

## DESIGN OF PASSIVE MAGNETIC CHANNELS

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### ABSTRACT

Passive magnetic channels are extensively used as magnetic shield or focusing devices for the extraction of beams from cyclotrons. The field properties in the beam aperture can be analyzed using a multipole expansion of the field in a two dimensional approximation, owing to the large length over aperture ratio.

Analytical formulas, assuming uniform saturation of the iron, are given to calculate the multipole coefficients. The basic configuration for dipole e/o quadrupole are evaluated and compared.

### 1. INTRODUCTION

Passive magnetic channels, providing partial shielding of the main field and radial focusing, have been introduced in the 60's for the beam extraction from sin-crocyclotrons<sup>1)</sup> and have been subsequently used also in many cyclotrons.

The most recent application of these devices is in the extraction system of superconducting cyclotrons like the K500, K1200 of MSU and of LNS/Milan.<sup>2)</sup> In these machines a considerable number of passive magnetic channels (up to 10) are spaced all the way along the beam extraction path. They provide partial shielding of the main field (up to 3 kG) and radial focusing (gradients up to 4 kG/cm).

The geometry of these channels (called also focusing bars) is similar to the one given in Fig. 3 and indicated as type I. Owing to the large value of the main field (in the range 15-50 kG) of superconducting cyclotrons uniform magnetization of the iron bars can be safely assumed.

Analytical formulas for the computation of the magnetic field produced by rectangular block of saturated iron are well known.<sup>3)</sup> An extensive analysis of the field properties of focusing bars is given in ref.<sup>4)</sup> in terms of the multipole coefficients obtained by numerical evaluation from the computed fields.

An alternative approach based on a direct analytical evaluation of the multipole coefficients is presented in the following.

### 2. MULTIPOLE GENERATION WITH SATURATED IRON BARS

K. Halbach has developed a technique for the generation of multipoles using permanent magnets.<sup>5)</sup> The multipole is made using equal blocks (rectangular, trapezoidal, ring sector) spaced around a circle and with a changing direction of the magnetization. The multipole coefficients are evaluated for a standard block placed on the X axis (see Fig.1) and for a given magnetization orientation.

The multipoles coefficients for all the other blocks are then obtained by the reference block considering the rotation of the block (angle  $\phi$  of Fig. 1) and the orientation of the magnetization.

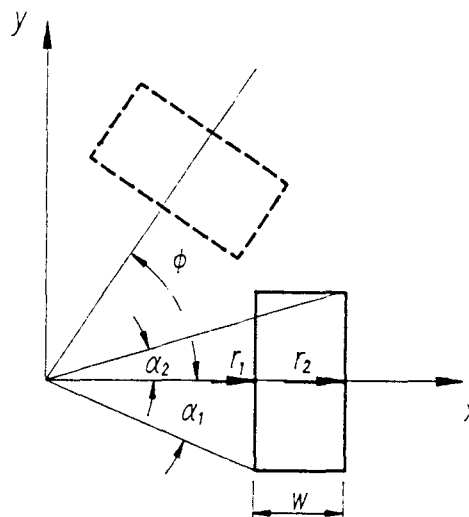


Fig. 1. Coordinate system and parameters of a standard block.

The calculation is two dimensional, i.e. the multipole is assumed to have infinite length in the direction of the beam path.

In the case of passive channels for cyclotrons the length over aperture ratio (typical values are 10 cm and 1 cm aperture diameter) justify the two dimensional approach; example are given in ref.<sup>4)</sup>

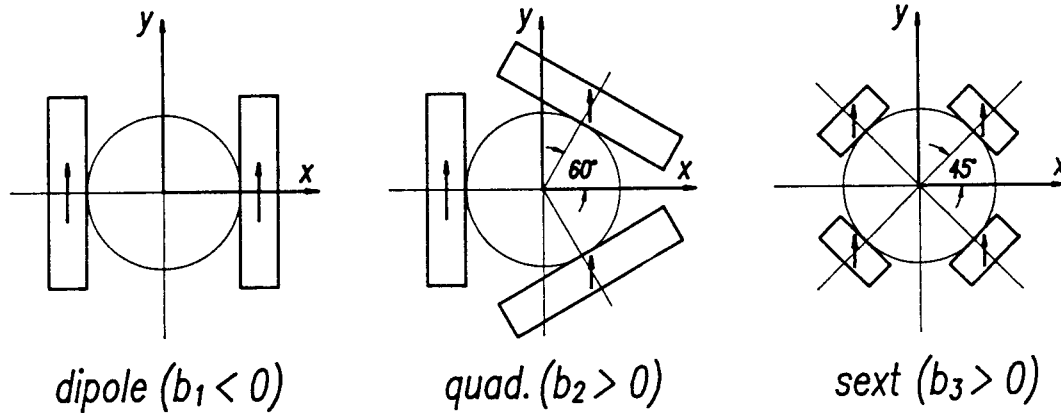


Fig. 2. Geometries of the bars for the generation of dipole, quadrupole and sextupole field.

