RESONANT DEPOLARIZATION EFFECTS FOR POLARIZED BEAMS IN THE INDIANA UNIVERSITY CYCLOTRON*

P. J. Moffa, A. D. Bacher, W. P. Jones and P. Schwandt
Physics Department, Indiana University, Bloomington, Ind. 47401

ABSTRACT

Resonant depolarization effects have been investigated for axially polarized beams accelerated in cyclotrons of the four-sector, separated-magnet configuration under construction at Indiana University. No intrinsic depolarizing resonances were encountered in the final cyclotron stage for beams of protons, deuterons, and ^{3}He ions. Depolarization probabilities for resonances present in the injector cyclotron stage were determined to be $< 10^{-6}$.

INTRODUCTION

In the course of the past decade, the use of intense beams of polarized ions has proven to be an important and fruitful addition to nuclear reaction studies with energetic particles. Extending these investigations of the spin-dependent features of nuclear interactions to higher energies is of immediate interest and should receive serious attention in the conception and design of accelerator facilities for nuclear physics research.

The Indiana University cyclotron facility can make a considerable contribution to these investigations because of the ease with which polarized beams from external ion sources can be accelerated over a wide range of energies. It is the purpose of the present report to show that polarized beams can be accelerated without significant depolarization in a separated-sector cyclotron of the Indiana design.

In cyclic accelerators employing aximuthally varying magnetic fields, loss of polarization may occur for an initially polarized beam due to a resonance interaction of the magnetic moment of the particle with time-dependent perturbing magnetic fields. Calculations of resonant depolarization have been carried out for more conventional (spiral-ridge), sector-focussed cyclotrons,[1,2] and the results have been reviewed[3] recently. The extension of the formalism[1,4,5] to the case of separated-sector cyclotrons presents no fundamentally new problems. The main differences encountered in the separated-magnet configuration are the substantially larger field gradients which produce larger perturbing fields and larger axial betatron oscillation frequencies. In the following sections we shall show that the characteristic properties of the separated-sector design combine to effectively reduce the depolarization probabilities.

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DEPOLARIZATION RESONANCES

For a particle with spin angular momentum $\vec{s}$ the non-relativistic equation of motion for $\vec{s}$ in the particle rest frame is

$$d\vec{s}/d\tau = G\vec{s} \times \vec{B}^{o} = \vec{s} \times \vec{\omega}_s \qquad (1)$$

where G is the gyromagnetic ratio and $\vec{B}^{o}$ is the applied magnetic field in the particle rest frame. According to (1), the spin precesses about the magnetic field with angular frequency $\vec{\omega}_s = G\vec{B}^{o}$.

For a beam of particles polarized along the z-axis, defined as perpendicular to the cyclotron median plane (plane of symmetry of $\vec{B}$), the torque on the spin vector depends on the radial and azimuthal components B_r and B_θ of the magnetic field. These horizontal perturbing fields are zero in the median plane ($z = 0$). In this plane, particles of constant energy move in an equilibrium orbit with an orbital frequency $\omega_c = qB/mc$, where q and m denote the particle's charge and mass. For small deviations of the particle path from the median plane, B_r and B_θ may be obtained from Taylor series expansions of the median plane field $B_z(r,\theta,z = 0)$ about the equilibrium orbit. Axial and radial betatron oscillations of the particle motion about the equilibrium orbit occur with fundamental frequencies ω_z and ω_r which are conventionally expressed in units of the orbit frequency as $\omega_z = \nu_z\omega_c$, $\omega_r = \nu_r\omega_c$.

If one carries out Fourier expansions of the median plane field B_z and of the axial and radial displacements z and Δr from the equilibrium orbit, one finds a frequency spectrum associated with the horizontal perturbing fields given by

$$\omega_f = n\omega_c + \ell\omega_z + m\omega_r = (n + \ell\nu_z + m\nu_r)\omega_c . \qquad (2)$$

In a cyclotron with perfect four-fold symmetry (i.e., four magnet sectors such as in the IU design) the field harmonic number n is restricted to multiples of 4. The harmonic number ℓ associated with the axial oscillations can take on any non-zero integer value (ℓ = 0 is not allowed by the assumed symmetry of $\vec{B}$ about the median plane) while there is no restriction on the integer values for the radial harmonic number m.

