

# ELECTRON COOLING SIMULATION FOR ARBITRARY DISTRIBUTION OF ELECTRONS

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## Abstract

Typically, several approximations are being used in simulation of electron cooling process, for example, density distribution of electrons is calculated using an analytical expression and distribution in the velocity space is assumed to be Maxwellian in all degrees of freedom. However, in many applications, accurate description of the cooling process based on realistic distribution of electrons is very useful. This is especially true for a high-energy electron cooling system which requires bunched electron beam produced by an Energy Recovery Linac (ERL). Such systems are proposed, for instance, for RHIC and electron – ion collider. To address unique features of the RHIC-II cooler, new algorithms were introduced in BETACOOOL code which allow us to take into account local properties of electron distribution as well as calculate friction force for an arbitrary velocity distribution. Here, we describe these new numerical models. Results based on these numerical models are compared with typical approximations using electron distribution produced by simulations of electron bunch through ERL of RHIC-II cooler.

## INTRODUCTION

A traditional electron cooling system employed at low-energy coolers is based on a uniform electron beam immersed in a longitudinal magnetic field of a solenoid. In this case the action of electron cooling on the ion dynamics inside a storage ring can be described using a few standard simplifications:

1. Angular deviation of the longitudinal magnetic field line is sufficiently less than the ion beam angular spread.
2. Ion transverse displacement inside the cooling section is small compared to the electron beam radius.
3. Ion beam temperature is substantially larger than electron one and ion diffusion in the electron beam can be neglected.
4. Electron beam has a cross-section with a round shape and uniform density distribution in the radial direction.

Under these assumptions and using analytic asymptotic representation of the friction force, the formulae for characteristic times of emittance and momentum spread decrease for electron cooling were obtained (see, for example, Ref. [1]). This model was used in several programs dedicated to electron cooling simulation, for example, in the first version of the BETACOOOL [2]. An uncertainty in predictions of the cooling rate based on such an estimate is typically a factor of two or three, which is acceptable for most designs of a traditional

cooling system. However, this model can not cover all possible versions of the electron cooling system design, and its accuracy is insufficient for a design of high energy electron coolers.

Recently, modifications of the usual configuration of the low energy electron cooling system were proposed. To avoid instability of the ion beam related to an extremely large density of the cooled beam it was proposed to use so called “hollow” electron beam – the beam with small density in the central part. The “hollow” electron beam is efficient for ion beam storage using cooling-stacking procedure. The low electron density in the stack region avoids overcooling of the stack and decreases (for heavy ions) recombination in the cooling section. Therefore, a few electron cooling systems with hollow electron beam were recently constructed.

Extension of the electron cooling method to the region of electron energy of a few MeV, which was successfully realized at Recycler in Fermilab and proposed for HESR (GSI, Darmstadt) and COSY (FZJ, Juelich), led to some changes of the electron beam properties. Accurate matching of an intensive electron beam with transport line in the cooling section is a complicated task in this energy range. The electron beam mismatch leads to fast decrease of the electron beam quality in the radial direction from the central part to the beam edge that can significantly affect the ion distribution under cooling.

Further increase of the electron energy is related with an RF acceleration of the electrons. In this case one can have Gaussian distribution of the electrons in radial plane and, if the electron bunch is shorter than the ion one, in longitudinal direction also. Simulation of the cooling process in this case requires modification both of the electron beam and the physical model.

Successful operation of the Recycler electron cooling system demonstrated good cooling efficiency in the case when the longitudinal magnetic field is used for electron beam transport only and does not influence the friction force significantly. This indicates that a strong magnetic field is unavoidable only in the case when a deep cooling of the ion beam is necessary (as, for example, for HESR in high resolution mode of operation). When the electron cooling is used for compensation of heating effects and stabilization of the ion beam phase volume at relatively large value, the non-magnetized cooling can be competitive as well. For example, although extensive studies of the magnetized cooling approach for RHIC showed that it is feasible [3] and would provide required luminosities for the RHIC-II, the baseline of the project was recently changed to the non-magnetized one. Application of electron cooling using the non-magnetized electron beam significantly simplifies the RHIC-II cooler

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design. Generation and acceleration of the electron bunch without longitudinal magnetic field allows low values of the emittance for the electron beam in the cooling section. Therefore, the cooling efficiency is comparable with the case of strong magnetic field application.

Other peculiarity of high-energy cooling system is a big length of the cooling section – up to about 20 - 100 m. To obtain very high accuracy of the magnetic field is difficult technical task, and cost of the cooling system utilizing the magnetized cooling will strongly depend on the required level of the accuracy. Therefore, before designing of the cooling section solenoid, one needs to investigate influence of the magnetic field line curvature on the cooling process.

