A COMPARISON OF VARIOUS GEOMETRIES OF A SEPARATED-SECTOR CYCLOTRON

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ABSTRACT

Using an analytical approximation for the magnetic field of a separated-sector isochronous cyclotron, the authors have analyzed various geometries of the cyclotron magnet system from the dynamical point of view. The following geometries are considered: a radial-sector magnet system having a constant vertical gap, a magnet having a variable gap and azimuthal sector width, and a system of radial sectors displaced relative to the center of the cyclotron. It is shown that the last case provides acceptable field characteristics and reasonable simplicity of geometry.

INTRODUCTION

Improved accelerated beam characteristics require ever greater precision of magnetic field spatial configuration in isochronous cyclotrons. Although, in general, modelling appears to be an unavoidable stage of field shaping, the volume of work on models may be considerably reduced if the accuracy of preliminary calculations is increased. The main purpose of calculations before modelling is the analysis of various magnet systems and the choice of an optimal geometry from the point of view of particle dynamics and construction considerations.

In the design of separated-sector cyclotrons, there is often used the rough "sharp edge" approximation (for example,[1]) or the somewhat more accurate "one sector" approximation [2]. We have used a more accurate method which includes both aforementioned approximations as particular cases. The advantage of this method is that it allows to reduce (and in some cases even exclude) modelling of the field. Using this method,one can obtain functional relations between magnetic field characteristics and betatron frequencies on the one hand and geometrical parameters of the magnet on the other. It makes possible a rapid comparison of various magnet designs and the determination of optimal pole shapes.

The method is based on conformal mapping; a detailed description and justification of the method has been published [3,4,5]. It allows generalization to the case of sectioned magnets having arbitrary pole excitation [4] and, in addition, taking into account the effect on field shape of current-carrying conductors located near the working region [4,5]. Modelling of the field of a separated-sector cyclotron [5] and a comparison of the calculations with experimental data for a three-sector cyclotron [2] have shown that the field approximation used is quite accurate.

MAGNETIC FIELD APPROXIMATION

Below we shall limit our treatment to the simplest case, when the current-carrying conductors are located sufficiently far from the working region. Under this condition, the vertical component of the magnetic field in the median plane is expressed in implicit form as follows:

$$\frac{\theta}{\theta_g}=\frac{1}{\pi}\left[\arctan\frac{1}{\mu}\sqrt{(1+\mu^2)f^2-\mu^2}+\frac{\mu}{2}\ln\frac{1+\sqrt{(1+\mu^2)f^2-\mu^2}}{1-\sqrt{(1+\mu^2)f^2-\mu^2}}\right] \quad (1)$$

where θ is the azimuthal angle measured from the center of the azimuthal gap between sectors, θ_g the azimuthal gap width and μ a parameter which is the most important characteristic of the field distribution. If θ_g is small one can take $\mu=2h/r\theta_g$ where $2h$ is the vertical gap of the magnet and r the radius. More generally, when θ_g is large and sector edges do not coincide with the radius (Fig.1):

$$\mu=\frac{h\sin\left(\frac{\pi}{N}-\frac{\theta_s}{2}\right)}{r\left(\frac{\pi}{N}-\frac{\theta_s}{2}\right)\sin\left[\frac{\pi}{N}-\frac{\theta_s}{2}+\arcsin\left(\frac{r_c}{r}\sin\frac{\theta_s}{2}\right)\right]} \quad (2)$$

where N is the number of sectors, r the radius measured from the magnet center, θ_s the azimuthal width of the sector and r_c the displacement of the sectors from the center. The azimuthal gap between sectors measured with respect to the common center is

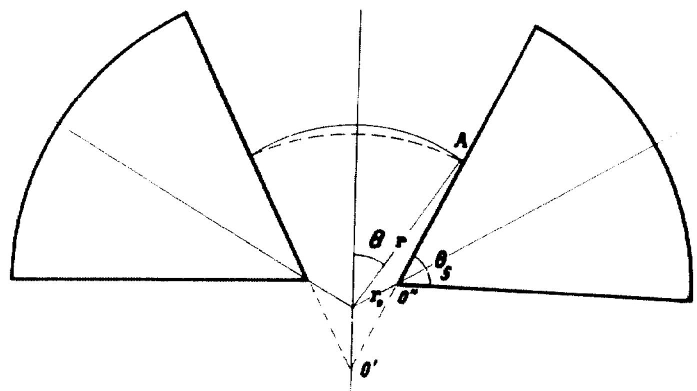

Fig.I. Geometry of sector cyclotron poles for the case when the sector boundaries do not coincide with radius. r -radius measured from the cyclotron center, θ_S -azimuthal width of the sector, r_c -displacement of the sectors relative to the center.

