CROSSING OF RESONANCES

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ABSTRACT

The equations for particle motion in accelerators are considered, taking into account energy gain per turn, for the investigation of integer and half-integer resonance crossing. Formulas are presented estimating amplitude behaviour when crossing resonances.

A correct analysis of integer resonance crossing in cyclotron or synchrotron accelerators leads to equations for the radial and vertical oscillations of the form[1,2]

$$r'' + Q_r^2 r + \frac{\gamma eV}{\overline{H}\sqrt{\gamma^2-1}} r' = -R\epsilon_{zs}\sin S\psi\,,$$

$$z'' + Q_z^2 z + \frac{\gamma eV}{\overline{H}\sqrt{\gamma^2-1}} z' = -R\epsilon_{rs}\sin S\psi\,, \qquad (1)$$

where r and z – radial and vertical coordinates of the particle, $r' = dr/d\psi$, $z' = dz/d\psi$, ψ – azimuthal coordinate, γ – relativistic factor, eV – energy gain per turn, $\overline{H}$ – the mean magnetic field at radius R, ϵ_{zs}, ϵ_{rs} – relative values of the S^{th} harmonic vertical and radial magnetic field components respectively.

In contrast to the shortened equations, equations (1) have a term of the form $\delta = \gamma eV/[\overline{H}\sqrt{\gamma^2-1}]$, which is a friction term ("electromagnetic" friction) and is caused by energy gain per turn in explicit form.[1,2]

The amplitudes of the oscillations excited when crossing the integer resonance are of the form[1,2]

$$r, z = \frac{R\epsilon_{z,r,s}}{\sqrt{(Q_{r,z}^2 - S^2)^2 + \delta^2 Q_{r,z}^2}}\,. \qquad (2)$$

In the case of a half-integer resonance crossing the equations for radial and vertical oscillations are of the form[1,2]

$$r'' + Q_r^2 r + \delta r' = -r_o\epsilon_{zs}\sin S\psi\,,$$

$$z'' + Q_z^2 z + \delta z' = -z_o\epsilon_{rs}\sin S\psi\,, \qquad (3)$$

where r_o and z_o are the initial radial and vertical coordinates.

Insofar as the half-integer resonance is far less hazardous than the integer one, shortened equations without the friction term can be used to calculate to a first approximation. In this case the maximum value of the amplitude excited in the half-integer resonance zone is approximately in explicit form[1,2]

$$y \approx 1.2 y_o \frac{\pi H_s}{\overline{H}_s}\left(\frac{E_o}{2eV}\right)^{1/2},$$

$$y = r\,, z \qquad (4)$$

where $H_s = H_{zs}$ is the amplitude of the S^{th} harmonic of the vertical magnetic field component in the case of radial movement and $H_s = H_{rs}$ is the amplitude of the S^{th} harmonic of the radial magnetic field component in the case of vertical movement; E_o is the particle rest energy.

Knowledge of the particle resonance crossing mechanism allows the particle energy in cyclotrons to be increased above E_o, the integer resonance to be used as the basis for resonant beam extraction, and the injected beam intensity to be increased.[3]

REFERENCES

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