Resonant depolarization may occur when the frequency of the oscillating perturbing fields in the particle rest frame, $\omega_f + \omega_c$, is equal to the spin precession frequency ω_s. Expressing ω_s in terms of the orbit frequency ω_c by $\omega_s = \frac{1}{2}g\omega_c$, where g is related to the gyromagnetic ratio G by $g = (2mc/q)G$, the relativistic condition for resonant depolarization then becomes

$$(\tfrac{1}{2}g - 1)\gamma = n + \ell\nu_z + m\nu_r \qquad (3)$$

where the factor γ results from the Lorentz transformation of the magnetic field from the laboratory system to the particle rest frame.

For small displacements from the equilibrium orbit, the Taylor series expansions of the horizontal perturbing fields can be limited to first-order terms in z and Δr. This leads to linear restoring forces for the particle motion about the equilibrium orbit. Since the azimuthal modulation of the force constant is rapid compared to the betatron oscillation frequencies, it is sufficient to consider only the fundamental harmonic components of the betatron oscillations. In this case ℓ and m are further restricted to the values $\ell = \pm 1$, $m = 0, \pm 1$. For the separated-sector configuration the appreciable deviation from a circular particle path is specifically included in the equilibrium orbit $r(\theta)$ and does not constitute any part of the radial betatron oscillation.

A search was made for possible depolarizing resonances in each of the two cyclotron stages for the entire range of p,d and ^{3}He beam energies. For each case values of ν_z and ν_r were calculated as a function of r for a median-plane magnetic field having an isochronous radial dependence and an azimuthal dependence fitted to measured field shapes. Assuming four-fold symmetry for the magnetic field ($n = 0,4,8....$) no intrinsic resonances are encountered in the final-stage cyclotron. In addition the effect of resonances which might arise from deviations of the magnetic field from perfect four-fold symmetry have been estimated to be small. Each of the particle beams considered were found to encounter a depolarization resonance in the acceleration through the injector stage. Labeled in the notation (n,ℓ,m), these resonances are $(4,-1,-1)$ for protons, $(0,+1,-1)$ for deuterons and $(-4,+1,-1)$ for $^3He^{++}$.

As an example, Figure 1 illustrates the radial dependence of ν_z and ν_r in the injector cyclotron for proton energies from 3 to 15 MeV (corresponding to final-stage energies from 40 to 200 MeV). Since $\gamma(g/2-1) = 1.793$ for non-relativistic protons, the $n = 4$ resonance is seen to occur at the radius where $\nu_z + \nu_r = 2.207$. The radial position of this resonance is virtually independent of the final proton energy

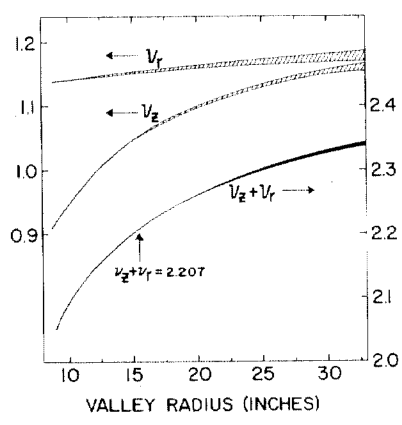

Fig. 1. Radial dependence of ν_z and ν_r for protons in the injector cyclotron (left ordinate). Shaded regions indicate the spread in values for proton energies between 3 and 15 MeV. The corresponding spread in values for $(\nu_z + \nu_r)$, which determines the proton resonance position, is also shown (right ordinate).

