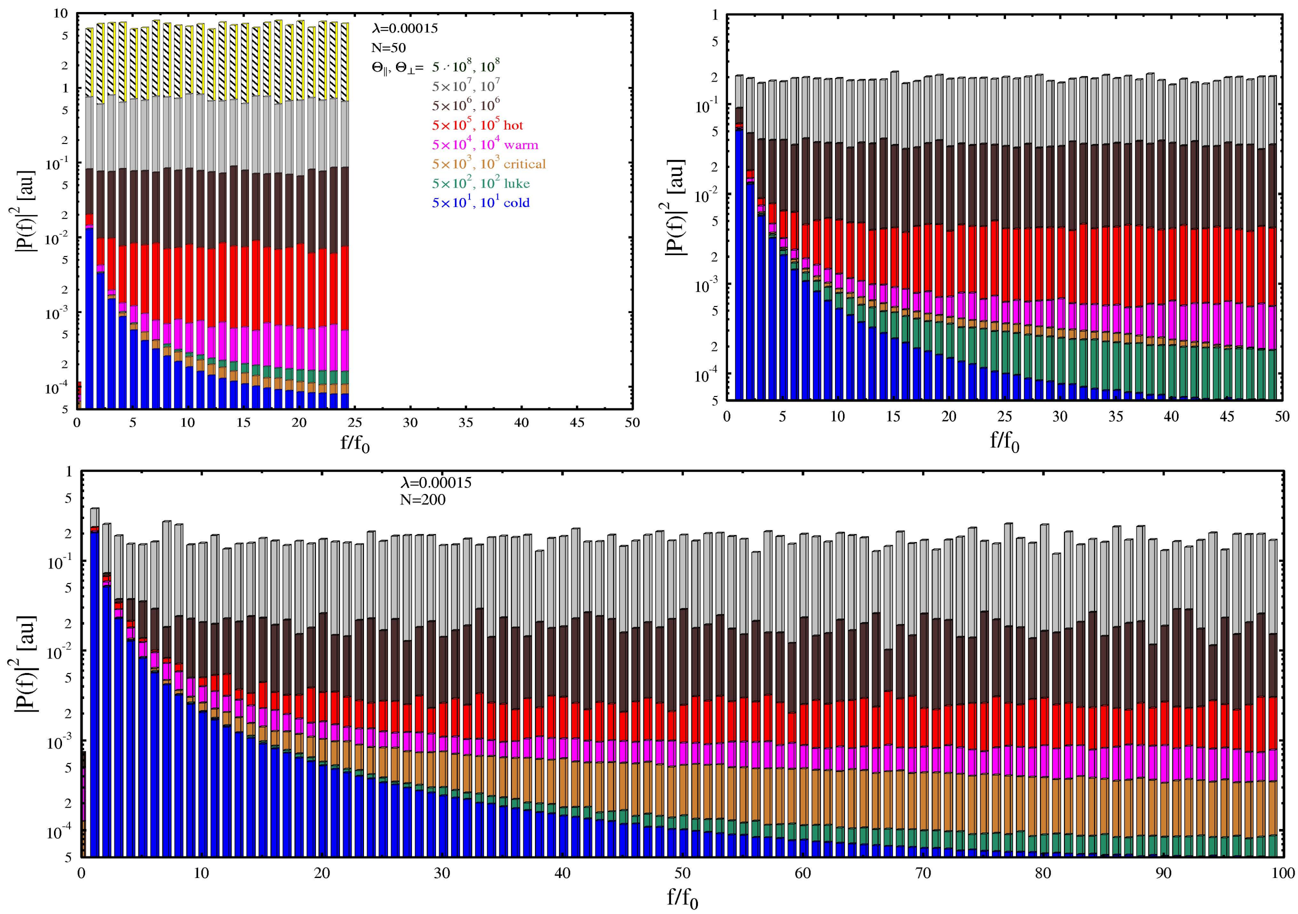

Figure 3: Calculated integrated Schottky spectra for the linar density $\lambda = 0.00015$ at various longitudinal and transverse temperatures and various number of particles N=50 (a: top left), N=100 (b: top right) and N=200 (c: bottom).

which, in turn converts to momentum spread by the relation $T_{||} = M(c\beta\delta p/p)^2/(8\log 2)$, where M is the mass and βc is the ion velocity. Note that it is independent of the measuring frequency (or harmonic number $n = f/f_0$), where f_0 is the revolution frequency and, certainly, independent of the number of ions N, see ref. [3].