Assuming midplane symmetry ( $y=0$ ), uniform saturation in the positive  $Y$  direction and the rectangular type block of Fig. 1, the formulas given by Halbach<sup>5</sup> can be simplified as

$$B_y(x, y=0) = \sum_n b_n \left( \frac{x}{\rho} \right)^{(n-1)} \quad (1)$$

where the field expansion is valid inside a circle of radius  $\rho$  not enclosing iron. (i.e.  $\rho \leq r_1$ ). The multipole coefficients  $b_n$  are given by ( $\rho = r_1$ ):

$$b_1 = \frac{B_s}{\pi} (\alpha_2 - \alpha_1) \quad (2)$$

$$b_n = -\frac{B_s}{\pi} \frac{1}{n-1} (\cos^{n-1} \alpha_1 \sin(n-1)\alpha_1 - \left( \frac{r_1}{r_2} \cos \alpha_2 \right)^{n-1} \sin(n-1)\alpha_2) \quad (3)$$

with  $B_s$  the saturated field in the positive  $Y$  direction. Rotation of an angle  $\phi$  of the reference block gives new values of the multipole coefficients

$$b_n(\phi) = b_n(\phi=0) \cos(n+1)\phi \quad (4)$$

The geometry of the basic multipoles, dipole, quadrupole, sextupole, are presented in Fig. 2 with the appropriate sign of the multipoles to obtain shielding ( $b_1 < 0$ ), radial focusing ( $b_2 > 0$ ) and to correct the sextupole component of the main field.

The general properties of the multipole using saturated iron are the following:

- to generate multipole of order  $N$  are required  $(N+1)$  bars
- maximum efficiency for given bar thickness is obtained for  $\alpha = \pi/(2(N+1))$  where  $\alpha = (\alpha_1 + \alpha_2)/2$ . These values are exact for block of trapezoidal shape and are only a good approximation for rectangular block

- only multipole coefficients of order  $N + k(N+1)$ ,  $k = 0, 1, 2 \dots$  are generated with decreasing values. Therefore:

$$\text{dipole } \alpha = \pi/4 \quad b_1, b_3, b_5, \dots$$

$$\text{quad } \alpha = \pi/6 \quad b_2, b_5, b_8, \dots$$

$$\text{sext } \alpha = \pi/8 \quad b_3, b_7, b_{11}, \dots$$

- The multipole coefficient of order  $N + (N+1)$  can be eliminated using  $\alpha = \pi/(2N+1)$

$$\text{dipole } b_3 = 0 \quad \alpha = \pi/3$$

$$\text{quad } b_5 = 0 \quad \alpha = \pi/5$$

$$\text{sext } b_7 = 0 \quad \alpha = \pi/7$$

with a modest reduction of efficiency.

- change of sign of the multipole coefficient of order  $N$  can be obtained rotating the all system by the angle of  $\pi/(N+1)$

### 3. FOCUSING BARS

As already mentioned passive magnetic channel providing shield ( $b_1 < 0$ ) and radial focusing ( $b_2 > 0$ ) are normally indicated as focusing bars.

The basic geometry is similar to the type I given in Fig. 3, although channels made with bars of circular cross section have been employed.

The desirable feature of these channel should be, beside providing dipole and quadrupole components, to eliminate or minimize the other main components, i.e. sextupole  $b_3$  and possibly octupole  $b_4$ .

Analytical evaluation of the multipole coefficients can be made, with the formulas presented in Section 2, considering the two bars on the right side (positive  $X$  axis) as the sum of a bar of height  $H$  with positive magnetization (positive  $Y$  axis) and a bar of height  $h$  with negative magnetization.

