IMPLEMENTATION OF SYNCHROTRON MOTION IN BARRIER BUCKETS IN THE BETACOOL PROGRAM*

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Abstract

The model of ion synchrotron motion in a stationary square wave barrier bucket was implemented into both main algorithms of the Betacool program: rms dynamics and Model Beam algorithm. In the frame of rms dynamics the calculation of the cooling and heating rates was modified in accordance with analytic expression for the ion phase trajectory in the longitudinal phase plane. In the Model Beam algorithm the generation of matched stationary particle array and simulation of the synchrotron motion were developed.

INTRODUCTION

Moving barrier RF bucket is an effective ion beam accumulation method used, for instance, in Fermilab's Recycler and proposed for NESR at FAIR project. A possible application of a stationary RF bucket is to compensate ionization energy loss in experiment with internal target. The ionization energy loss is the main physical effect limiting the experiment duration. The barrier bucket application permits to sufficiently decrease of required power of a cooling system when a high resolution in experiment is necessary. It is essentially true for dense internal target, for instance a pellet target. So an application of stationary RF bucket for WASA at COSY experiment can allow sufficiently decrease requirements for maximum electron current in proposed high voltage electron cooling system [1].

The mean energy loss can be compensated by usual sinusoidal RF system at relatively small voltage amplitude; however this leads to sufficient increase of intrabeam scattering (IBS) growth rates. Even at long length of the bunch the particle density in its central part increases significantly in comparison with a coasting beam. At a barrier RF bucket application the particle density inside the bucket is almost uniform. Therefore the IBS growth rates increase by a factor equal to ratio of the ring circumference to the bucket length only. This advantage of the barrier bucket is of great importance when the experiment requires high momentum resolution and, correspondingly, the ion beam momentum spread has to be as small as possible.

Recently a new program was developed for barrier RF bucket simulation for FAIR rings [2]. To compare predictions of different models and to estimate efficiency of the barrier bucket application in internal target experiments the new algorithms were implemented into Betacool program [3] also.

The general goal of the BETACOOL program is to simulate long term processes (in comparison with the ion revolution period) leading to the variation of the ion distribution function in six dimensional phase space. Therefore the Betacool is not a tracking code, and simulation of transverse and longitudinal ion motion is based on analytical expressions for the phase trajectories.

Evolution of the second order momenta of the ion distribution function is realized in so called "rms dynamics" algorithm based on assumption of Gaussian shape of the distribution. Here all the heating and cooling effects are characterized by rates of emittance variation or particle loss.

The investigation of the beam dynamics at arbitrary shape of the distribution is performed using multi particle simulation in the frame of the Model Beam algorithm. In this algorithm the ion beam is represented by an array of model particles. The heating and cooling processes involved into the simulations lead to a change of the particle momentum components and particle number.

Therefore implementation of the new model required corrections in algorithms for heating and cooling rate calculation, development of algorithms for generation of the model particle array matched with RF system and simulation of the model particle synchrotron motion. New tools for the data post processing and visualization of the results were developed also.

General behaviours of the synchrotron motion are determined by integrated RF pulse strength, and essential physics is independent on the exact shape of the barrier RF wave. Simplest analytical solution for the phase trajectory can be obtained at square wave barrier bucket; therefore this model was implemented into the program at the first step. In future we plan to develop the algorithms for synchrotron motion simulation at arbitrary RF shape and moving barrier bucket.

SINCHROTRON MOTION IN SQUARE WAVE BARRIER BUCKET

The RF voltage time dependence at square wave barrier bucket can be written as

$$V(t) = \begin{cases} sign(\eta)V_0 & if \quad -(T_2+T_1)/2 \le t \le -(T_2-T_1)/2 \\ -sign(\eta)V_0 & if \quad (T_2-T_1)/2 \le t \le (T_2+T_1)/2 \\ 0 & otherwise \end{cases},$$

where V_0 is the voltage height, T_1 is the pulse width, T_2 is the gap duration, η is the ring off-momentum factor.

