EFFECTS ON A BEAM OF PROLONGED WEAVING AROUND THE $\nu_r=2\nu_z$ RESONANCE*

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Abstract

For certain special operating conditions in the (nearly completed) K800 superconducting cyclotron, the beam will weave around and remain close to the $\nu_r=2\nu_z$ coupling resonance over a significant fraction of its acceleration history. The resultant effects were investigated using the Z^4 Orbit Code which is specifically designed to provide realistic data on such nonlinear phenomena. These investigations show that while the phase space distributions tend to become very chaotic, the total growth of radial and vertical emittances remains fairly well within the upper limits suggested by theory. In addition, theoretical expectations regarding the radial and vertical action variables are also found to be reasonably accurate.

Introduction

The importance of the $\nu_r=2\nu_z$ coupling resonance has been recognized for a long time since it occurs in the extraction region of most cyclotrons where it severely limits the achievable turn separation. That is, if the radial displacements become too large, the resultant coupling action will cause the vertical height of the beam to grow beyond the allowed aperture with a resultant loss in current.

In superconducting cyclotrons the values of ν_z generally rise when the extraction energy for a given ion is lowered. One therefore finds operating conditions where the beam repeatedly crosses the $\nu_r=2\nu_z$ resonance well below its final energy, and the effects produced in one such case are presented here.

Since $\nu_r-2\nu_z=0$ is a "difference" resonance, its effect is limited according to theory by an approximate conservation rule which states :[1]

$$J_x + \frac{1}{2} J_z = J_o \quad , \qquad (1)$$

where J_x (or J_z) is the instantaneous value of the radial (or vertical) action variable for a given orbit, and J_o is a constant. Basically, J is a canonical variable proportional to the square of the oscillation amplitude, and $\nu_r J_x$ (or $\nu_z J_z$) is the oscillation energy when the motion is linear. The values of J_x and J_z are especially important since in the absence of resonances, they are both adiabatic invariants. Moreover, $2\pi J$ is the area of the eigenellipse on which the orbit is instantaneously located, and this area is proportional to the corresponding emittance.

Since J_o is determined by the initial conditions of an orbit, the above relation shows that J_x and J_z will have maximum values given by:

$$J_x < J_o \quad , \text{ and } \quad J_z < 2J_o \quad , \qquad (2)$$

provided certain conditions are fulfilled. These require that $|\nu_r-2\nu_z|$ be very small and that all other resonances can be neglected.

The Z^4 Orbit Code developed here is specifically designed for studies of nonlinear coupling resonances.[2] The equations of motion on which it is based are correct to fourth order in z and can be derived from a single hamiltonian (which is another requirement for the validity of (1) above).

Magnetic Field and Initial Conditions

The magnetic field chosen here is designed for a charge Q/A=0.5 with a final energy (per nucleon) of 142 MeV. The resultant values of ν_r and $2\nu_z$ are plotted in Fig. 1, and we find, in particular, that between 55 and 115 MeV the average value of $(\nu_r-2\nu_z)$ is only 0.012, while the rms value is 0.026. This energy range corresponds to 300 out of the total 700 turns executed by the beam.

All of the orbits described here start well below the resonance at 30 MeV and go 500 turns to 131 MeV, about 50 turns before extraction. A central ray was determined by starting on the accelerated equilibrium orbit (AEO) with the optimum phase, and the subsequent deviation of this ray from the AEO for the entire 500 turns was negligible (<0.03 mm). The starting times for the other orbits were all adjusted to give the same average phase values for the first 15 turns, and we

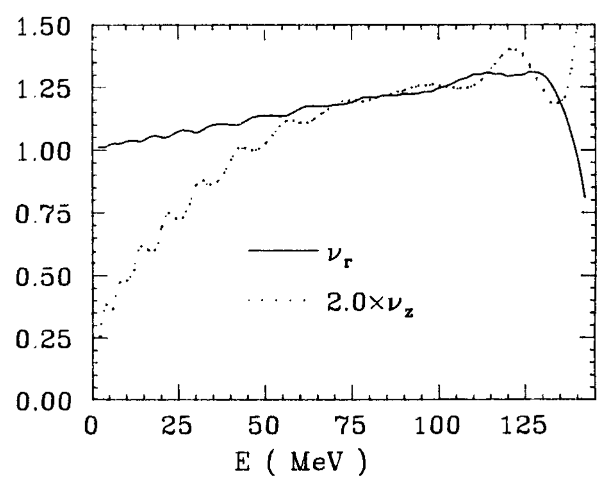

Fig. 1--Plot of ν_r and $2\nu_z$ versus energy showing that for this field, the operating point weaves around and remains close to the $\nu_r=2\nu_z$ coupling resonance over a wide range of energies.

then found that the energy differences remain remarkably small, being less than 5 keV out of 100 MeV at the end of 500 turns.

