

## ION\_TRACKER, A TRACE TRACKING PROGRAM FOR CYCLOTRON DESIGN

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### ABSTRACT

ION\_TRACKER is an interactive program for the design of axial injection and central region of cyclotron. The ions can be tracked from the beginning of the cyclotron axis through a single gap buncher and an inflector, and then inside the cyclotron until to extractor. Tracking can be done separately in every part or through the whole system at once. It uses the predictor-corrector method to solve equation of motion of charged particle in a cyclotron. We shall introduce restrictions of the program and show some examples of its use.

### 1. INTRODUCTION

As is well known to cyclotron builders, numerical orbit studies are an important step in cyclotron construction. In fact, orbit studies serve initially as the basis of the design of central region and extraction elements. Furthermore, the support from the numerical orbit studies to the beam development work during the machine operation helps to guide over the possible problems considering about beam optics.

The beam optics of central region and axial injection of the K130 cyclotron have been extensively studied numerically<sup>1)</sup> for the purpose of design and understanding of the beam injection. For the numerical calculations we have used a self written program ION\_TRACKER.

In this report, we present the program ION\_TRACKER which has been developed during the design of the K130 cyclotron. We shall introduce the method to solve the equation of motion of a charged particle in the electromagnetic field of a cyclotron. In particular, we draw attention to the macro programming feature of the program.

### 2. INTEGRATION METHOD AND FIELD CALCULATION

ION\_TRACKER integrates numerically the equation  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$  using a cartesian coordinates with a time as independent variable. Used integration method is a predictor-corrector method of order  $h^5$ . Integration

method provides one measure of the error being made at each step and it is used to control the step size employed in the integration. Another attractive feature of this integration method is that only two evaluations of the force  $\vec{F}$  need to be computed at each integration step. More details about predictor-corrector methods see for example R.W. Hamming, "Numerical Methods for Scientists and Engineers".<sup>2)</sup>

Ions can be tracked all the way from the beginning of the axial injection line to the extractor of the cyclotron. Magnetic and electric fields of the cyclotron are given to the ION\_TRACKER in field maps. Using predictor-corrector method with variable integration time step we need to have a method to calculate the fields at arbitrary point. Fields and their first derivative, at locations lying between the given data points, are obtained through a double three-point Lagrange interpolation.<sup>3)</sup> This interpolation scheme provides that the fields as well as their first derivative will be continuous.

Since ION\_TRACKER has been developed for the beam optics study of the K130 cyclotron it has some "machine dependent" restrictions which will be outlined below. Some attempts has been paid to make the code, especially for field calculations part of it, so that it is easy to modify if needed.

#### 2.1. Axial Injection Line

The axial injection line may be consist of axially symmetric magnetic fields, like axial magnetic field of the cyclotron and solenoids, and a single acceleration gap buncher. The buncher can be excited by combining an rf sine wave and its second harmonic.

The electric field of the buncher is obtained by using a hard edge approximation. Electric field is assumed to be zero outside of the gap, and inside to have only axial component which is calculated from the formula  $E_z = V_i/d$ , where  $d$  is the gap length and  $V_i$  is the voltage of the  $i$ :th harmonic.

The axial magnetic field is given along the magnetic axis of cyclotron and it can be combined from three subfields. The subfields, for example the axial magnetic field

of the cyclotron and two solenoids, can be replaced and rescaled interactively while running the program. Outside of the axis of the cyclotron the magnetic field will be calculated from field expansions, which are derived using Maxwell's equations and assuming the perfect axial field symmetry. We shall get the expansions

$$\begin{aligned} B_r(r, z) &= \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!(n+1)!} B^{(2n+1)}(z) \left(\frac{r}{2}\right)^{2n+1} \\ B_\theta(r, z) &= 0 \\ B_z(r, z) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!^2} B^{(2n)}(z) \left(\frac{r}{2}\right)^{2n}. \end{aligned}$$

ION\_TRACKER uses the axial magnetic field and its first three derivatives to calculate magnetic field outside of the symmetry axis.

## 2.2. Spiral Inflector

The electric field of the spiral inflector is computed from an electric potential map. We have used RELAX3D<sup>4)</sup> to calculate the potential map. Our RELAX3D calculations are done with the aid of an unnamed program which prescribes to RELAX3D the inflector electrode locations around an analytical central ray. The assumption of the hard edge field has been used in solving the shape of inflector electrodes. Since a real inflector will have fringe fields, which increase its effective length, we have included to the program the possibility to shorten the electrodes.

The magnetic field is calculated in the same way as in axial injection line.

## 2.3. Cyclotron

ION\_TRACKER is limited to use of two rf systems with straight dees and the acceleration gaps have to be in constant azimuth angles. The electric field produced by the dees in the median plane of the cyclotron is calculated in two different ways. In the central region the electric field is obtained from a three dimensional rectangular potential map which can be calculated by using RELAX3D. Outside of this region the field will be calculated from a gaussian approximation.<sup>5)</sup> In Fig. 1 are shown the electric field components calculated in both methods along the ion path for one gap crossing.