The equations of the synchrotron motion of the ion at charge eZ written in the variables (s-s$_0$, $\delta = \Delta p / p$) are

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$$\begin{cases} \dfrac{d(s-s_0)}{dt} = |\eta|\beta c\delta_A \\ \dfrac{d\delta}{dt} = 0 \end{cases}$$

inside the gap of the RF voltage, and

$$\begin{cases} \dfrac{d(s-s_0)}{dt} = |\eta|\beta c\delta \\ \dfrac{d\delta}{dt} = -\dfrac{ZeV_0}{Cp_0} \end{cases}$$

when the particle crosses the cavity during the voltage pulse. Here δ_A is the amplitude of the momentum deviation; C is the ring circumference, p_0 – synchronous momentum.

From solution of the motion equations one can obtain main parameters of the bucket. The maximum momentum deviation (the barrier height) can be calculated from the following equation

$$\delta_{A,\max}^2 = \frac{2T_1}{T_0}\frac{ZeV_0}{\beta c p_0|\eta|}, \quad (1)$$

where T_0 is the revolution period. Inside the bucket the period of the synchrotron oscillations is equal to

$$T_s = \frac{2T_2}{|\eta|\delta_A} + \frac{4Cp_0\delta_A}{ZeV_0}, \quad (2)$$

and the trajectory length is given by

$$S_b = T_2\beta c + \frac{|\eta|\beta c C p_0 \delta_A^2}{ZeV_0}. \quad (3)$$

Rms momentum deviation σ_δ relates to the amplitude as follows

$$\sigma_\delta^2 = \frac{(2/3)|\eta|Cp_0\delta_A^4 + ZeV_0T_2\delta_A^2}{2|\eta|Cp_0\delta_A^2 + ZeV_0T_2}. \quad (4)$$

RATE CALCULATION

Modifications in the rms dynamics algorithm are related mainly to calculation of IBS heating and electron cooling rates. At simulation of an interaction with an internal target the mean energy loss is ignored and characteristic growth time of the momentum spread is calculated using expression for the energy loss fluctuations.

The IBS process in the first approximation can be simulated using formulae for coasting beam at substitution of the bucket length instead the ring circumference. The bucket length is calculated in the following steps. For given rms momentum spread the amplitude of the rms particle oscillations is to be found from the equation (4). Introducing

$$\xi^2 = \frac{ZeV_0T_2}{2|\eta|Cp_0} \quad (5)$$

one can obtain

$$\delta_A^2 = \frac{\sqrt{9(\xi^2-\sigma_\delta^2)^2 + 12\xi^2\sigma_\delta^2} - 3(\xi^2-\sigma_\delta^2)}{2}. \quad (6)$$

For this amplitude the bucket length is calculated in accordance with (3).

The cooling rate calculation is based on averaging of the friction force acting on the "rms ion" in the cooling section over phases of betatron and synchrotron oscillations [4]. To realize this procedure one needs to recalculate the ion co-ordinates (s-s_0, δ), obtained at the exit of the cooling section, into its rms momentum deviation. For the particle at given longitudinal co-ordinates (s-s_0,δ) the amplitude of oscillations is calculated in accordance with the phase trajectory equation. The result is obvious, when (s-s_0) lies in the gap between RF waves: $\delta_A = \delta$. Inside the wave the amplitude is equal to

$$\delta_A^2 = \delta^2 + \frac{2ZeV_0}{|\eta|\beta c C p_0}(s-s_0), \quad (6)$$

where (s-s_0) is measured from the beginning of the wave. The corresponding rms momentum deviation is given by:

$$\langle\delta^2\rangle = \frac{\delta_A^4 + 3\xi^2\delta_A^2}{3(\delta_A^2+\xi^2)}, \quad (7)$$

where ξ is determined by (5).

GENERATION OF MATCHED MODEL PARTICLE ARRAY

The beam dynamics simulation in the Model Beam algorithm is started with a model particle array with Gaussian distribution in all degrees of freedom.

Generation of initial ion distribution in the longitudinal phase plane, generation of new model particle at losses are based on procedures for generation of individual ion co-ordinates and matching of the ion with the ring lattice in the generation position. Generation of the ion longitudinal co-ordinates in the case of barrier bucket application is realized in the following steps.

