Evaluation of Third-Harmonic Voltage Flattopping for a Superconducting Cyclotron*

M.M. Gordon and Dong-o Jeon
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI48824-1321,U.S.A.

ABSTRACT

As demonstrated in the PSI-SIN 600 MeV cyclotron, third harmonic voltage flattopping can lead to great improvements in the phase width and energy homogeneity of the beam. The field of that cyclotron is, however, almost perfectly isochronous, and to determine whether this condition is a prerequisite for flattopping, we have investigated whether similar results could be obtained for superconducting cyclotrons like ours where the phase deviations can become quite large in the nonisochronous extraction region. Our calculations show that for phase deviations up to 30°, the beam improvements are quite comparable to those obtained for a perfectly isochronous field provided the third harmonic amplitude can be increased by about 25%. However, the benefits of flattopping decrease significantly and the third harmonic amplitude increases rapidly as the maximum phase deviations rise to about 60°.

1. Introduction

Among cyclotron designers who aim at single turn extraction, the use of third harmonic voltage flattopping has long been recognized as a valuable tool for achieving higher beam currents while fulfilling the requirement of energy homogeneity within the extracted beam.[1] A very successful application of this technique has been achieved in the 600 MeV cyclotron at PSI(SIN) near Zurich.[2] More recently, experimental results have been reported on the use of a fourth harmonic voltage at the TRIUMF cyclotron.[3]

The K500 and K1200 superconducting cyclotrons at our laboratory have three magnet sectors with three dees in the intervening valleys. The interiors of two of these dees are occupied by cryopanels, while the remaining dee is vacant so that this space could be used to house a smaller dee that would provide a third harmonic voltage. The calculations reported here were designed to determine whether this project might be worth pursuing.

We used the simple differential equations for the longitudinal motion to find the energy E and the phase ϕ as a function of turn number from given starting conditions. We then determined the range of initial phases ϕ_i for which the energy values on the final turn exceed the maximum energy of the previous turn. Within this phase range, the functional dependence of the final energy on the initial phase was optimized by adjusting the three available parameters: the amplitude and the relative phase of the third harmonic voltage , and the fundamental RF frequency. The availability of three parameters (rather than one) enables one to overcome the difficulties arising from the nonisochronous character of the magnetic field.

The electrode structures at the center of our cyclotrons are designed using the CYCLONE program which integrates orbits starting from the source (or inflector) using realistic electric fields. This process is based on the concept of a"central ray". That is, the central ray orbit is the one having the particular starting time which leads to the best overall energy-gain and which ends up with optimum centering. We assume that the current density as a function of starting time is symmetric about this value, and that the peak current density is associated with the central ray. When we compared the actual phase versus energy curve obtained from the CYCLONE program with curves obtained from the longitudinal motion equations for different starting phases, we found that the best match occured for an assumed central ray starting phase $\phi_{ci} = -10°$. The calculations reported here therefore assumed this value. We also carried out calculations using $\phi_{ci} = 0°$, but the results were not significantly different.

2. Procedure

The deviation of the magnetic field from isochronism, as determined from the equilibrium orbit code, is specified by the frequency error:

$$\Delta(E) = \omega_0/\omega - 1 = \nu_0\tau(E) - 1, \qquad (1)$$

*Work supported by the National Science Foundation under Grant No. 89-13815

where $\nu_0 = \omega_0/2\pi$ is a given reference frequency, and $\tau(E) = 2\pi/\omega$ is the orbit period for a given energy. Figure 1 shows a plot of this function as derived from the K1200 field data for two cases:

$$E_f/A = 40 \text{ MeV with } q/A = 0.25, \text{ and}$$

$$E_f/A = 200 \text{ MeV with } q/A = 0.5,$$

where E_f is the final energy, qe is the ion's charge, and A is its mass number. These curves show especially large values of $\Delta(E)$ near $E = E_f$ which result from the ions being accelerated out into the non-isochronous edge region of the field during the pre-extraction process. Such curves can, of course, be shifted up or down by changing the reference frequency ν_0.

During cyclotron operation, the RF frequency is adjusted as part of the optimization of the extracted beam current. We therefore set

$$\nu_{rf} = h(1+\epsilon)\nu_0, \tag{2}$$

where h is the harmonic number and ϵ is an adjustable parameter. Note that ϵ values are generally very small and are given below in parts per million(ppm). Although we assume that the magnetic field is fixed, one could choose instead to make such adjustments by shifting the central field level.

The nonzero values of $\Delta(E)$ and ϵ produce a phase change per turn given by

$$\begin{aligned}\frac{d\phi}{dn} &= \omega_{rf}\tau(E) - 2\pi h \\ &= 2\pi h\left[(1+\epsilon)\,\Delta(E) + \epsilon\right].\end{aligned} \tag{3}$$

This equation would have an additional term if the dee voltage depended on the energy (or radius),[4] but we decided to ignore this effect for the present. Also note that $h = 1$ for the cases considered here.