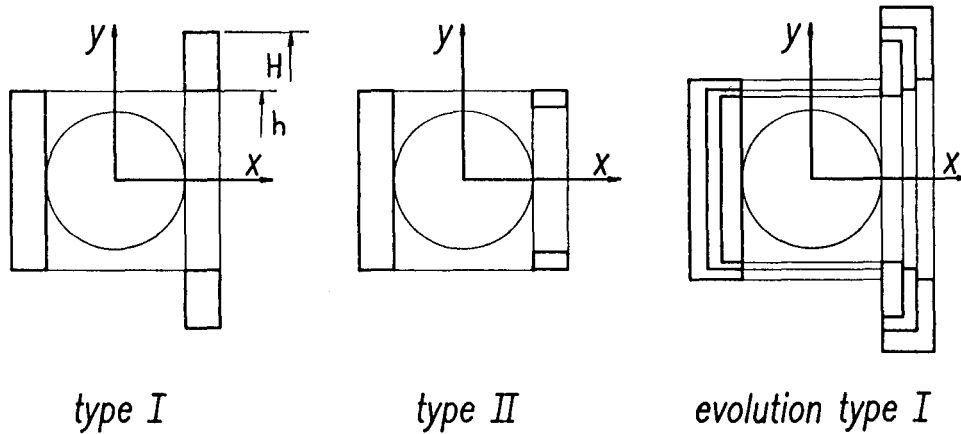


Fig. 3. Geometries of the focusing bars producing dipole (shield) and quadrupole fields.

Imposing the condition  $b_3 = 0$ ,  $b_4 = 0$  the solution can be numerically found and generally implies an height of the left bar not equal to  $h$ .

Table 1. Multipole coefficients (kG) for various magnetic channels

n	type I	type II	dipole	quad
1	-0.845	-0.944	-1.691	0.0
2	1.228	0.848	0.0	3.174
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.028	0.080	0.057	0.0
6	-0.192	-0.119	0.0	0.0
7	-0.005	-0.043	-0.017	0.0
8	0.018	0.002	0.0	-0.387

Three specific cases can however be evaluated quite easily;

- setting  $H = \infty$  a pure quadrupole field ( $b_2 > 0$ ,  $b_1 = b_3 = 0$ ) is obtained. The height  $h$  can be chosen to have  $b_4 = 0$ . This corresponds to  $\alpha = 45^\circ$  for the left bar; the value is exact for block of trapezoidal shape and only a good approximation for the rectangular shape.
- for the type I configuration the sextupole component  $b_3$  depend only on the value of  $H$ ; therefore  $b_3 = 0$  using  $\alpha = 60^\circ$ . The value of  $h$  can be chosen to obtain  $b_4 = 0$ ; this corresponds to  $\alpha = 47^\circ$
- for the type II configuration the octupole coefficient  $b_4$  depends only on the value of  $h$ ; selection of  $\alpha = 45^\circ$  gives  $b_4 = 0$ . The value of  $H$  can be chosen to have  $b_3 = 0$ , i.e.  $\alpha = 50^\circ$ .

Type I and type II focusing bars represent practically the extremes of the useful range for this type of configu-

ration. Type I is limited by the total height  $H$  which increase with the bar thickness as shown in Fig. 3. Type II is instead limited by the small vertical dimension  $(H - h)$  of the bars on the right side for reasonable values of bar thickness.

The multipole coefficients up to  $b_8$  for type I and type II focusing bars are given in Table 1 for a selected case : aperture  $r_1 = 0.6$  cm, bar thickness 0.2 cm and saturation field  $B_s = 21.4$  kG.

The main difference between type I and type II geometry is the value of the quadrupole component which is almost 50% higher for type I case. The dipole component is instead 10% higher for the type II. For both cases the main imperfection multipole is the dodecapole component  $b_8$ .

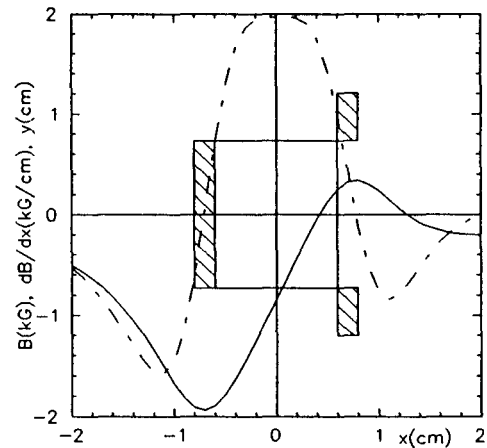


Fig. 4. Fields and gradients for type I geometry. Solid line field, dashed line gradient.