Most of the effects can be taken into account by numerical solution of the ion motion equations inside the cooling section. For such simulations, an electron beam model should calculate the friction force components as a function of the ion velocity and the ion position inside the electron beam. The required local parameters of the electron beam can be calculated using some analytical model describing electron density and velocity spread as functions of co-ordinates, or, in a general case, from electron distribution imported from output file of a program dedicated to the electron dynamics simulations. In this report, we describe a few analytical models of the electron beam realized in the BETACOOOL program and the algorithm of the friction force calculation from an electron array calculated with PARMELA code [4] for RHIC electron cooling system.

## FRICION FORCE MODELS

In the particle rest frame (PRF) the friction force acting on the ion with a charge number  $Z$  passing through an electron beam of density  $n_e$  can be evaluated in the absence of magnetic field by numerical integration of the following formula [5]:

$$\vec{F} = -\frac{4\pi n_e e^4 Z^2}{m} \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{|\vec{V} - \vec{v}_e|}{|\vec{V} - \vec{v}_e|^3} f(v_e) d^3 v_e \quad (1)$$

where  $e$  and  $m$  are the electron charge and mass,  $V$  and  $v_e$  are the ion and electron velocities, respectively. The Coulomb logarithm is kept under the integral because the minimal impact parameter depends on electron velocity:

$$\rho_{\min} = \frac{Ze^2}{m} \frac{1}{|\vec{V} - \vec{v}_e|^2}.$$

At a given value of the ion velocity the maximum impact parameter  $\rho_{\max}$  is constant and determined by either the dynamic shielding radius or by the time of flight of an ion through the cooling section. Inside the ion vicinity of radius equals to the maximum impact parameter the electrons are distributed over velocities in accordance with the function  $f(v_e)$ . In a practical case, the maximum impact parameter is sufficiently less than the electron beam radius and all the friction force formulae presume that the electron density inside the vicinity of the ion is uniform.

The formula (1) is valid also in the case when a magnetic field of a small value is used for the electron beam focusing in the cooling section. The magnetic field does not affect the force value when a radius of the electron gyration  $\rho_{\perp}$  is larger than the maximum impact parameter. However, even in this case the magnetic field modifies the electron distribution over velocities due to coupling between transverse planes. For a standard assumption that the thermal velocity distribution is Gaussian, the distribution function can be presented as

$$f(v_e) = \left(\frac{1}{2\pi}\right)^{3/2} \frac{1}{\Delta_{\perp}^2 \Delta_{\parallel}} \exp\left(-\frac{v_{\perp}^2}{2\Delta_{\perp}^2} - \frac{v_{\parallel}^2}{2\Delta_{\parallel}^2}\right), \quad (2)$$

where  $\Delta_{\perp}$  and  $\Delta_{\parallel}$  are the electron rms velocity spreads in the transverse and longitudinal direction correspondingly. Therefore, the friction force can be presented as a sum of radial and longitudinal components. In the absence of longitudinal magnetic field in the cooling section the electron motion in transverse planes is uncoupled. The electron beam can have different thermal velocity in the horizontal and vertical planes. In this case the force is a vector with all three different components.

For large magnetic field (when  $\rho_{\perp} \ll \rho_{\max}$ ) the integrand in Eq. (1) is modified. In the framework of binary collision model the transverse and longitudinal components of the friction force for strong magnetization can be approximated by the formulae [6]:

$$F_{\parallel} = -\frac{4\pi Z^2 e^4 n_e L_M}{m} \int \frac{V_{\perp}^2 (V_{\parallel} - v_e)}{(V_{\perp}^2 + (V_{\parallel} - v_e)^2)^{5/2}} f(v_{\parallel}) dv_{\parallel}$$

$$F_{\perp} = -\frac{2\pi Z^2 e^4 n_e L_M}{m} \int \frac{V_{\perp} (V_{\perp}^2 - 2(V_{\parallel} - v_e)^2)}{(V_{\perp}^2 + (V_{\parallel} - v_e)^2)^{5/2}} f(v_{\parallel}) dv_{\parallel} \quad (3)$$

where the Coulomb logarithm is equal to  $L_M = \ln\left(\frac{\rho_{\max}}{\rho_{\perp}}\right)$ .

In these formulae it is assumed that the transverse electron motion is completely suppressed by the magnetic field. The binary collision model for finite value of the magnetic field was developed by Erlangen University group [7]. The obtained formulae contain dependence on both the longitudinal and transverse electron velocities.