In order to describe the gaussian approximation we shall define the right handed cartesian coordinate system, which has its origin at the middle of the straight accelerating gap. The X and Y axis will lay on the median plane of the cyclotron and the Z axis is oriented axially. X axis is parallel and Y axis is normal to the gap.

The normal field component in the median plane is obtained from the gaussian function

$$E_y(y) = E_{\max} e^{-1/2(y/\Delta y)^2},$$

where the width  $\Delta y$  is related experimentally to the gap width  $W$  and the dee-aperture  $H$  by

$$\Delta y = 0.2H + 0.4W.$$

Assuming the perfect median plane symmetry the axial field component can be calculated from the formula

$$E_z(y, z) = E_y(y) \frac{yz}{\Delta y^2}.$$

The parallel field component is assumed to be zero.

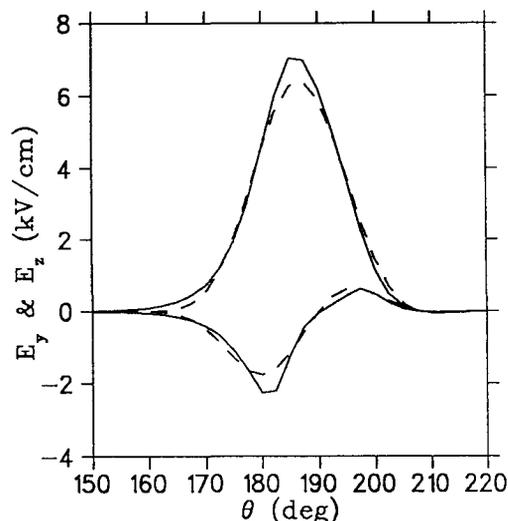


Fig. 1. Electric field components with time modulation along the ion path for one gap crossing. In solid line are drawn the field components obtained from potential map data and in dashed line from gaussian approximation.

The cyclotron magnetic field is given on the median plane in a polar map. Outside of the median plane the magnetic field will be calculated from field expansions. Using Maxwell's equations and the assumption of the median plane symmetry we shall get the expansions

$$\begin{aligned} B_r(r, \theta, z) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{\partial B^{(n)}(r, \theta)}{\partial r} z^{2n+1} \\ B_\theta(r, \theta, z) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{\partial B^{(n)}(r, \theta)}{\partial \theta} z^{2n+1} \\ B_z(r, \theta, z) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} B^{(n)}(r, \theta) z^{2n}, \end{aligned}$$

where  $B^{(0)}(r, \theta) = B(r, \theta)$ ,  $B^{(1)}(r, \theta) = \nabla^2 B(r, \theta)$ ,  $B^{(2)} = \nabla^2 B^{(1)}(r, \theta)$  and so on.

In ION\_TRACKER the magnetic field off the median plane can be obtained correctly to first or fourth order in  $z$ . For the fourth order calculation in addition to the median plane magnetic field map  $B(r, \theta)$  the corresponding maps for  $B^{(1)}(r, \theta)$  and  $B^{(2)}(r, \theta)$  are required.

These maps must be calculated and stored in advance. Since the maps  $B^{(1)}$  and  $B^{(2)}$  contain high derivatives of the  $B$ , they exhibit much more noise than the original field data. Even though the integration routine will act as averaging process in filtering out the high frequency components of the noise some caution is required.

We should note that for the fourth order calculation the used field expansions are the same as in code SPIRAL GAP.<sup>6)</sup>

### 3. PROGRAM INPUT AND OUTPUT

ION\_TRACKER is a command driven program. The terminal interface routine, that closely mimics the DCL command recall facility, as well as many other general purpose routines, has been taken from TRIUMF libraries. For this reason the program has a lot of common features with the graphic code PLOTDATA.<sup>7)</sup> Some examples of the features provided by ION\_TRACKER:

- basic mathematical functions
- expression evaluation
- allow scalar variables (100)
- dynamic and static 20 line input recall buffers
- macro capability with parameter passing
- looping, branching and conditional statements in macros
- create your own commands by "aliasing"
- allow you to define keypad keys

All input is fed to ION\_TRACKER by commands. Field maps and other long input like equilibrium orbit and central region geometry data are read from files. The commands can be entered from three sources:

1. interactively via the terminal keyboard
2. non-interactively via a DCL command file
3. non-interactively via an ION\_TRACKER command macro file

In interactive mode the status of the input of ION\_TRACKER can be displayed on terminal screen when desired.

An important part of the beam optics calculation is the graphical visualization of the numerical data. ION\_TRACKER itself has no graphics but it writes an input file for PLOTDATA. The absence of the graphics is not a real disadvantage: the orbit data can be read and drawn in PLOTDATA easily and fast.