The midplane fields and gradients for the two type of focusing bars are plotted in Figs. 4,5. Uniformity of the gradients is within 10% or better inside a region covering

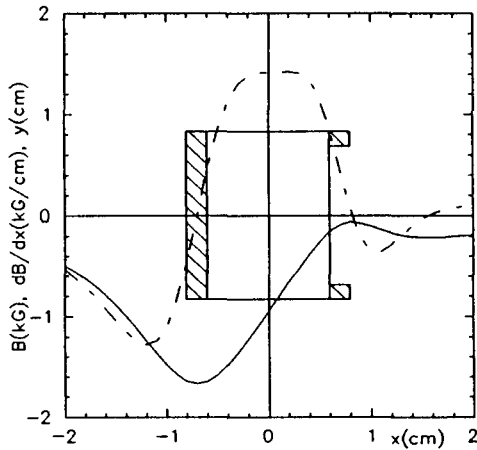


Fig. 5. Fields and gradients for type II geometry. Solid line field, dashed line gradient.

2/3 of the aperture. The uniformity normally increases (up to 5%) with increasing bar thickness.

The possible ranges of  $b_1$ ,  $b_2$ , varying the bar thickness, for type I and type II configuration are presented in Fig. 6 for the reference case; maximum bar thickness is 1 cm.

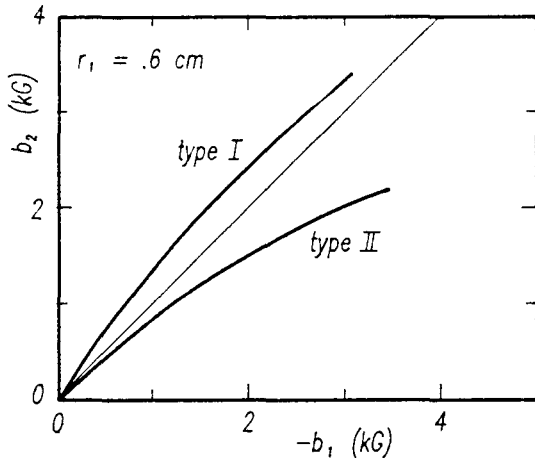


Fig. 6. Dipole ( $b_1$ ) and quadrupole ( $b_2$ ) multipole coefficients produced by type I and II focusing bars, varying the bar thickness.

#### 4. COMPARISON OF THE GEOMETRIES

For the extraction of the beam from the cyclotron partial shielding of the main field as well as radial focusing is needed. They can be provided separately, using a set of multipoles of the Halbach type, or combined in a single device like in the case of the focusing bars.

In Table I are reported the multipole coefficients of the dipole and quadrupole channel of the Halbach type

(see also Fig. 2) for an aperture  $r_1 = 0.6$  cm and bars thickness 0.2 cm.

The multipole components of order  $N + (N + 1)$  has been eliminated by appropriate selection of the bar height as indicated in Section 2.

For the dipole there is a gradient of 0.100 kG/cm at 2/3 aperture. In the case of the quadrupole the uniformity of the gradient is better than 10% for 2/3 of the aperture.

As apparent from the Table, the type I focusing bars has half strength of the pure dipole and one third strength of the pure quadrupole.

The use of individual multipoles seems therefore more efficient, for a given aperture and bar thickness, of the type I focusing bars. Furthermore, the focusing bars are not flexible enough for the independent selection of the dipole and quadrupole strength.

The obvious, and perhaps dominant, advantages of the focusing bars are the simplicity of the device and the possibility to combine the two components (dipole, quadrupole) in a single channel of very short length.

#### 5. REFERENCES

- 1) Danilov, V.I., Savchenko, O.V., Instr. Exp. Tech. **3**, 363 (1959); Suwa, S. et al., Nucl. Instr. and Meth. **5**, 189 (1959).
- 2) Fabrici, E.M. et al, Nucl. Instr. and Meth., **184**, 301 (1981); Fabrici, E.M., Salomone, A. in **Proceedings of the IX International Cyclotron Conference** (Caen, 1981) p. 501.
- 3) Carbonel, B. et al, Nucl. Instr. and Meth., **84**, 144 (1970).
- 4) Gordon, M.M. and Taivassalo V., Nucl. Instr. and Meth. **A247**, 423 (1986).
- 5) Halbach K., Nucl. Instr. and Meth. **169**, 1 (1980), and Halbach K., Nucl. Instr. and Meth. **189**, 213, (1982).