DEPOLARIZATION PROBABILITIES

Although the resonance condition (3) was found to be satisfied in the injector stage for each particle type, the occurrence of appreciable depolarization depends on the amplitude of the perturbing fields (resonance strength) and on the time spent near the resonance (resonance width). Under the assumption that the frequency difference $\Delta\omega = \omega_f - \omega_s$ varies linearly with time near a resonance characterized by (n,ℓ,m), the transition probability for a particle with spin $s = \frac{1}{2}$ from the state $m_s = \pm\frac{1}{2}$ to the state $m_s = \mp\frac{1}{2}$ is given[1] by

$$P(\tfrac{1}{2},-\tfrac{1}{2}) = 1 - \exp(-2\pi\, b_n{}^2/\Gamma_{\ell m}). \qquad (4)$$

The quantity b_n is proportional to the amplitude of the nth harmonic component of the perturbing field which in turn depends on both the magnetic field gradients and the amplitudes of the betatron oscillations about the equilibrium orbit. $\Gamma_{\ell m}$ is the time rate of change of the frequency difference $\Delta\omega$ and hence is proportional to the time rate of change of $\ell\nu_z + m\nu_r$. For particles with spin $s = 1$, the probability for a transition between states with different spin projections can be calculated using an expression[6] due to Majorana which relates $P(1,0)$ and $P(1,-1)$ to the value of $P(\frac{1}{2},-\frac{1}{2})$ that one would obtain assuming the particles to have spin $s = \frac{1}{2}$.

Table I gives the calculated depolarization probabilities $P(m,m')$ for particles in the injector cyclotron under the assumption of betatron oscillation amplitudes of 0.25 cm and a uniform energy gain per turn for a total of 100 turns. Here the resonance strength and resonance width are presented in dimensionless combination with the orbit period T_c. For small depolarization, b_nT_c gives an average value for $[P(m,m')]^{1/2}$ per turn while the resonance width as given represents the number of cyclotron turns required to accomplish 90% of the total depolarization. As presented, these quantities, and hence the depolarization probabilities, are independent of the final beam energy, as one would expect for the non-relativistic particle energies encountered in the injector stage.

Table I Summary of depolarization probabilities

Particle	Resonance Strength b_nT_c	Resonance Width $(2\pi/\Gamma_{\ell m}T_c{}^2)^{\frac{1}{2}}$	Depolarization Probability $P(\frac{1}{2},-\frac{1}{2})$	
p	4.25×10^{-5}	14	3.5×10^{-7}	
$^3He^{++}$	1.75×10^{-4}	7.5	1.75×10^{-6}	
			P(1,0)	P(1,1)
d	3.5×10^{-6}	11	3.0×10^{-9}	2.25×10^{-18}

DISCUSSION

We have considered the acceleration process for polarized beams of protons (40-200 MeV), ^{3}He (30-300 MeV), and deuterons (30-130 MeV) in both cyclotron stages of the Indiana facility. In the injector stage intrinsic depolarizing resonances (arising from azimuthally varying fields with perfect four-fold symmetry) have been located for each beam. The depolarization probabilities have been found to be negligibly small in spite of the fact that the resonance width typically involves one-tenth of the total number of turns and the azimuthal field gradients which give rise to the perturbing fields are quite large near the sector-magnet boundaries. The smallness of the depolarization probabilities can be attributed to two effects. First, the azimuthal perturbing field B_θ is more sharply localized in the separated-sector design than in conventional sector-focussed cyclotrons, with the result that the lower-order Fourier components ($n = 0,4$) of B_θ may actually be smaller. Second, the amplitudes of the betatron oscillations can be considerably smaller in the separated-sector design than in conventional cyclotrons. This latter property has been verified by initial studies of the beam dynamics in the injector stage.

No intrinsic depolarizing resonances are found for the same beams in the main-stage cyclotron. Estimates of the effects of depolarizing resonances which might arise from departures of the magnetic field from its assumed four-fold symmetry indicate that these imperfection resonances can also be neglected.

These results[7] show that a four-sector cyclotron of the Indiana design is well-suited to the acceleration of a variety of polarized beams. Other effects which can lead to a degrading of the beam polarization have been considered. Depolarization induced by the electric field acting on the beam at each dee crossing has been estimated to be even smaller than that due to the perturbing magnetic fields. Close attention is being paid to the design of the inflection and extraction apparatus for each cyclotron stage as well as to the magnetic elements in the beam line in order to insure that the entire facility will be well-matched to an experimental program employing polarized particles as a nuclear probe.

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