To calculate the friction force components using analytical formulae, the electron beam model have to calculate the following local (in a given ion position inside electron beam) parameters:

- electron density,
- electron longitudinal velocity spread  $\Delta_{\parallel}$ ,
- electron transverse velocity spread  $\Delta_{\perp}$  to calculate the force in presence of a magnetic field,
- electron horizontal and vertical velocity spreads in absence of the magnetic field.

When the electron beam is presented as an array of particles, the distribution function over velocities is given as a series of  $\delta$ - functions:

$$f(v) = \frac{1}{N_{loc}} \sum_{j=1}^{N_{loc}} \delta(\vec{v} - \vec{v}_j), \quad (4)$$

where  $N_{loc}$  - number of the electrons in small vicinity of the ion position.

## ANALYTICAL MODELS OF ELECTRON BEAM

A few analytical models for electron beam representation are now available in BETACOOOL: uniform cylinder, Gaussian cylinder, Gaussian bunch, “hollow” beam and electron beam with parabolic density distribution. The cooler model takes into account variation of the electron beam position and angular deviation along the cooling section (it can be caused, for example, by magnetic field errors in the cooling section). For this purpose, the co-ordinates of the electron beam trajectory inside the cooling section can be read from an additional external file, and the ion motion equations are solved numerically inside the cooler. The friction force can be calculated in accordance with one of the analytical models or using results of numerical calculations imported from an external file.

As an illustration of the principle, we briefly describe the electron beam model for the parabolic density shape developed for simulation of antiproton cooling at the Recycler. Input parameters of the model are the beam radius  $a$  and the beam current  $I_e$ . The electron density  $n$  as a function of radial coordinate  $r$  is calculated in accordance with

$$n(r) = n_0 \left( 1 - \left( \frac{r}{a} \right)^2 \right), \quad (5)$$

when  $r < a$  and equal to zero in opposite case. The central density  $n_0$  is calculated from the beam current as

$$n_0 = \frac{3I_e}{\pi a^2 v e}, \quad (6)$$

where  $v$  is the electron velocity.

The effect of the beam mismatch in the cooling section is taken into account as a linear increase of the electron transverse velocity spread with the radial position

$$\Delta_{\perp} = \Delta_{\perp,0} + \frac{d\Delta_{\perp}}{dr} r, \quad (7)$$

where  $\Delta_{\perp,0}$  - is the velocity spread at the beam axis, and the velocity gradient  $d\Delta_{\perp}/dr$  is an input parameter. The longitudinal velocity spread is assumed to be independent on the radial co-ordinate.

## ELECTRON ARRAY

The distribution of electrons can be generated with an external code. For example, in the case of RHIC-II simulations such electron distribution is an output of PARMELA code [6], which is used to simulate electron beam transport to the cooling section. The distribution of electrons is then read into BETACOOOL and is referred to as “electron array”.

For the friction force calculation, the local model uses local parameters within electron array calculated as a function of the ion coordinates. The program first finds local number of electrons  $N_{loc}$  which have minimum

distance to the ion position (the value of  $N_{loc}$  is an input parameter). For  $N_{loc}$  found, the program calculates mean and root mean square parameters for all the coordinates and velocity components, which are used to calculate the local density of the electrons.

The local density and rms parameters found can be used in calculation of the friction force with analytic formulas and assumption of Gaussian velocity distribution. Such a model is called here “local-Gaussian”. In most cases, this approach is sufficient. It allows to compare cooling process based on local characteristics of electron distribution with the one expected based on the friction force calculation using projected global rms parameters of the whole distribution.

In another local model, an assumption that local velocity distribution is Gaussian is not used. The velocity components are calculated directly which allows us to study friction force for an arbitrary velocity distribution. Here, we refer to this model as “local-arbitrary”. The distribution function of the local electrons is given by formula (4). For non-magnetized cooling it leads to the following expression for the friction force components

$$F_{\alpha} = \frac{4\pi m_e Z^2 e^4}{m} \times \frac{1}{N_{loc}} \sum_{j=1}^{N_{loc}} \frac{(V_{\alpha} - v_{j,\alpha}) L_{C,j}}{\left( \sqrt{(V_x - v_{j,x})^2 + (V_y - v_{j,y})^2 + (V_z - v_{j,z})^2} \right)^3} \quad (8)$$

where  $V_{\alpha}$  are the components of ion velocity in the particle rest frame,  $v_{j,\alpha}$  - the velocity components of  $j$ -th electron ( $\alpha = x, y, z$ ). The minimum impact parameter in the Coulomb logarithm  $L_{C,j}$  is calculated via velocity of  $j$ -th electron.