On the other hand, if the ions are very cold and are ordered along a string, then

$$|P_{\text{string}}|^2 \propto \frac{N^2}{n^2}\,, \tag{2}$$

which is, certainly, independent of temperature. For N ions there exist N/2 different harmonic Fourier transforms which are continued in a mirrored fashion so that $|f_{\text{string}}|^2(n = 2N) = |f_{\text{string}}|^2(n = N)$ which gives additional effects in the Schottky spectra from low particle numbers not contained in eq. (2). For intermediate temperatures no analytical results are available.

In the following we study this important transition region by particle simulations of intermediate temperatures and reasonably low number of particles.

SIMULATIONS

For that purpose we use the code *RODS* (*R*eorganization *O*f *D*ynamical *S*ystems)[2] which was developed in the context of (1D or 3D) Coulomb crystals [12] and was successfully applied to derive the necessary conditions for the existence of Coulomb strings [4, 8] and to studies of the exchange of energy from transverse to longitudinal degrees of freedom [9] in an ion beam.

Since Coulomb systems scale nicely, the density can just be characterized by the dimensionless quantity $\lambda = a_{\text{WS}}/d$, where d is the average linear distance in the string region and $1.8a_{\text{WS}}$ would be the average 3D distance in an ion gas and $a_{\text{WS}} = (3q^2/2M\omega_\beta^2)^{1/3}$ being the Wigner-Seitz radius (q is the charge, M the mass and ω_β the betatron frequency). $\lambda = 0.00015$ is a typical value in the string region, see [4]. Longitudinal, $\Theta_{||}$, and transverse, $\Theta_\perp$, relative temperatures are the respective kinetic energies measured in units of $\epsilon_0 = q^2/d$.

The program places at random a number N of ions with linear density λ at transverse temperature $\Theta_\perp$and longitudinal temperature, in general $5\Theta_{||}$, in a cylinder of length

[2] The Windows® program can be downloaded from the website *http://www.gsi.de/~hasse*.

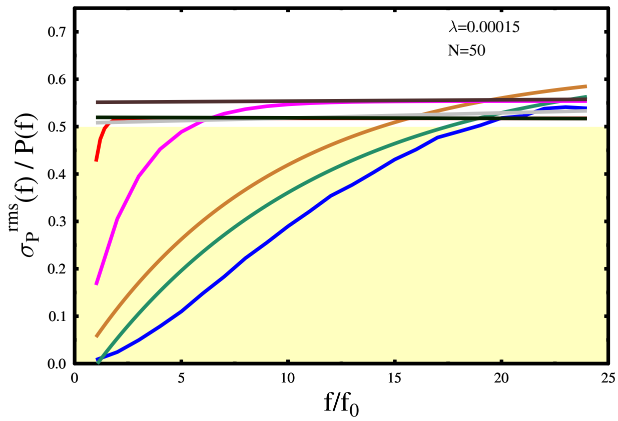

Figure 4: Rms width over average value of the integrated Schottky spectra of Fig. 3a for various temperatures (same color code).

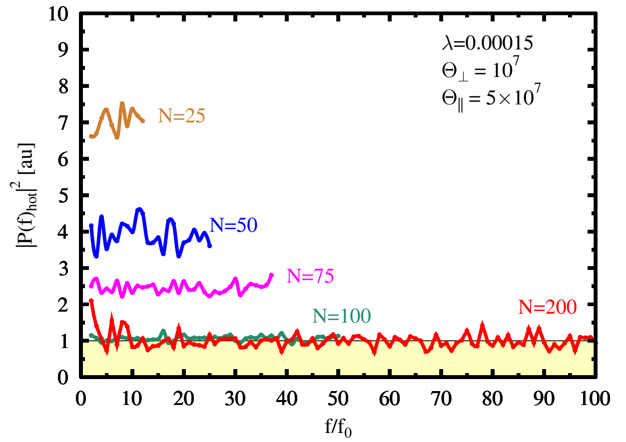

Figure 6: Particle number dependence of hot integrated Schottky signals vs. harmonic normalized to unity for $N \rightarrow \infty$.

