THE EFFECT OF CERTAIN MAGNETIC IMPERFECTIONS ON THE BEAM QUALITY IN TRIUMF

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ABSTRACT

This paper discusses the tolerances imposed on certain imperfections in the TRIUMF magnetic field by the resonances $\nu_x = 1/1$, 2/2, 3/2 and 6/4, $\nu_z = 1/2$ and $\nu_x - \nu_z = 1$, and by the height of the equilibrium orbit. A method of calculating harmonic coil settings to centre a beam of wide phase width is described.

INTRODUCTION

The permissible amplitudes of the radial betatron oscillations are determined by the mode of cyclotron operation required.[1] At 30 MeV, where slits may be installed, separated turn extraction requires an amplitude less than 0.03 in., standard operation (±600 keV energy resolution at 500 MeV) requires amplitudes less than 0.14 in., and the mode where the largest possible duty factor is more important than energy resolution requires amplitudes less than about 0.40 in. The latter limit is set by the acceptance of the beam transport system.[2] The limitation on the vertical amplitude at extraction is about 0.4 in. and is set by the external beam transport system also.

Fig. 1 shows the variation of the working point (ν_x, ν_z) with energy for one of the 1:10 model magnet fields studied together with several low order resonance lines. This paper reports some of the calculations that we have made concerning the effect of some of these resonances on the beam quality. All calculations were made assuming the fundamental component only of the radio-frequency to be present.

RADIAL MOTION

$\underline{\nu_x = 1/1}$. It has been shown[3,4] that TRIUMF is very sensitive to a first harmonic component in the magnetic field. In the region between 40 in. and 80 in., where conditions are not adiabatic, a component of 0.1 G can give rise to a coherent radial betatron oscillation amplitude of 0.1 in. In this region we have three sets of harmonic coils which can be powered to produce a first harmonic component of varying amplitude and phase; the coils are approximately 17 in. apart. It is well known that it is possible to choose a setting for one harmonic coil so that a beam of narrow phase width will find itself on centre when entering the adiabatic region.

In the case of beams of large phase width, ions of different phase make a different number of turns under the influence of any residual first harmonic component left in the magnet after shimming and end up with different displacements from their ideal centre or quasi-fixed point (QFP).[5] We have evolved the following procedure to enable us to choose settings for three harmonic coil sets to minimize the average displacement from the centre for beams of wide phase width. It is assumed that it is possible to measure the absolute position in (R,p_R) space of a beam of ions of given RF phase to an accuracy of, say, 0.05 in.

For a particle with given phase and energy we describe the displacement from the QFP that results when one set of these harmonic coils is excited, by a vector $\vec{E}$, whose amplitude is linearly proportional to the strength of the first harmonic produced by the coils and where phase is shifted by a constant amount with respect to the first harmonic phase. If the first harmonic is denoted by a vector $\vec{A}$, then

$$\vec{E} = \Gamma \vec{A} \tag{1}$$

where the elements of Γ are called the coil coefficients. This linear relationship holds for 0 deg phase, any non-linearities in the magnetic field are unimportant for displacement less than a few inches; however, longitudinal $(R-\phi)$ coupling can cause deviations from linearity for other phases. For a phase of 25 deg the deviation is about 10% and at the present time this is ignored.

The coil coefficients are obtained by running ions of a given phase from injection to 35 MeV, well into the adiabatic region, using the general orbit codes PINWHEEL and GOBLIN.[6] Each run had a known first harmonic field superimposed from one of the coils. The resulting displacement from the QFP was determined at several energies; hence the coil coefficients could be calculated for this phase, coil and energy. The coefficients were calculated for three phases, 0, 25 and 40 deg. A simulation test was made by superimposing a bump, supposed to be the first harmonic component left after shimming, on the field, and the code was run again to find the values of $\vec{E}$ due to this imperfection for the same phases and energies for which coil coefficients were calculated. In practice these values of $\vec{E}$ would be measured experimentally. We then calculated the values of $\vec{A}$ required to minimize

$$\sum_{I} W(I) \left(E(I) + \sum_{J} \Gamma(I,J)\ A(J)\right)^2. \tag{2}$$

The W(I) are weighting factors, the energies in the adiabatic region being weighted more heavily. The problem was somewhat ill conditioned so more displacements were calculated (I) than there were free parameters (J).