Examples of azimuthal distributions of the field determined by equation (I) are given in Fig.2. The aforementioned approximations are obtained from (I) in two limiting cases: I) $\mu = h = 0$ -"sharp edge" approximation 2) $\mu \to 0$, $\theta_g \to \infty$ (h is finite)- "one sector" approximation.

From equation (I) one can obtain by integration expressions for the constant component (average value) of the field f_c and for the flutter $F = \langle H^2 \rangle / \langle H \rangle^2 - 1$

$$f_o = 1 + \frac{N\theta_g}{2\pi}\left(\frac{\mu}{\pi} \ln \frac{1+\mu^2}{\mu^2} - \frac{2}{\pi} \arctan \frac{1}{\mu}\right) \qquad (4)$$

$$F = \frac{1 - \frac{N\theta_g}{\pi^2} \arctan \frac{1}{\mu}}{f_c^2} - 1 \qquad (5)$$

Eq. (4) and (5) allow calculation of the betatron oscillation frequencies ν_r , ν_z , the required ampere-turns of distributed windings and the determination of the optimal geometry of the magnet. In interesting practical cases, the value f_o is determined to an accuracy of better than I% and for small azimuthal gaps 0.I%.

If one compares (4) and (5) with the "sharp edge"

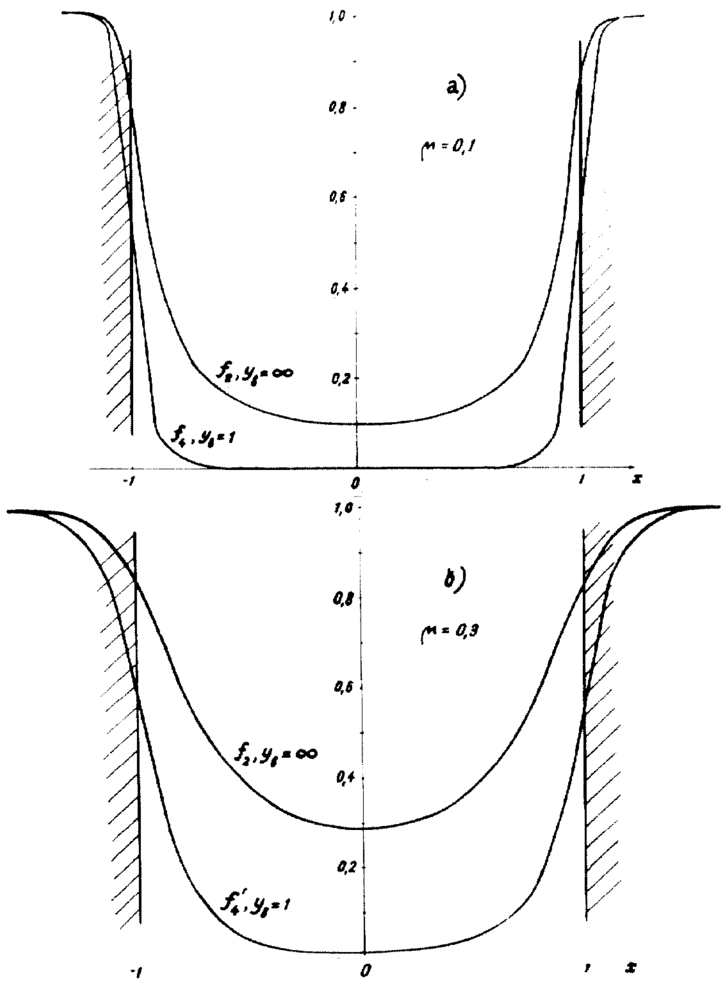

Fig.2. Dependence of field on azimuth $x=\theta/\theta_g$ for two values of parameter μ. The upper curves ($y_\ell=\infty$) are represented by equation (I), valid for the case when current-carrying conductors are located far from the working region (the height above the median plane is more than 1½–2 vertical gaps). The lower curves ($y_\ell = 1$) were obtained by using formulae taking into account the proximity of the conductors [4] and plotted for the case when they are at the edge of the pole.

formulae:

$$f_c = 1 - \frac{N\theta_g}{2\pi} \; ; \quad F = \frac{1}{2\pi/N\theta_g - 1} \; , \tag{6}$$

one can see that approximation (6) gives an error of about 20-30% in the determination of f_c even for small $\mu \simeq 0.1-0.2$, and may err by a factor 1.5-2.0 in the determination of ν_z. Below, using eq.(4) and (5), a comparison is made of several geometries of a separated-sector cyclotron. The calculations are for the parameters of spectrometric cyclotron [6].

RADIAL-SECTOR SYSTEM HAVING A CONSTANT VERTICAL GAP

Such a magnet, whose sectors have constant azimuthal width, is geometrically the simplest and is considered in a number of designs [7,8]. Its distinguishing feature is that the average field decreases with radius instead of increasing as required for isochronism.