1. Initially the momentum deviation is generated in accordance with Gaussian law at standard deviation equal to the rms momentum spread.
2. For the given momentum deviation the maximum momentum deviation is calculated in accordance with the formula (1).
3. The period of the ion synchrotron oscillations is calculated in accordance with (2).
4. At given maximum deviation the values of the momentum deviation and longitudinal co-ordinate are calculated using equations of the phase-space trajectory at the moment of time uniformly distributed from 0 to T_s.

Repetition of this procedure is resulted in a stationary particle array (without oscillations of the particle density during synchrotron motion) with Gaussian distribution over the momentum deviation.

SIMULATION OF LONGITUDINAL MOTION

The Model Beam algorithm realizes Monte-Carlo method for solution of Langevin equation based on an assumption, that the integration step over time is sufficiently longer than the decoherence period (a few millions of revolutions). In this case the phase advance of betatron and synchrotron oscillations during the integration step is an arbitrary number. In the transverse phase space the betatron motion is simulated using linear transformation maps.

In the case of a barrier bucket application the period of synchrotron oscillation is determined by the particle momentum deviation. At low momentum spread it can be of the order of a few seconds (see, for example, Table 1 below). Correspondingly, the decoherence time can be compared or even longer than the step of the integration over time. To take into account this peculiarity of the synchrotron motion two algorithms were developed: at random and regular phase advance during the integration step.

At the random phase advance the amplitude of the particle momentum deviation is calculated from its actual momentum and longitudinal co-ordinate using formulae (6) and (7). Thereafter the phase of synchrotron oscillations is generated uniformly between 0 and 2π and new co-ordinate and momentum are calculated as functions of the phase.

At regular phase advance the phase of the synchrotron oscillations of the particle is calculated from its co-ordinate and momentum deviation. The phase advance for each particle is calculated from the period of its synchrotron oscillations as $\Delta\varphi = 2\pi \cdot \Delta t / T_s$, where Δt is the integration step. New particle coordinates are obtained from the phase trajectory equation.

EXAMPLE OF HESR SIMULATION

Possibilities of the new algorithms can be illustrated by example of the antiproton cooling simulation in High Energy Storage Ring of FAIR project (GSI).

One of the main goals of electron cooling application at the HESR is to reach equilibrium relative momentum spread at the level of 10^{-5} in the energy range from 2 to 8 GeV. Challenge of the project is to compensate a beam heating due to interaction with an internal hydrogen pellet target at the thickness of about $4\cdot10^{15}$ atoms/cm^2, which is required to obtain the design luminosity. The electron cooling system was designed in TSL for the cooling section length of 20 m and electron current up to 1 A. In [5] it is shown that this system permits to obtain equilibrium 90% relative momentum spread of the antiproton beam at the level of $4\cdot10^{-5}$.

An additional benefit in the beam quality can be provided by compensation of mean ionization energy loss in the target using a barrier bucket system. To prove this fact the 8.9 GeV/c antiproton beam dynamics was simulated at barrier bucket parameters listed in the Table 1. Number of the antiprotons was equal to 10^{10} and the cooling process was simulated with account of IBS.

Table 1: Parameters of barrier RF bucket used in the simulations.

Pulse width, T_1/T_0		0.1
Gap duration, T_2/T_0		0.7
Voltage amplitude	V	20
The barrier height, $\delta_{A,max}$		$1.14\cdot10^{-4}$
Rms bucket length at $\delta = 2\cdot10^{-5}$	m	405.4
Synchrotron period at $\delta = 2\cdot10^{-5}$	s	3.93

The transverse beam emittance was stabilized at the level of $5\cdot10^{-8}$ π·m·rad by the electron beam tilt in respect to the antiproton orbit. At 8 m of the beta-function in the target position it provides good overlap with the pellet target. Ionization energy loss in the target was simulated in accordance with Urban model [6] taking into account a probability of large fluctuations. The transverse particle momentum variation was calculated in accordance with plural scattering model.