Nd with periodic boundary conditions. Then the equations of motion are solved by molecular daynamics, hereby summing up all long-range Coulomb interactions by Ewald summation, see ref. [12]. After sufficient time when equilibrium has been reached the process is repeated a few hundred times in order to yield good statistics.

With this procedure was recorded whether or not the particles were repelled when approaching each other and the reflection probability as function of density was calculated. As shown in [4] exactly at a number of particles corresponding to the gap between the experimental red (hot) and cold (blue) points of Fig. 2 this calculated reflection probability jumps from zero to 100%.

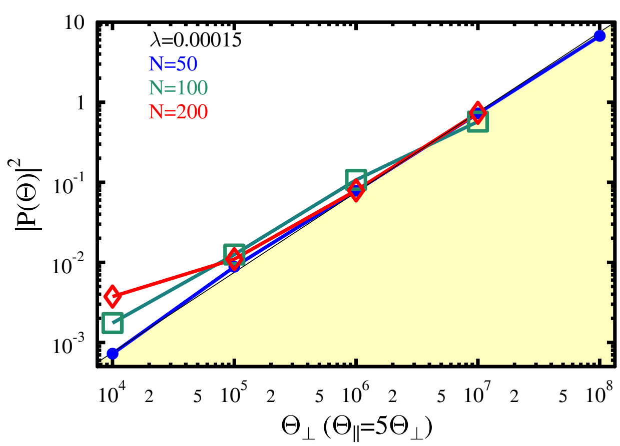

Figure 5: Temperature dependence of hot integrated Schottky signals for various particle numbers.

RESULTS

This procedure is now extended to also record the Schottky signals. They are Fourier transformed and integrated over one harmonic to yield the Schottky frequency spectra $|P(f)|$ of Fig. 3. Fig. 4, in addition, also shows the ratio of rms width to average value of the integrated spectra of Fig. 3 vs. harmonic number.

In Fig. 3 one observes the transition from cold (ordered) over critical to warm (random) systems which manifests itself in such a way that the $1/n^2$ behaviour of eq. (2) gradually goes over into a flat distribution of eq. (1) which is independent of frequency and particle number. The critical temperatures where this transition occurs correspond roughly to the critical temperatures derived in ref. [8]. The widths of Fig. 4 reflect the same behaviour: For hot systems they level off to a value around 0.5 independent of temperature as is the case for thermal equilibrium. For the cold string-like systems the width is almost zero for low temperatures, thus also indicating very small momentum spreads.

PARTICLE NUMBER DEPENDENCE

The linear temperature dependence for hot systems is extracted from Fig. 3 and presented in Fig. 5. The proportionality to temperature of eq. (1) (averaged over the plateau harmonics) is reproduced over the three hot decades in temperature. Only for cold systems a slight dependence on par-

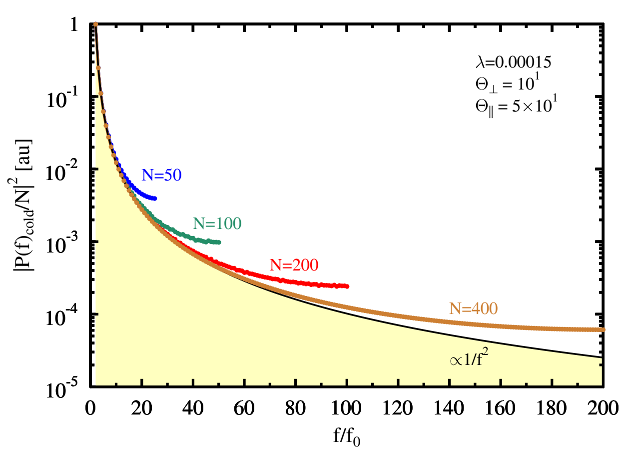

Figure 7: Particle number dependence of cold integrated Schottky signals (divided by N^2)vs. harmonic.

ticle number is found. Different behaviours are found for the frequency dependence of the hot and cold systems of of Figs. 6,7, respectively. Here for systems with only a few particles as is the case in the ultracold region, the Schottky signals depend strikingly on N. Most strikingly, only for more than 100 particles the trend with N^2 is reproduced for the hot systems. This might be used for the determination of the actual particle numbers in the rings.

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