In Fig.3 are shown the radial dependencies of f_c, ν_r and ν_z for a magnet having $2h = 10$ cm and $\theta_s = \theta_g$. Without correcting turns, $f_o \simeq 0.7$ at radius $r = 50$ cm and then decreases by 30% as r increases to 300 cm ("sharp edge" approximation gives $f_o = 0.5$ everywhere). At a final proton energy of 80 MeV about 40 per cent of the field must be produced by distributed turns. At initial radii, ν_z is less than 0.5 and then increases, crossing this value. It is necessary to correct ν_z at initial radii or to increase the injection energy.

GENERAL CASE: VARIABLE SECTOR WIDTH AND VARIABLE VERTICAL GAP

Using (4) and (5) one can find a geometry for which f_o and F (and, hence, ν_z) will vary in accordance with a given law. This problem may be reduced to solving at every radius the following transcendental equation with respect to μ

$$f_o(\rho) = \frac{N}{\pi^2} \cdot \frac{r_o}{\mu\rho} \arctan\frac{1}{\mu} \cdot \left\{ \frac{\mu}{\arctan\frac{1}{\mu}} \ln\frac{1+\mu^2}{\mu^2} + \right. \tag{7}$$

$$\left. + \left[1 + \frac{\pi^4}{N^2} \cdot \frac{\mu^2 \rho^2 f_o^2(\rho)\,(F+1)}{r_c^2 \left(\arctan\frac{1}{\mu}\right)^2} \right] - 1 \right\} \; ,$$

($\rho = r/r_e$ and $2\eta = 2h/r_e$ –dimensionless radius and vertical gap, and η_e and r_e are constant) and then calculating from μ the corresponding sector width

$$\theta_s(\rho) = \frac{\pi}{2}\left\{1 - \frac{\pi}{\arctan\frac{1}{\mu}}\left[1 + \left(1 + \frac{\pi^2}{N^2}\cdot\frac{\mu^2\rho^2 f_c^2(\rho)(F+1)}{\eta_0^2\,(\arctan\frac{1}{\mu})^2}\right)^{1/2}\right]^{-1}\right\} \tag{8}$$

and vertical gap

$$\eta(\rho) = \mu\rho\left[\frac{\pi}{N} - \frac{\theta_s(\rho)}{2}\right] \tag{9}$$

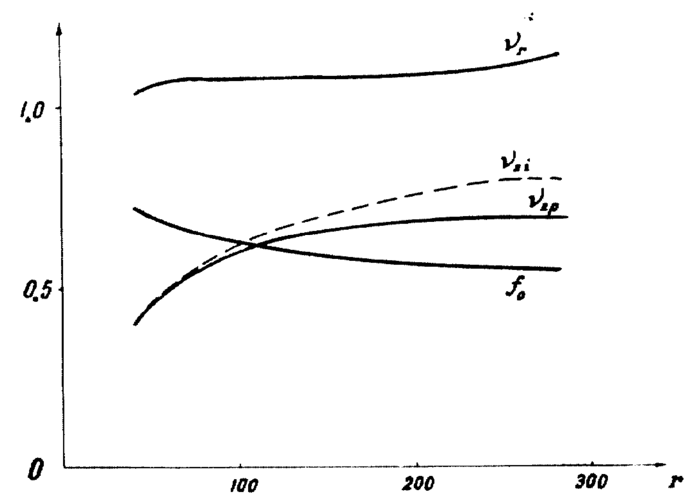

Fig.3. Dependence of average field f_c and betatron frequencies ν_z, ν_r on the radius in a sector cyclotron with $N=4$. Radial – sector system, $2h$ =10cm, $\theta_s=\pi/4$. Subscripts p and i in ν_z signify protons and heavy ions, respectively.

Calculated results are presented in Fig.4. Isochronous $f_c(r)$ for constant F is obtained if the sector width θ_s and the vertical gap 2η vary; this variation must be particularly large in the region of small radii. Beginning at a certain radius, the azimuthal gap remains constant. Since shaping poles in both directions results in additional technological difficulties, it is natural to consider the case when only the sector width varies.