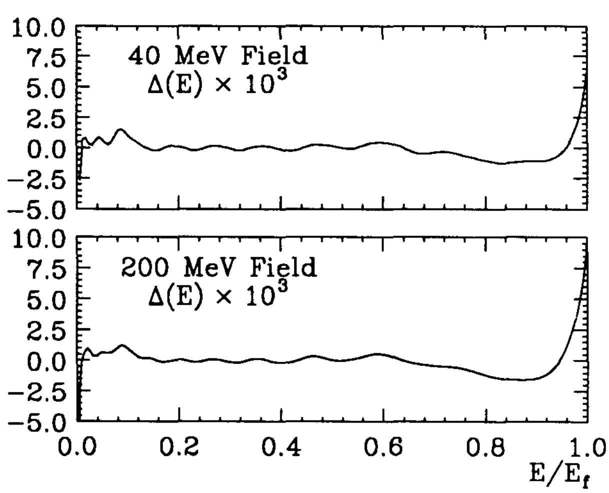

Fig. 1. Plots of the frequency error $\Delta = \omega_0/\omega - 1$ as a function of energy for two fields used to produce ions having the following final energies and charges: E_f/A=40 MeV with q/A=0.25(top), and E_f/A=200 MeV with q/A=0.5(bottom). The 200 MeV field is the least isochronous of all our fields.

When the effect of a third harmonic voltage is included, the increase in energy per turn is given by

$$\frac{dE}{dn} = qV_1\left[\cos\phi - \lambda\cos(3\phi - \psi)\right], \tag{4}$$

where V_1 is the peak voltage gain per turn produced by the first harmonic, $\lambda = V_3/V_1$ is the voltage ratio of the third to the first harmonic, and ψ is the relative phase. Because of the electrode structures in the center of the cyclotron, the turn pattern is approximately fixed. We therefore set

$$qV_1 = E_f/n_0, \tag{5}$$

where n_0 is the nominal turn number. For the present calculations, we chose both $n_0 = 500$ and $n_0 = 800$, which are approximately the values appropriate for the K500 and K1200 cyclotrons. Also, changing n_0 from 500 to 800 allowed us to investigate the effect of increasing the phase deviations without changing the magnetic field.

The above differential equations for ϕ and E were integrated starting from $n = 0$ with $E = 0$ and initial phase values ϕ_i which were distributed symmetrically about the central ray value ϕ_{ci}. For any parameter set $(\lambda, \psi, \epsilon)$, the integration proceeds until the central ray energy exactly matches the given final energy(e.g., 40 MeV), and this determines the final turn number n_f. The energy values for $n = n_f$ and different ϕ_i values are then examined to see if they meet certain criteria, and if not, a linear projection technique is used to obtain an improved parameter set. The entire process is iterated until the desired results are obtained.

For the special case $\lambda = 0$ where the third harmonic is completely absent, only the frequency parameter ϵ is available, and this is adjusted so that the central ray has the maximum energy on the final turn. This condition has long been recognized as providing the best situation for single turn extraction.[5]

As λ increases above zero, the curve showing the final turn energy as a function of ϕ_i becomes broader and flatter, and when λ exceeds a critical value λ_c, this curve exhibits two peaks rather than one. In this case, the parameters ψ and ϵ are chosen so that the two peaks have the same height with the central ray in the valley midway between them. These conditions are consistent with our previously noted assumption concerning the symmetry of the current density as a function of starting time.

Finally, when λ increases to λ_{max}, the central ray's final energy declines enough to reach the maximum energy on the previous turn. At this point, the calculation stops. Although results were obtained over the complete range of λ values, only those of particular interest will be discussed below.

3. Results

Single turn extraction requires turn separation only at the final energy where the ions must clear the septum and enter the extraction channel. This radial separation of the turns at the final energy can be obtained provided the ion bunches are clearly separated in energy. That is, even if these bunches overlap spatially because of the radial oscillations, they can in principle be separated by a suitable dispersive process. In our cyclotrons as in many others, turn separation is generated by passage through the $\nu_r = 1$ resonance, and the amount of this separation is restricted by vertical stability requirements. This process has therefore limited dispersion.

In presenting our results here, we shall assume an energy spread within the bunch on the final turn, δE, which is just one-half of the peak energy gain on this turn. In this case, the energy gap between successive bunches is equal to δE.

Consider first the results obtained when the third harmonic is absent, i.e., $\lambda = 0$. For an isochronous field and a central phase $\phi_c \equiv 0$, one finds that the acceptable initial phase width is given by

$$\delta\phi_i = 2/\sqrt{n_0} \text{ (in radians)}, \tag{6}$$

where n_0 is defined in Eq. 5 above. For the two cases considered here, $n_0 = 500$ and $n_0 = 800$, this yields 5.1° and 4.1°, respectively.