Starting conditions for (x,p_x) and (z,p_z) were chosen to be the four points on the major and minor axes of eigenellipses having prescribed areas, as shown in Fig. 2. These areas correspond to an extracted beam having final emittances of 10 mm-mrad radially and 20 mm-mrad vertically, which are about twice the values expected for the K800 cyclotron as determined from the measured emittances for the K500 cyclotron and the momentum ratio for the two machines.

Because of median plane symmetry, the points (z,p_z) and $(-z,-p_z)$ are equivalent starting conditions,

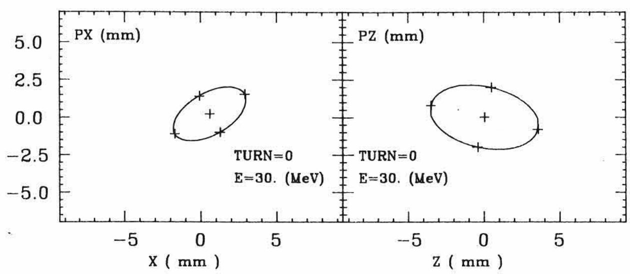

Fig. 2--Initial conditions for (x,p_x) and (z,p_z) on the two eigenellipses at 30 MeV corresponding to emittance values with J_x=10 mm-mrad, and J_z=20 mm-mrad. Note that the (x,p_x) eigenellipse is always centered on the "accelerated equilibrium orbit".

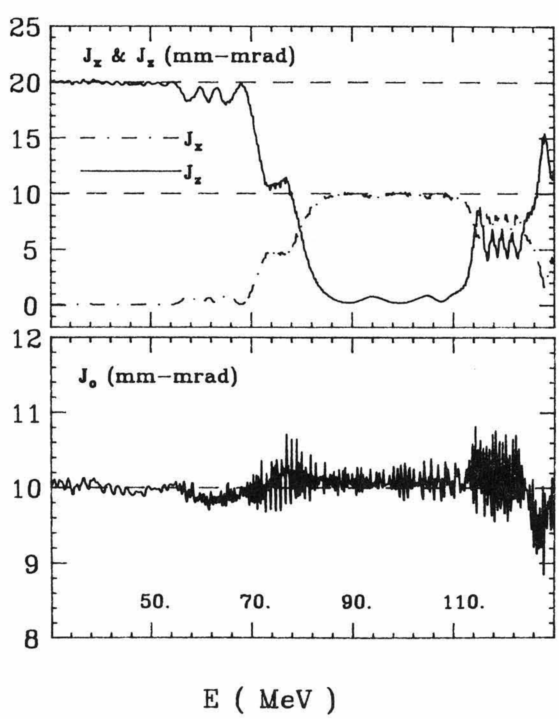

Fig. 3--Plots of J_z, J_x, and J_o versus energy for one orbit that starts with J_z=20 and J_x=0. The broken line at J_x=10 is the theoretical maximum for J_x. Also, J_o is theoretically a constant. See Eq. (3) for units.

so that only one orbit of the pair was computed. Each point on the (x,p_x) eigenellipse (and the central ray) are therefore represented by three different orbits, one in the median plane and the other two starting on the major and minor axes of the (z,p_z) eigenellipse. On the other hand, each initial (z,p_z) point corresponds to five different orbits. Thus, only fifteen orbits were computed altogether. Although this is a relatively small sample, it is sufficient to show the main aspects of the resonance phenomena.

Results

The action J is used here to characterize a given orbit and to make the value more meaningful, we express J values in mm-mrad by using a constant scale factor. That is, we make the replacement:

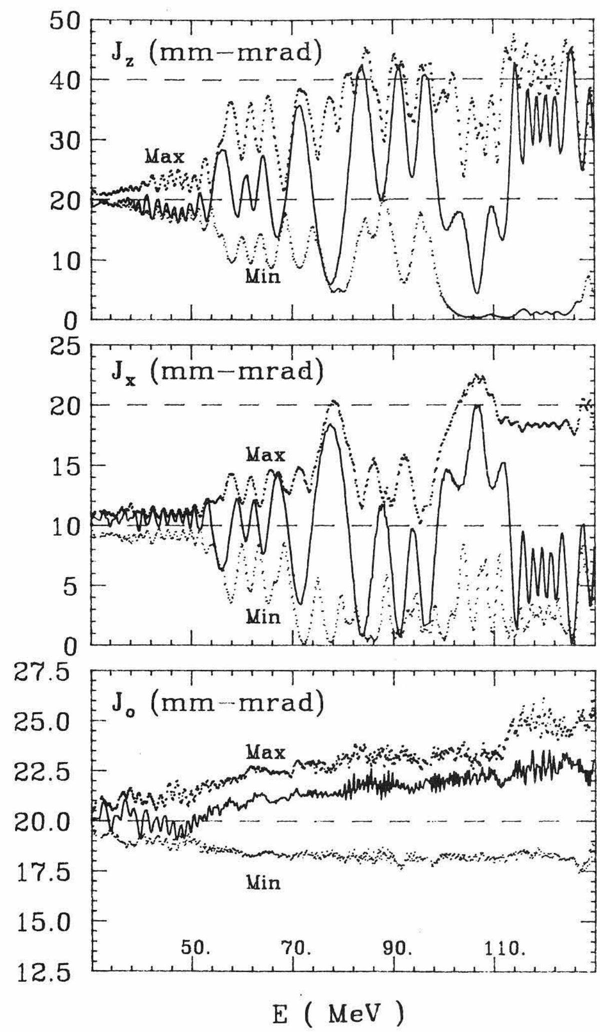

Fig. 4--Plots of J_z, J_x, and J_o versus energy summarizing the data for eight orbits starting with J_z=20 and J_x=10 shown in Fig. 2. Solid curves are for one representative orbit, while dotted curves give maximum and minimum values for all eight orbits. Broken lines at J_z=40 and J_x=20 show theoretical limits. Here J_o is theoretically equal to 20 for all these orbits. See Eq. (1).

$$2\pi J/p_f \rightarrow J \quad , \tag{3}$$

where p_f is the final momentum corresponding here to 142 MeV. Thus, the initial values are J_x=10 (or 0) and J_z=20 (or 0). We should also note that output data were recorded once per turn at Θ=0, and that the average energy gained per turn is 202 keV.

The four median plane orbits with J_x=10 (J_z=0) show remarkably little deviation from linearity and adiabaticity, since for the entire 500 turns, the maximum deviation from J_x=10 was less than 3 %. Further evidence for linearity is provided by the very small energy differences for all 15 orbits, as noted above.

On the other hand, the two orbits that start on the central ray with J_z=20 (J_x=0) show very characteristic coupling effects. That is, the initial energy of the vertical oscillations is transferred almost entirely to the radial oscillations and then back again. This is shown for one of these orbits in Fig. 3 where J_z, J_x, and J_o are plotted as a function of energy. As can be seen, J_x rises to a plateau near J_x=10 while J_z drops down close to zero, and this plateau persists from about 85 to 110 MeV. Such plateaus result from the orbit remaining close to the unstable fixed point that corresponds to median plane motion. At the same time, the results show that J_o remains nearly constant deviating from 10 by less than 8 %, in good agreement with theory.

The results obtained for the remaining eight orbits are more difficult to interpret. These orbits start with both J_x=10 and J_z=20, and the subsequent values of J_z, J_x, and J_o are summarized in Fig. 4. The motion is fairly linear from 30 to 50 MeV where the deviations of J_z and J_x from their initial values are not very large. From 50 MeV onward, however, the coupling action takes hold quite strongly, as can be seen from the rapid changes in the maximum and minimum values.

The maximum values are particularly interesting since they exceed the theoretical limits (J_x=20, and J_z=40) at several places, reaching peak values of J_x=23 and J_z=47 near 110 MeV. Moreover, the deviations of J_o from 20, the theoretical value, grow progressively larger, reaching extreme values of 26 and 17.5 near 123 MeV. Nevertheless, all these deviations are less than 30% which is not unreasonable.

Phase plots are shown in Fig. 5 for the radial and vertical motion of all 15 orbits at turns 100, 150, 200, and 500. The results at turn 100 verify that the motion remains fairly linear from 30 to 50 MeV since the plotted points remain bunched together close to the original eigenellipses. Between 100 and 200 turns (50-70 MeV), however, the distribution becomes progressively more disorganized and then remains quite chaotic out to turn 500. The theoretical limits for the growth in areas are shown by the broken curves, and although these limits are not exceeded on the particular turns shown here, the data of Fig. 4 indicate that some points move outside these limits temporarily between turns 300 and 500.