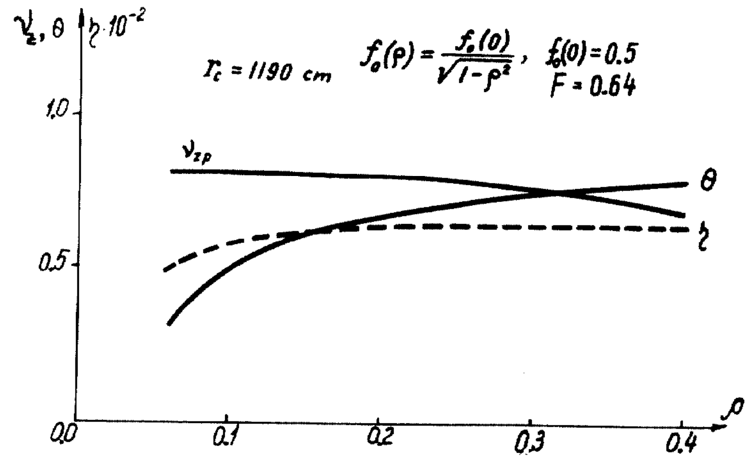

Fig.4. Dependence of azimuthal width θ_s, dimensionless vertical half-gap η and vertical betatron frequency ν_z on the dimensionless radius ρ for isochronous average field $f_c(\rho)$ and constant flutter F =0.64.

CONSTANT VERTICAL GAP, VARIABLE SECTOR WIDTH

The dependence $\theta_s(r)$ is determined from the condition that the average field is isochronous. In such a case, the flutter is not constant; it decreases at large as well as small radii but the decrease of ν_z at small radii is not so abrupt as for constant θ_s. Hence, a sector having such a geometry allows one to obtain a good approximation of the isochronous field and to some degree correct ν_z at small radii.

The realization of a magnet in which the sector width varies strictly in accordance with calculations is hardly expedient, since different laws govern the acceleration of different ions, so that correcting coils are unavoidable for retuning the field. However, the sector width should be as close as possible to the calculated value in order to minimize the field deviations which must be compensated for by correcting windings.

"DISPLACED" SECTORS

As the simplest approximation of the case considered in the previous section, let us consider a sector having straight azimuthal boundaries. In this case, one can satisfy the isochronous condition at two radii. Geometrically, such a method of field shaping may be represented as the displacement of a sector having constant azimuthal width θ_s by a distance r_c from the center of the system (see Fig.I). An analogous method is used in the three-sector cyclotron [2].

If r_1 and r_2 are the radii at which the average field is equal to that of the isochronous, then from (3) we obtain two equations for θ_s and r_c :

$$\frac{r_1}{r_2} = \frac{\sin\frac{\theta_s+\theta_2}{2}}{\sin\frac{\theta_s+\theta_1}{2}} \;; \qquad \frac{r_c}{r_1} = \frac{\sin\frac{\theta_s-\theta_1}{2}}{\sin\theta_s/2} \;, \tag{IO}$$

where $\theta_1 = \theta_s(r_1)$ and $\theta_2 = \theta_s(r_2)$ are determined via $f_0(r_1)$ and $f_0(r_2)$, respectively. Solving these equations, one can determine the parameters θ_s and r_c determining the magnet geometry, and then the magnetic field distribution and its deviation from the "optimal". The results of such calculations are presented in Fig.5. One can see that deviations from the isochronous field

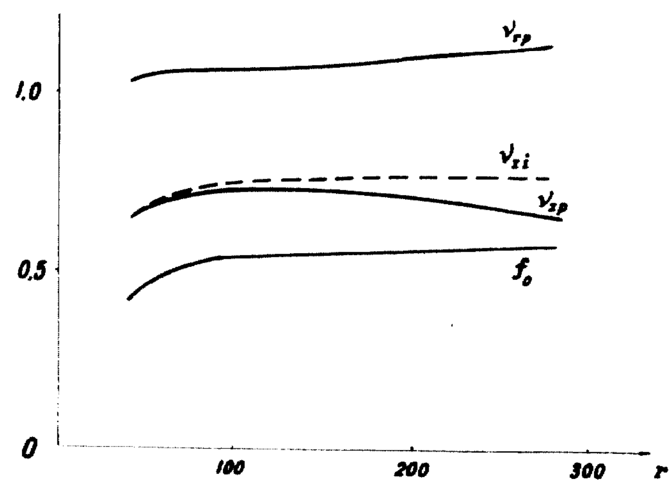

Fig.5. Dependence of average field $f_0(r)$ and betatron frequencies ν_r, ν_z on the radius in a sector cyclotron with N =4. "Displaced" sectors, θ_s =0.9, r_c =30cm $2h$ =IOcm.

and the variation of ν_z with radius are less than for a sector having constant azimuthal width. Hence, a magnet having radially "displaced" sectors has better characteristics than the radial-sector system considered above. Thus, using a magnet having "displaced" sectors seems to be most expedient from the point of view of simple geometry and acceptable field characteristics.

CONCLUSION

The final choice of magnet geometry depends in many respects on the maximum energy and the types of ions being accelerated. To accelerate ions having different charge to mass ratios, e/m, one must change the $f_c(r)$ law and, hence, use a system of retuning turns. In such a case, it is desirable to select a pole geometry providing an isochronous law for some "intermediate" value of e/m.

The field of the selected magnet may be calculated by means of the more accurate formulae [9], taking into account the location of windings.

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