The corresponding results were computed for the 40 MeV and the 200 MeV fields whose deviations from isochronism are shown in Fig. 1. For $n_0 = 500$, we found $\delta\phi_i = 4.8°$ and 4.7° for the 40 MeV and 200 MeV cases, while for $n_0 = 800$, we obtained $\delta\phi_i = 3.2°$ and 2.8° for the same two cases. Considering how small these values are, and considering also that the beam current is roughly proportional to $\delta\phi_i$, one can easily recognize the potential value of flattopping.

The greatest amplification of $\delta\phi_i$ is obtained for values of λ in the range $\lambda_c < \lambda < \lambda_{max}$ where the curve showing the final energy as a function of ϕ_i has two peaks. Following the procedure described near the end of the last section, and imposing the condition discussed above on the energy spread δE within the bunch, we found the results that are summarized in the tables below, one for $n_0 = 500$ and the other for $n_0 = 800$. In each table, data are presented for the two fields (40 MeV and 200 MeV) described in Fig. 1, and as a reference, for an isochronous field ($\Delta \equiv 0$).

Note that the actual final turn number n_f exceeds n_0 as λ increases, mainly because the energy gain per turn is roughly proportional to $qV_1(1-\lambda)$. Following n_f in the tables are the values of λ, the fractional third harmonic voltage parameter, and the relative phase ψ of the third harmonic. These are followed by the values of ϵ, the fractional change in the main RF frequency. The next row lists the resultant values of $\delta E/qV_1$, where δE is the energy spread within the final bunch as well as the energy gap between bunches, and qV_1 is the peak energy gain per turn produced by the main harmonic. Finally, the values of $\delta\phi_i$ are given.

The values of λ given in the tables show that the required percentage of third harmonic voltage rises from 12 for the isochronous field up to 14 or 15 for the two other fields when $n_0 = 500$. For $n_0 = 800$, however, this percentage rises from 12 up to between 18 and 20 in the same cases. These differences are quite significant when one considers the RF power requirement, and are due mainly to differences in the phase deviations. That is, the central phase ϕ_c ranges from −10° to +10° for the isochronous field, and from about −30° to +20° for the two nonisochronous fields when $n_0 = 500$. For $n_0 = 800$, on the other hand, the values of ϕ_c range from about −50° to +30° for the latter two fields.

Table 1 Results for $n_0 = 500$

Field	Isoc	40 MeV	200 MeV
n_f	570.1	579.1	582.3
λ	0.123	0.140	0.146
ψ(deg)	0.0	2.0	2.9
ϵ(ppm)	97.5	91.6	101.3
$\delta E/qV_1$	0.438	0.430	0.430
$\delta\phi_i$(deg)	35.4	34.2	33.5

Table 2 Results for $n_0 = 800$

Field	Isoc	40 MeV	200 MeV
n_f	910.5	962.0	980.4
λ	0.122	0.180	0.201
ψ(deg)	31.4	24.9	22.8
ϵ(ppm)	61.1	66.5	85.9
$\delta E/qV_1$	0.439	0.421	0.397
$\delta\phi_i$(deg)	31.4	24.9	22.8

The most impressive numbers in these tables are the values of $\delta\phi_i$. For $n_0 = 500$, the values for the two nonisochronous fields are less than 2° below the $\delta\phi_i$ for the isochronous field, and for all three fields, the values are about <u>seven</u> times larger than those given above for $\lambda = 0$. On the other hand, for $n_0 = 800$, there are significant differences between the $\delta\phi_i$ values for the isochronous and nonisochronous fields. One finds nevertheless that all these values are roughly <u>eight</u> times larger than those cited above for $\lambda = 0$ in the corresponding cases.

To illuminate the results further, plots of the final energy versus the initial phase are shown in Fig. 2 for all three fields when $n_0 = 800$. These curves show the two peaks of equal height with the central ray point ($\phi_{ci} = -10°$) in the valley midway between them. A horizontal broken line divides the full height into two equal halves each of height δE, as defined above. That is, the top half represents orbits within the final bunch while the bottom half shows the gap between successive bunches. The width of the curve where the straight line crosses it then defines $\delta\phi_i$, the acceptable initial phase width given in Table 2. Including the central ray itself, we therefore find three orbits which end up at $n = n_f$ with exactly the same energy.