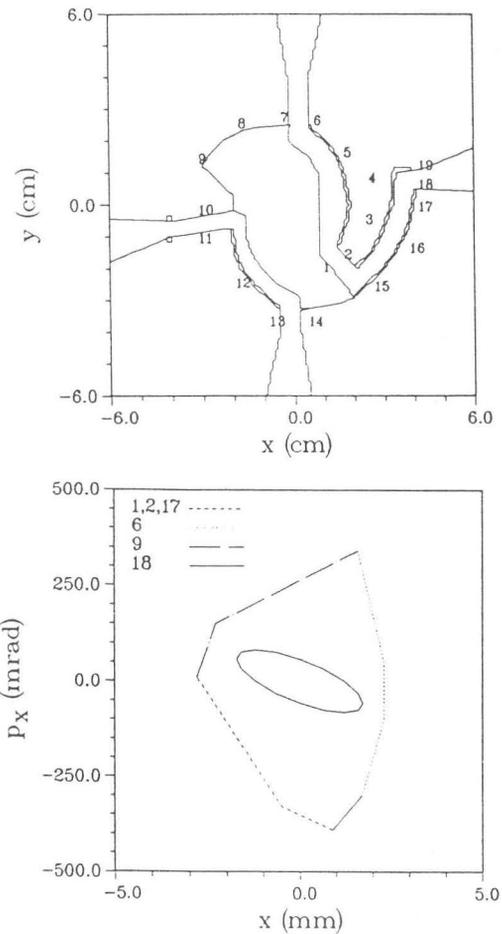


Fig. 2. In the upper picture is shown the layout of the central region. The obstacles on the central region are specified in places numbered in the picture. In the first harmonic acceleration mode the puller electrode is the dee with a 45 degrees nose. In the lower picture is shown the result of the acceptance calculation. The different line types indicate the limiting places in the central region. The matched beam,  $\epsilon = 100\pi$  mm $\times$ mrad, is drawn in solid line.

## 4. EXAMPLES

### 4.1. Acceptance Calculation

ION\_TRACKER can calculate transverse acceptances of the central region. Radial and axial acceptances are calculated independently and they are determined by calculating a group of ion trajectories starting from the boundary of the acceptance. In radial acceptance calculation the central region geometry is given as input. The obstacles on the central region are specified in azimuth angles  $\theta_i$ . When the ion reaches the angle  $\theta_i$  it will be checked if it passes the obstacle or not. Into the

output file ION\_TRACKER writes the starting values of the ions which have been able to go through the central region and in radial case also the place where the "near-by" ion has been lost. In Fig. 2 we have an example of the result of a radial acceptance calculation of the first harmonic acceleration mode in the K130 cyclotron.

#### 4.2. Macro Programming

Especially macro programming feature makes ION\_TRACKER powerful and fast to solve various kind of problems. Macros can be applied for example to generate different kind of beams, to search accelerated equilibrium orbits, to calculate spiral acceptance and to tune cyclotron parameters.

ION\_TRACKER can utilize calculated orbit data. Orbit data is not stored in a memory while tracking the ions but it can be read from the output file of ION\_TRACKER. From the orbit data several statistics can be calculated. In addition to the one variable statistical computation, you can do a fit of regression line and ellipse to the given data. The results of the computations will be assigned to the specified variables.

Calculated orbit data can be made use of a feedback to tune cyclotron parameters like strengths of the axial solenoids, buncher parameters, spiral voltage and rf-frequency. It is essential that those parameters are well tuned when studying the beam dynamics in the cyclotron. The tuning is straight forward but it can be time consuming and tedious work. If macros are applied to do the tuning, then it can be done by entering a single command.

As an example of beam simulation we shall present the part of macro program which generates realistic dc-beam and track it through the axial injection line into the cyclotron. Results of this calculation are presented elsewhere in this Proceedings.<sup>1)</sup> The beam has been generated and tracked by executing following macro loop:

```
DO I=1,1000
  EXE BEAM.TCOM 100 2.5 0.25 100 2.5 0.25
  TAU = RAN*360
  LOCATION X Y -160.0
  TRANSVER XP YP
  LONGITUD 9.7E-3 TAU
  RUN AIC
ENDDO
```

The command macro file BEAM.TCOM is executed on the first line of the loop. In this macro one point (X,XP,Y,YP) is picked randomly from a four dimensional gaussian distributed phase space. The macro has six parameters which defines the Twiss' parameters of the uncorrelated subspaces (X,XP) and (Y,YP). In this specific case both subspaces have the same Twiss' parameters:  $\epsilon = 100\pi$  mm×mrad,  $\beta = 2.5$  mm/mrad and  $\alpha = 0.25$ . On the second line of the loop a random number from the interval 0 - 360 is assigned to the variable

TAU. RAN is a random number function which will get a value from 0 to 1. In the following three lines the starting values which have been calculated on the previous lines will be set to the ion. Starting location is 160 cm from the median plane of the cyclotron, injection energy is 9.7 keV and phase is TAU. The tracking will be done on the last line of the loop. The parameter AIC (Axial Inflector Cyclotron) of the command RUN specifies the tracking area.

#### 5. FUTURE DEVELOPMENTS

We have been planning to implement also electrostatic deflector, electromagnetic channel and focussing bars to our code. If we succeed in that we shall have a single code which can handle all the beam dynamics from the beginning of the axial injection line to out of the cyclotron.

Another aim is to make better documents and user guide to the program. Up to now we have just up dated our command reference manual and made sufficient comments in the code.

#### 6. ACKNOWLEDGEMENTS

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