By this means coil settings were found to reduce the displacements due to a simulated first harmonic error of 2 G, which is much larger than anticipated, from a maximum of 2.0 in. to a maximum of 0.14 in. for a beam of 40 deg phase width. The same technique has been used to centre to 0.04 in. a beam that was offset due to phase-dependent electrical forces. If calculations were to be made with a third harmonic component of RF added, or a different magnetic field used, then new coil coefficients should be calculated.

$\nu_x = 2/2$. Radial instability can be caused by the presence of a second harmonic field component $B_2(R)$ with a radial gradient. Calculations made with the equilibrium orbit code CYCLOP[6] are in agreement with the predictions of Hagedoorn and Verster[7] and show that a tolerance of $\partial B_2/\partial R < 0.5$ G/in. and $B_2 < 5.0$ G should restrict instability to the first turn and that the effect on accelerated orbits would be negligible. The effect of a large second harmonic in TRIUMF is discussed in Ref. 8.

$\nu_x = 6/4$. This is an intrinsic resonance in TRIUMF occurring at about 435 MeV. A static phase plot, made 10 MeV below the resonance, is shown in Fig. 2. The motion is stable inside the approximate quadrilateral defined by the four unstable fixed points; the region of linear motion, where the particles precess around an ellipse, extends over about one-third of the stable region. As the energy changes the size of the stable region also changes, shrinking to zero on the resonance; the behaviour of the four fixed points is shown in Fig. 3. For this particular model field the maximum stable amplitude is smaller than 0.5 in. over a region corresponding to about five turns; the beam traverses this region so rapidly that no distortion takes place.

$\nu_x = 3/2$. This resonance is, in first order, driven by a third harmonic imperfection with a radial gradient. The action of a third harmonic gradient is to render the radial motion unstable, with flow lines running in two opposite directions in phase space and the rate of flow being amplitude dependent. Thus the presence of this imperfection tends to stretch the phase space ellipse occupied by the beam.

We have made calculations using a third harmonic bump with three free parameters, the radial gradient G_3, the phase and bump starting radius. Calculations of the influence of the three parameters on the width of the stop band and the rate of growth of amplitude were in agreement with previous studies.[9]

Using the phase angle that gave the largest instability and a bump that started 10 in. before the resonance, we accelerated particles from well below the resonance to well above it. The effect on the emittance of a radial gradient G_3 of 0.4 G/in. is contrasted with the unperturbed case in Fig. 4 at an energy above the resonance. The stretched emittance rotates about the origin of the (x,p_x) axes and is enclosed by the dotted ellipse. The phenomenon of precessional mixing will cause the dotted ellipse enclosing the stretched emittance to be the effective emittance presented to an extraction mechanism for beams of large phase width.

It can be seen from Fig. 4 that the effective emittance area is increased by a factor 5.5 for a third harmonic gradient of 0.4 G/in.; a gradient of 0.1 G/in. increases the area by a factor of 0.3, and the tolerance is set at the latter figure.

VERTICAL MOTION

Equilibrium Orbit Displacement. One can show, using the smooth approximation, that if a horizontal field component $B_R(\theta)$ of the form

$$B_R(\theta) = \overline{B}_R + \sum_n \left(H_R^n \cos n\theta + G_R^n \sin n\theta\right) \tag{3}$$

is present everywhere in the cyclotron, the particles oscillate about an equilibrium orbit displaced

$$z_{eo}(\theta) = \frac{R}{\overline{B}_z} \left(\frac{\overline{B}_R}{\nu_z^2} + \sum_n \frac{1}{(\nu_z^2 - n^2)} \left(H_R^n \cos n\theta + G_R^n \sin n\theta\right)\right) \tag{4}$$

above its position in the absence of such a field B_R. If ν_z is not close to an integer value, the displacement is largely determined by the average field $\overline{B}_R$. The code CYCLOP has been modified to search for equilibrium orbits in (z,p_z) space as well as (R,p_R) space and gives results that agree closely with the predictions of Eq. (4) at the centre of the cyclotron. In regions of large spiral (4) is still a useful guide.

With a large radius and a low magnetic field TRIUMF is sensitive to errors of the type $\overline{B}_R$, especially at radii smaller than 60 in. where the motion is not adiabatic and a coherent oscillation can be developed. However, we estimate that an asymmetric excitation of 50 At, less than 20% of capacity, of all trim coils together will give a correcting field $\overline{B}_R$ of about 3 to 4 G.