Plots of the phase ϕ as a function of energy from $E = 0$ to $E = E_f$ are shown in Fig. 3 for the same three cases used in Fig. 2. The three solid curves correspond to the three orbits just mentioned which end up with the

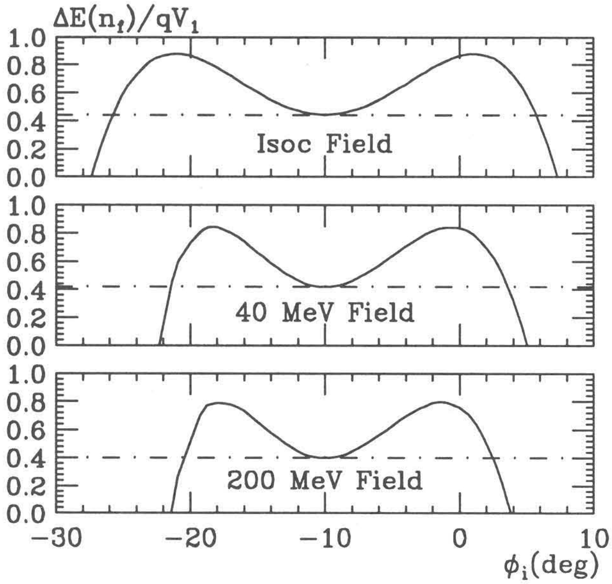

Fig. 2. Plots of the final energy as a function of the initial phase ϕ_i obtained under optimized conditions using the RF parameters given in Table 2 for three different fields: isochronous(top), 40 MeV(center), and 200 MeV(bottom). Here ΔE is the difference between the energy on the final turn and the maximum energy on the previous turn, and $qV_1 = E_f/n_0$ with $n_0 = 800$.

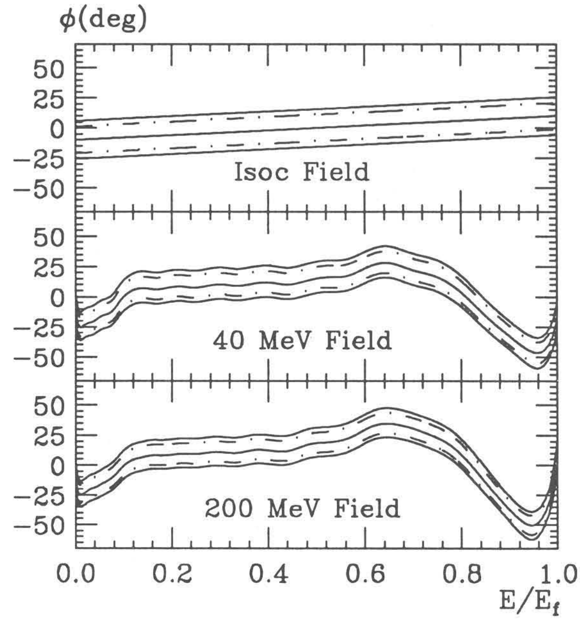

Fig. 3. Plots of the phase ϕ as a function of energy obtained for the same fields and RF parameters as those used for Fig. 2. The three solid curves all lead to the same final energy as defined by the horizontal broken line in Fig. 2, and the two broken curves lead to the two energy peaks in Fig. 2.

same final energy. The two intervening broken curves correspond to the orbits that end up with the peak energy. These sets of curves show in detail the evolution of the phase deviations mentioned above. Curves similar to those shown in Fig. 2 and Fig. 3 were obtained for $n_0 = 500$, but these are less dramatic and are omitted here to save space.

4. Conclusions

While the K1200 cyclotron is fully occupied at present with carrying out nuclear physics experiments, the K500 cyclotron is available and could be used to test the effect of a third harmonic voltage on beam properties. As noted above, this cyclotron is characterized by a $n_0 = 500$ turn geometry and would, according to Table 1, require a third harmonic voltage which is about 14% of the first harmonic. Since only one of the three dees is available for the third harmonic voltage, this percentage rises to 42%. The design of a possible RF system is now under way.

We are indebted to H. Blosser and F. Marti for many useful discussions and to the latter for assistance with the field data. We should also acknowledge that somewhat similar calculations were carried out for the TRIUMF cyclotron by C. Kost as reported by M. Craddock.[6]

5. REFERENCES

1) M.M. Gordon, Nucl. Instr. & Meth. **58**, 245(1968).
2) S. Adam, et al, IEEE Trans. **NS-28**, 2721(1981); W. Joho, **Proc. Ninth Int'l. Cyclotron Conf. at Caen** (Les Editions de Physique, Paris, 1981)337.
3) R.E. Laxdal et al, Conf. Record of the 1991 Particle Accelerator Conf. (IEEE 91 CH3038-7)810.
4) W. Joho, Particle Accel. **6**, 41(1974).
5) M.M. Gordon, IEEE Trans. Nucl. Sci. **NS-13**, 48(1966).
6) M. Craddock, **Proc. Fifth All-Union Nat'l. Conf. on Particle Accel.**, Dubna, Oct. 1976.