## EXAMPLE OF RHIC-II SIMULATION

The high-energy electron cooling system for RHIC-II requires bunched electron beam. Electron bunches are produced by an Energy Recovery Linac (ERL), and cooling is planned without longitudinal magnetic field.

Simulations of electron beam dynamics were performed including compensation of space-charge defocusing in the cooling section. Electron distributions used in these studies were obtained by starting with the uniform cylinder (beer-can) at the cathode and tracking it through the ERL.

In general, electron beam dynamics simulations are aimed at minimization of global projected rms emittance and momentum spread while local rms velocity spread could have smaller values which would enhance cooling.

Another feature of electron distribution resulting from RF acceleration is that longitudinal rms velocity spread of electrons is not Gaussian, as shown in Fig. 1.

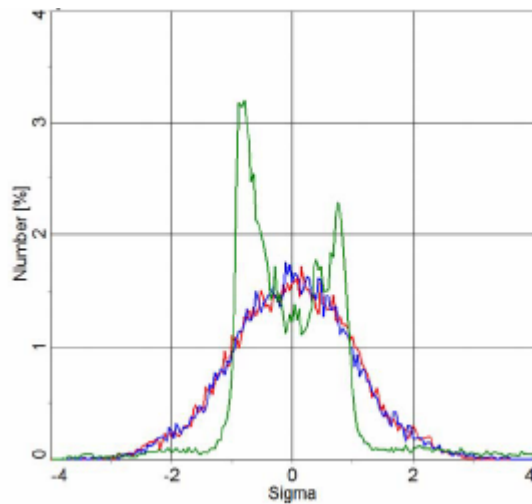


Figure 1: Histogram of velocity distribution of electrons at the start of the cooling section. Red and blue – horizontal and vertical; green – longitudinal.

As a result, different cooling dynamics can be observed in simulations when assumption of Gaussian velocity distribution is used instead of calculation using Eq. (8). This is shown in Figs. 2 and 3, where only electron cooling was included in simulations, with all other effects turned off.

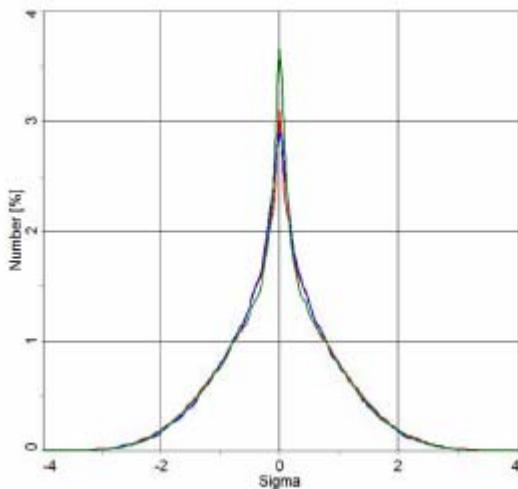


Figure 2: Horizontal (red), vertical (blue) and longitudinal (green) ion beam profiles based on assumption of Gaussian velocity distribution in all three planes.

During simulations electron beam was kept in a fixed position with respect to the center of the ion bunch. In Fig. 3, one can see that the hollow longitudinal velocity distribution (Fig. 1) prevents collapse of the distribution core which happens for Gaussian velocity distribution in Fig. 2, otherwise.

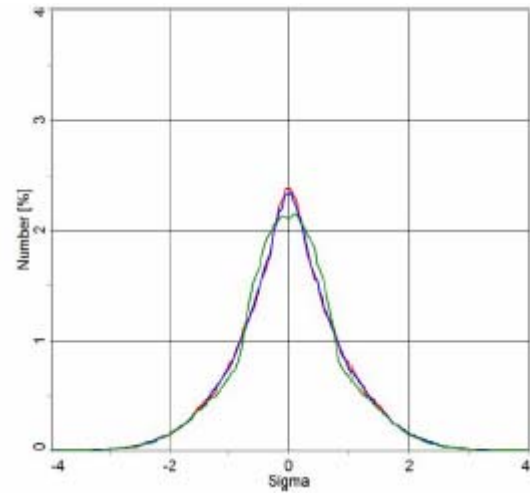


Figure 3: Horizontal (red), vertical (blue) and longitudinal (green) ion beam profiles after 5 minutes of cooling based on velocity distribution shown in Fig. 1.

The effect of hollow velocity distribution is less pronounced when intrabeam scattering is included in simulations, which prevents core collapse as well. However, the local model shows more effective cooling since most of the electrons in the core of the distribution have small angular spread [8].

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