TRANSFER MATRIX METHOD FOR COMPUTING DEPOLARIZATION EFFECTS OF A POLARIZED PROTON BEAM IN A TRANSPORT SYSTEM

D. Rapin, R. Hess, J.-C. Niklès, D.W. Werren
University of Geneva

ABSTRACT

We discuss transfer matrices T used in the expression $\vec{S}_{out} = T \cdot \vec{S}_{in}$ for computing the spin precession of relativistic protons. In particular we give the matrix elements to first order approximation for quadrupole, bending magnet with inclusion of entrance and exit path, and a solenoid.

INTRODUCTION

With the recent development of polarized ions sources, polarized proton beams of several hundred nAmp will soon be available. The evaluation of depolarization effects in the beam transport system may be important because of the high degree of polarization of these beams. In particular our study was motivated by an application to the polarized proton beams of the swiss meson factory SIN.

SPIN TRANSFER MATRICES

We follow here closely R.R. Stevens and G.G. Ohlsen[1)] but generalize their results to the relativistic case. The spin direction is referred in a frame attached to the beam line. The z axis is along the beam line, the y axis is vertical and the x axis is chosen to form a right handed coordinate system. We assumed the same first order approximations as in the beam optic[2)] treatment. Thus a beam transport element rotates the entrance spin vector $\vec{S}_{in}$ into a new direction $\vec{S}_{out}$ according to

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_{out} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \cdot \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_{in}$$

Clearly the matrix T should be unitary. Note that the t_{ij} elements are depending on the particles coordinates, and that the trajectories should be first computed.

In a magnetic field $\vec{B}$ the equation of motion for the spin vector of a relativistic particle is given by[3]

$$\frac{d\vec{S}}{dt} = \vec{S}\ \frac{e}{m\gamma}\left(\vec{B} + (\frac{g}{2} - 1)(\gamma\vec{B}_\perp + \vec{B}_\parallel)\right) \quad \text{(in the Lab system)}$$

$\vec{B}_\parallel = (\hat{v}\cdot\vec{B})\cdot\hat{v}$, is the field component parallel to the particle trajectory, $\vec{B}_\perp = \vec{B} - \vec{B}_\parallel$. $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ where $\beta = v/c$, m is the proton rest mass, and $(\frac{g}{2} - 1)$ is the anomalous magnetic moment in units of $\frac{e\hbar}{2m}$, g(proton) = 5.58.

The table I gives the transfer matrices T for several beam elements of interest. In our derivation[4] we approximated the field at the end of the solenoid according to N.D. Edwards and B. Rose[5]. Actually these matrices are not exactly unitary so that the spin vector has to be normalized at the end of the computation.

A comparison of these matrices with those used for beam optics shows that trajectory focusing terms have their analogues in the spin matrices, that is exactly the same terms multiplied by the factor $G = \left(1 + (\frac{g}{2} - 1)\gamma\right)$. This is obvious in quadrupole lens or entrance path of a bending magnet.

Furthermore in the horizontal bending magnet the t_{13} term $(= -\sin\omega_2)$ corresponds to the expected mean spin rotation around the y-axis, including a focalization effect X·G. The rotations around the x-axis (t_{23}-term) or z-axis (t_{12}-term) are proportional to $\omega_3 \sim (\frac{g}{2} - 1)(\gamma - 1)$, and thus differ from zero for relativistic particles only. These terms are produced by the fact that the magnetic field in the reference frame of the particle is different from that of the laboratory system. For a proton of 600 MeV, the quantities $(\gamma-1)X'$ or $(\gamma-1)Y'$ are small and thus square terms like $((\gamma-1)X')^2$ are neglected in the given table. However, in the detailed derivation[4] of these matrices we include such terms which may have importance at higher energies.

Similar considerations can be made for the homogeneous part of the solenoid. The same approximations to the relativistic terms have been used.

Depolarization effects for the entire beam transport system may now be evaluated using a Monte-Carlo method. Here the initial beam is generated, i.e. trajectories and spin directions, and then propagated particle after particle through the whole system.

In the case where this formalism is used to evaluate small depolarization effects, a different approach may be more convenient. Writing $\vec{S} = \vec{S}_{av} + \Delta\vec{S}$ we may consider $\vec{S}_{av}$ as the mean polarization of the beam and treat $\Delta\vec{S}$ as additional coordinates. Neglecting second order terms like $\Delta\vec{S}\cdot X$, $\Delta\vec{S}\cdot Y$, ..., the spin computation may be included in an extended optical matrix.

$$\begin{pmatrix} \vec{P} \\ \\ \Delta\vec{S} \end{pmatrix}_{\text{out}} = \left(\begin{array}{c} \text{conventional} \qquad 0 \\ \text{optics} \\ \hline \\ \text{spin} \end{array}\right) \cdot \begin{pmatrix} \vec{P} \\ \\ \Delta\vec{S} \end{pmatrix}_{\text{in}}$$

where the elements of the spin part are depending on $\vec{S}_{av}$.