$\nu_z = 1/2$. This can be driven by the gradient of a first harmonic component of magnetic field. If the coils are excited to their maximum capacity in a first harmonic mode they can produce a gradient G_1 of about 1.0 G/in. CYCLOP calculations indicate that gradients of this magnitude render the vertical motion unstable if $0.45 \leq \nu_z \leq 0.55$. Since it is intended to keep $\nu_z < 0.45$, we expect no problems with this resonance.

$\nu_x - \nu_z = 1$. This resonance was studied by assuming that two sectors, 180 deg apart, were rotated vertically through angles $+\alpha$ and $-\alpha$. Static and accelerated GOBLIN calculations gave results for the maximum rate of growth of the oscillation amplitudes and for the maximum amplitude acquired that were in agreement with the predictions of Joho.[10]

If it is assumed that the condition for separated turns requires that the maximum increase in radial amplitude be less than 0.01 in.

for a beam with vertical amplitude 0.4 in., then α should be less than 0.001 rad. This corresponds to the rotated sectors being 0.35 in. higher at the outside than the inside.

This tolerance, and that for $\nu_x = 3/2$, differs from previously published values[4] because the latter were obtained with a code that had a mispunched DATA statement.

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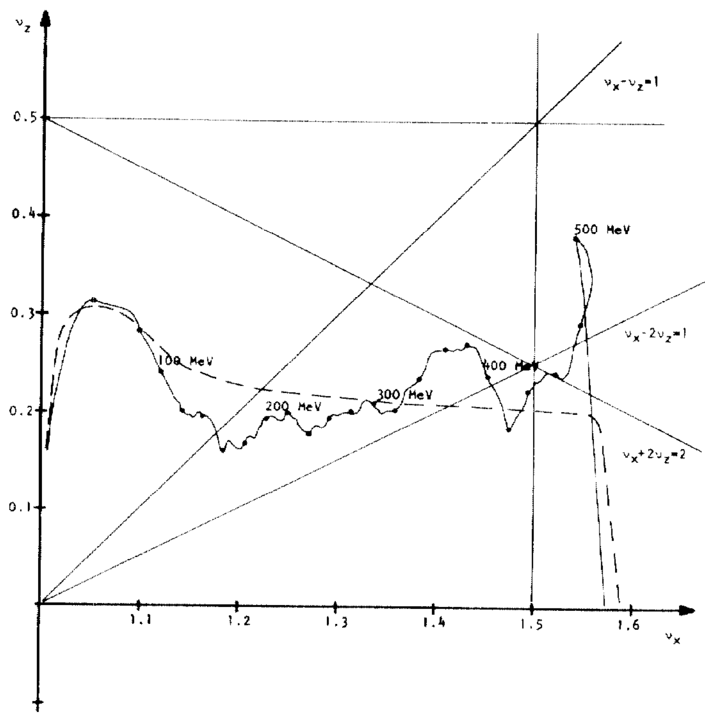

Fig. 1. Plot of ν_x vs ν_z, for field 01/30/70; the dashed line indicates the original design aim.

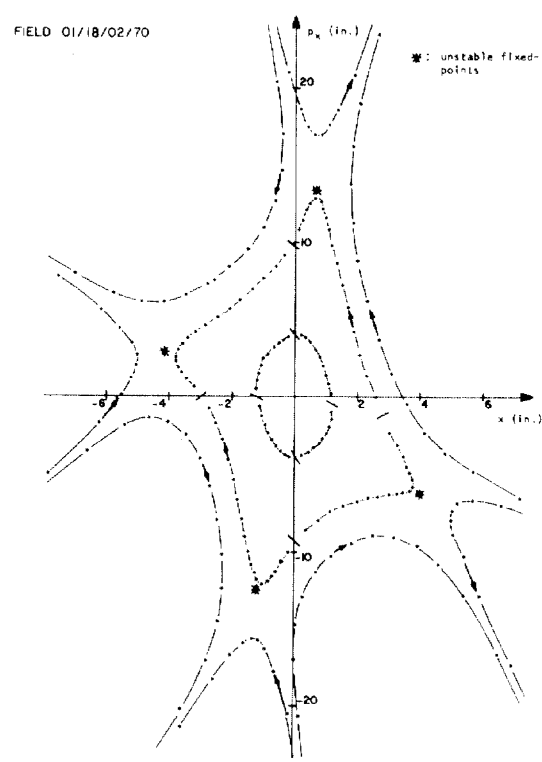

Fig. 2. Phase space plot for 6/4 resonance; ν_x = 1.487, point every second turn.