The program also records the maximum height Δz on each turn, and the resultant values show that Δz=10 mm initially, reaches a peak value of 14 mm, and ends up at 12 mm at 131 MeV. This growth in vertical height is quite consistent with theoretical expectations and the J_z values reported above.

One should, of course, keep in mind that this coupling resonance is nonlinear so that its effects depend quite strongly on the conditions of the beam prior to the resonance. Although we have assumed here initial emittance values which are about twice those expected for the K800 cyclotron, we have at the same time assumed that the orbits are very well centered and have the same initial phase. Orbit centering is known

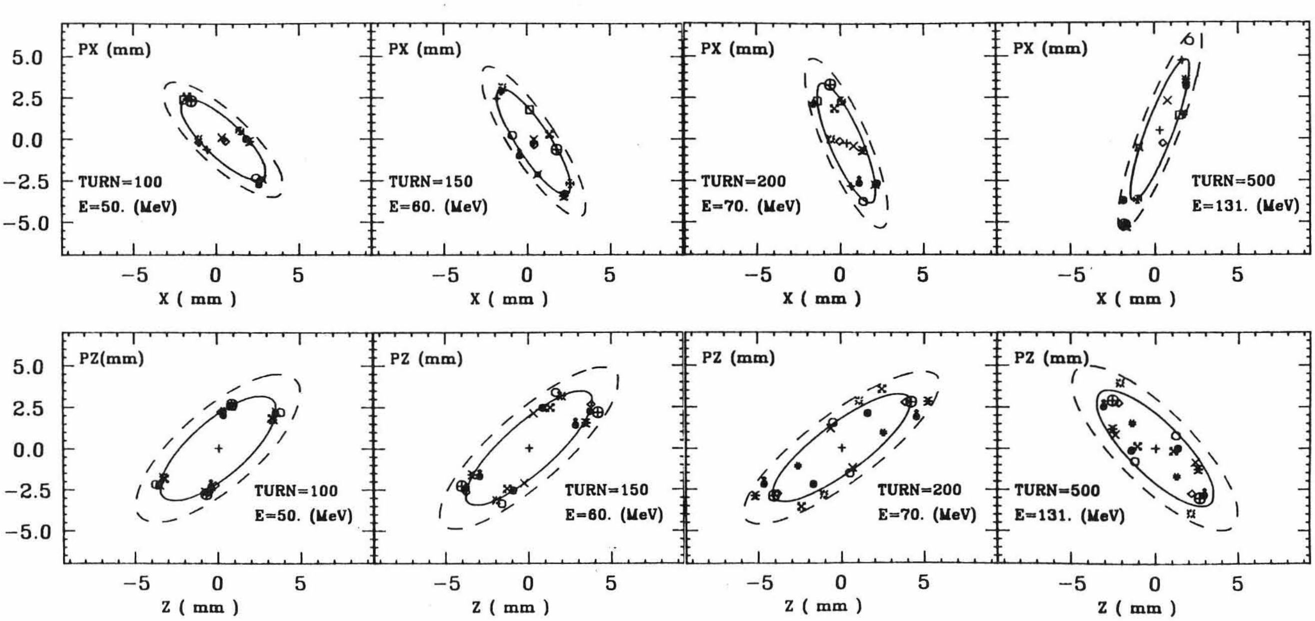

Fig. 5--Plots of p_x versus x (top) and p_z vs z (bottom) for 15 orbits on turns 100, 150, 200, and 500. These plots show evolution of points from an initially ordered state to a chaotic one. Solid eigenellipses have the same areas as the initial ones in Fig. 2, while broken eigenellipses represent theoretical upper limits for the growth in these areas.

to depend on phase, and as shown in a previous study, the spread in phase values definitely influences the resultant growth in vertical emittance.[3] The present calculations are, however, the first to be carried out here (and perhaps elsewhere) with a proper treatment of all of the coupling effects.

References

1. See e.g., A.A. Kolomensky and A. N. Lebedev, Theory of Cyclic Accelerators (John Wiley, New York, 1966), Sec. 3.6.
2. M. M. Gordon and V. Taivassalo, Nucl. Inst. and Meth. A247 (1986) 423.
3. B. F. Milton and H. G. Blosser, Proc. 10th Int. Cyclotron Conf.,(East Lansing) 52, (1984).

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