REFERENCES

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(2) K.L. Brown. SLAC report No 75 (1969)

(3) G. Besnier - CERN - 70-11

(4) D. Rapin. Internal report - University of Geneva (1972)

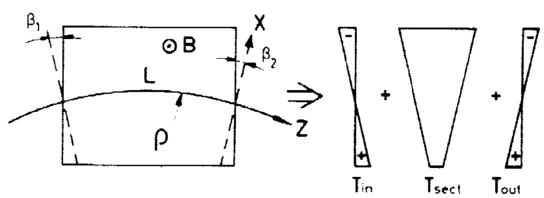
(5) N.D. Edwards and B. Rose. Nucl. Instr. Methods 7 (1960) 135

TRANSFER MATRIX ELEMENTS

Quadrupole

$$T_q = \begin{pmatrix} 1 & 0 & G(X'_L - X'_0) \\ 0 & 1 & G(Y'_L - Y'_0) \\ -G(X'_L - X'_0) & -G(Y'_L - Y'_0) & 1 \end{pmatrix}$$

Horizontal bending Magnet



$$T_{\substack{in\\out}} = \begin{pmatrix} 1 & \pm\frac{g}{2}\frac{Y}{\rho} & G\frac{X}{\rho}\,\mathrm{tg}\,\beta \\ \mp\frac{g}{2}\frac{Y}{\rho} & 1 & -G\frac{Y}{\rho}\,\mathrm{tg}\,\beta \\ -G\frac{X}{\rho}\,\mathrm{tg}\,\beta & G\frac{Y}{\rho}\,\mathrm{tg}\,\beta & 1 \end{pmatrix}$$

$\beta_1, \beta_2 > 0$ in that case

Bending $\sphericalangle$: $\frac{L}{\rho}$ $\qquad \frac{1}{\rho} = \frac{eB}{p}$

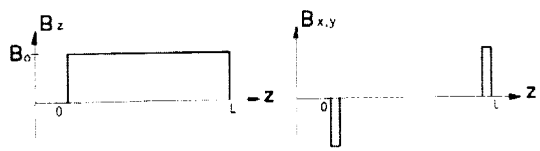
Rotation $\sphericalangle$ of polarization $\frac{L}{\rho}\gamma\,(\frac{g}{2}-1)$

$$\left.\begin{aligned} \omega_2 &= G(X'_0 - X'_L) + \frac{L}{\rho}\gamma\,(\frac{g}{2}-1) \\ \omega_3 &= -(\gamma-1)\,Y'\frac{L}{\rho}(\frac{g}{2}-1) \end{aligned}\right\} |\omega_3| \ll |\omega_2|$$

$$T_{sect} = \begin{pmatrix} \cos\omega_2 & \frac{\omega_3}{\omega_2}\sin\omega_2 & -\sin\omega_2 \\ -\frac{\omega_3}{\omega_2}\sin\omega_2 & 1 & \frac{\omega_3}{\omega_2}(1-\cos\omega_2) \\ \sin\omega_2 & \frac{\omega_3}{\omega_2}(1-\cos\omega_2) & \cos\omega_2 \end{pmatrix}$$

$Y' = Y'_L = Y'_0$ for horizontal sector bending magnet

Solenoid



$$T_{\substack{in\\out}} = \begin{pmatrix} 1 & 0 & \pm G\frac{Y}{2}K \\ 0 & 1 & \mp G\frac{X}{2}K \\ \mp G\frac{Y}{2}K & \pm G\frac{X}{2}K & 1 \end{pmatrix}$$

$K = \frac{eB_o}{p}$

$\theta_p = \frac{g}{2} KL$: rotation $\measuredangle$ of the transverse polarization of the beam

$\theta_L = (\frac{g}{2} - 1) KL$

$S_L = (\gamma - 1) \sin\theta_L$

$C_L = (\gamma - 1)(1 - \cos\theta_L)$

$$T_{homog} = \begin{pmatrix} \cos\theta_p & \sin\theta_p & -X'_L C_L + Y'_L S_L \\ -\sin\theta_p & \cos\theta_p & -X'_L S_L - Y'_L C_L \\ -X'_0 C_L - Y'_0 S_L & X'_0 S_L - Y'_0 C_L & 1 \end{pmatrix}$$

Drift

$$T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

TABLE I

$G = (\frac{g}{2} - 1)\gamma + 1$

L : effective length

$\vec{P} = (X, X', Y, Y')$ coordinates vector of the particle trajectory

$\vec{P}_o$ ($\vec{P}_L$) is defined at the entry (exit) of the section corresponding to the matrix T.