Initial value of the rms relative momentum spread was chosen to $2\cdot10^{-5}$. First 20 seconds of cooling the momentum spread decreases to equilibrium and thereafter stays a constant during long period of time (Fig. 1). (The “sigma” at horizontal axis in the Figures 1, 2 corresponds to relative momentum deviation of $2\cdot10^{-5}$.)

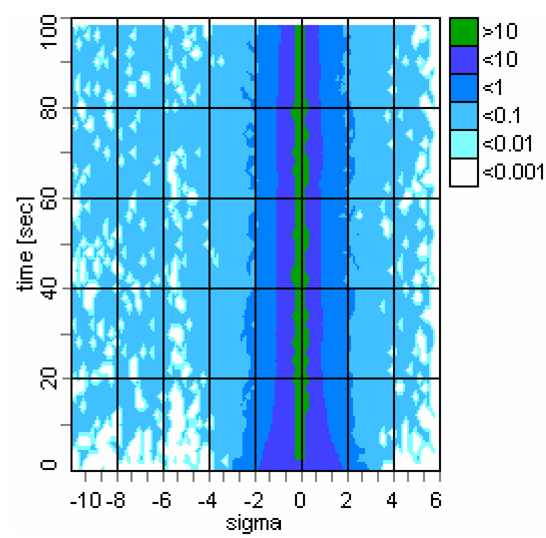

Figure 1: Beam momentum spread evolution during first 100 seconds of the electron cooling. The different colors indicate the relative particle number in logarithmic scale.

Inside the bucket the particle distribution over the momentum deviation (Fig. 2) is close to Gaussian at the rms width of about 10^{-5}. Long low energy tail (below momentum deviation of $-5\sigma_0$, which corresponds to the bucket height) is formed by the particles lost from the bucket. Intensity of the tail is below 10^{-3} and, in principle, it can be suppressed by increase of the voltage height.

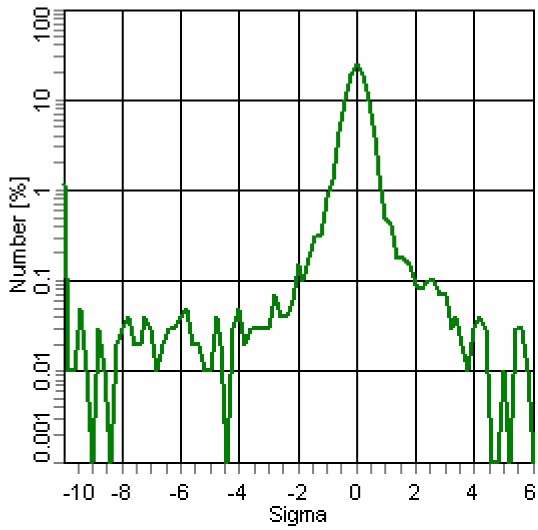

Figure 2: Ion distribution over the momentum deviation after 100 seconds of cooling.

From the particle distribution in the longitudinal phase plane presented in the Fig. 3 one can see that the particle density is practically uniform along the bucket.

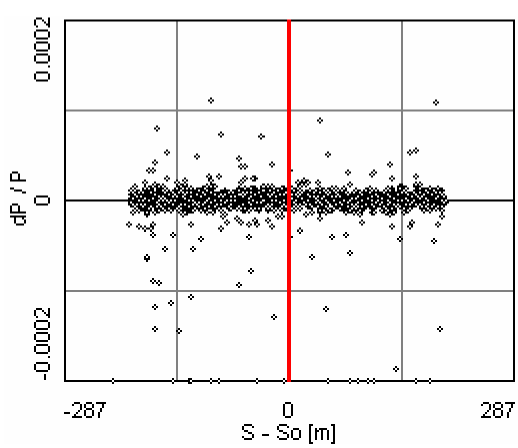

Figure 3: Particle distribution in the longitudinal phase plane after 100 seconds of cooling.

Comparison between cooling in the case of the coasting beam and the barrier bucket application shows that the barrier bucket system permits to reach the same value of the momentum spread at electron current value less by about 1.5 times. The burrier bucket system increases the bunching factor insufficiently, and permits to provide acceptable peak to mean luminosity ratio.

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