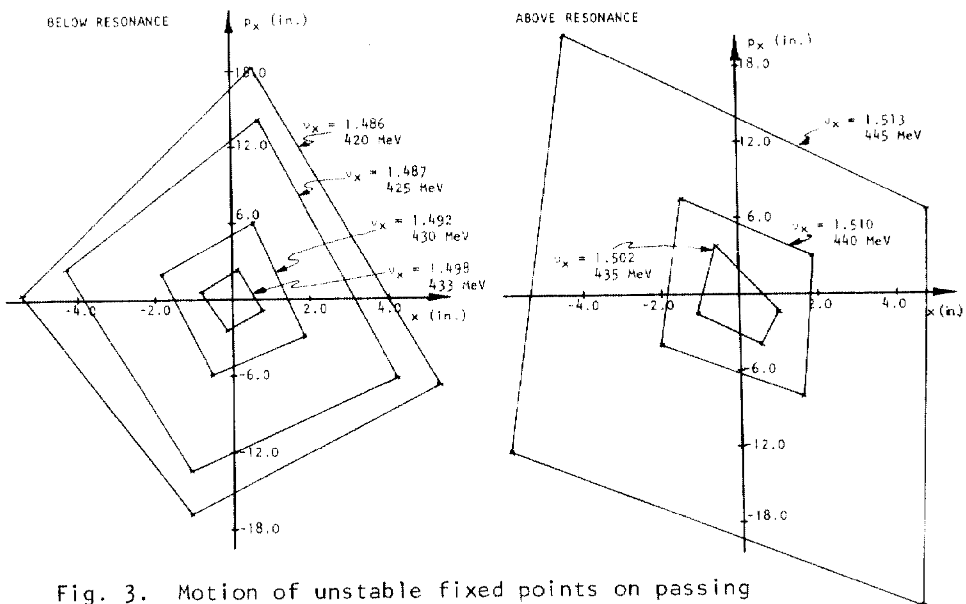

Fig. 3. Motion of unstable fixed points on passing through 6/4 resonance.

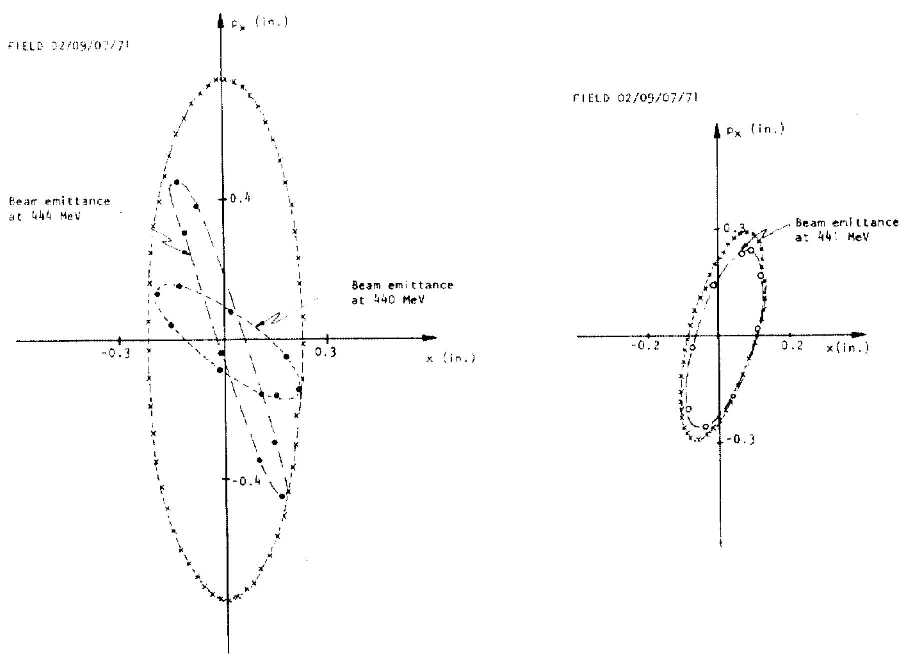

Fig. 4. Effective phase space area occupied by the beam at 441 MeV:
a) when a third harmonic quadrant of 0.4 G/in. amplitude is present,
b) when no third harmonic component is present.