OPTIMIZATION OF THE PHASE ACCEPTANCE OF THE TRIUMF CYCLOTRON

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ABSTRACT

The horizontal and vertical beam behaviour in the TRIUMF cyclotron has been calculated numerically up to 20 MeV. Effects limiting the cyclotron phase acceptance for an extracted beam with good emittance and energy resolution are discussed, as well as ways of overcoming these effects. Two effects which are critical because of their strong phase dependence are vertical electric focusing at the dee gaps and coupling between the radial and longitudinal motions. With only the RF fundamental present it is shown that the second of these effects can be reduced considerably by the use of local bumps in either the average magnetic field or its second harmonic component, or both. The addition of a third harmonic RF component suitably phase shifted from the fundamental results in the phase dependence of both effects being considerably reduced over a wide phase range. For an initial beam emittance of 0.5π in.-mrad and an extracted energy resolution of ±600 keV the net phase acceptance is expected to be ∿30 deg with fundamental RF only and ∿60 deg with 24% third harmonic RF added leading 8 deg in phase.

INTRODUCTION

At TRIUMF we are trying to provide on the one hand a very good energy resolution beam ($\Delta E/E = \pm 3 \times 10^{-4}$) by reducing the phase interval and the oscillation amplitude with a slit system placed near the centre,[1] while on the other hand providing a beam with the widest phase interval and maximum intensity compatible with an energy resolution of $\pm 10^{-3}$ for experiments where large intensities and duty cycles are preferred.[1,2] The addition of a third harmonic component to the fundamental RF[3] should for the first case allow separated turn extraction with reasonable tolerances on the magnetic field stability, and for the second case allow a wider phase range to be accepted if the proper amplitude and phase shift with respect to the fundamental are chosen. Since the third harmonic may present practical difficulties, central region studies and optimization have been done for both the "fundamental mode" and the "added third harmonic mode".

Low energy spread, low duty cycle beams have already been obtained with slits in other cyclotrons, and due to the large size of the TRIUMF orbits good results should be obtained, providing the required stability and accuracy specifications are reached for the various machine parameters. On the other hand, a reasonable resolution for a large phase interval, high intensity beam appears to be more difficult to obtain and seems to depend critically on the detailed structures of the electric and magnetic fields in the centre. Therefore, our efforts in the past year have been directed mainly towards achieving a large phase acceptance.

Limits for the phase acceptance interval are set for negative phases by lack of vertical focusing, and for positive phases by energy resolution deterioration due to increased radial centring difficulties[2] and greater emittance stretching caused by radial-longitudinal (r-ϕ) coupling.[4] (Our phase convention is that ions with negative phases enter the dee gap first.) Other factors limiting the phase range could arise from vertical dee misalignments or from first harmonic errors in the magnetic field. For these factors we assume that, if the tight tolerances cannot be achieved in practice, one can reduce their effects to acceptable values by appropriate corrective means.[5,6] Ways for improving the vertical performance for leading phases and the radial performance for lagging phases will be described below.

BEAM BEHAVIOUR USING RF FUNDAMENTAL ONLY AND AN ISOCHRONOUS MAGNETIC FIELD

Fig. I shows the central electrode layout that will be used for the first beam tests. With the injection gap along the dee gap centre-line, good centring is obtained over a wide phase range, provided the field is isochronous and first harmonic errors are kept low.

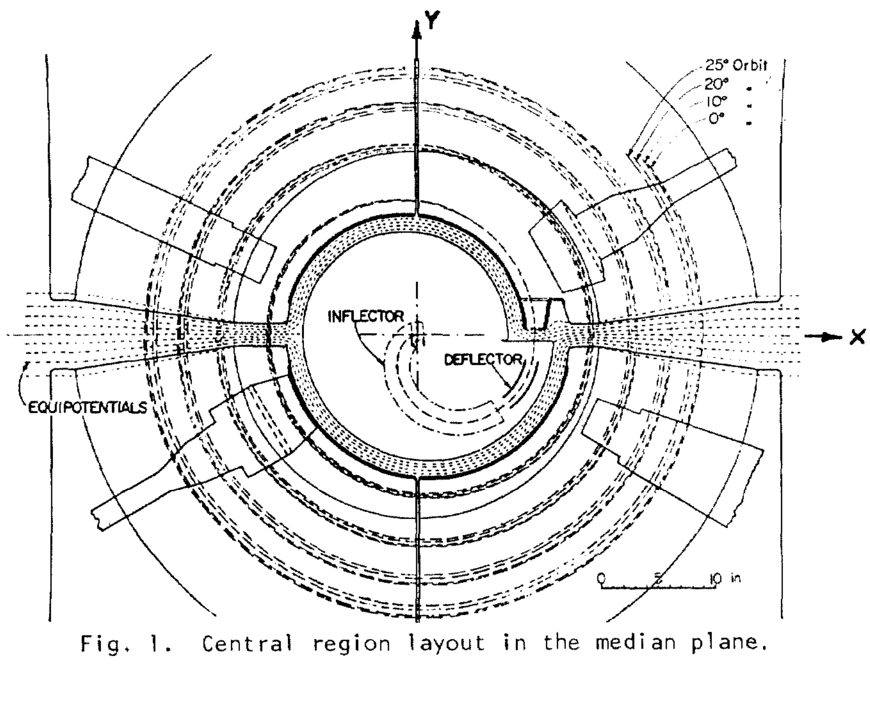

Fig. 1. Central region layout in the median plane.

The equipotential lines shown in Fig. 1 were obtained with program RELAX[7-8] following an iterative dee-shaping procedure whose aim was to eliminate as much as possible any distorting effects due to transverse E_x electric field components, which are generally phase dependent. Trajectories corresponding to 0, 10, 20 and 25 deg phases, obtained with program PINWHEEL[9] assuming fundamental RF only, are also shown and can be seen to spiral outwards without cross-overs and with radial spacings gradually increasing with azimuth, demonstrating good centring.

Small errors in central trajectory centring caused by residual electric E_x distortions have been reduced by using the method proposed by J. Bolduc and G. Mackenzie[6] to correct for magnetic first harmonic errors. In general, centring errors can be kept below 0.04 in., no matter what their cause, provided they are initially below 0.2 in. and one can measure the beam position in the central region within ±0.02 in.

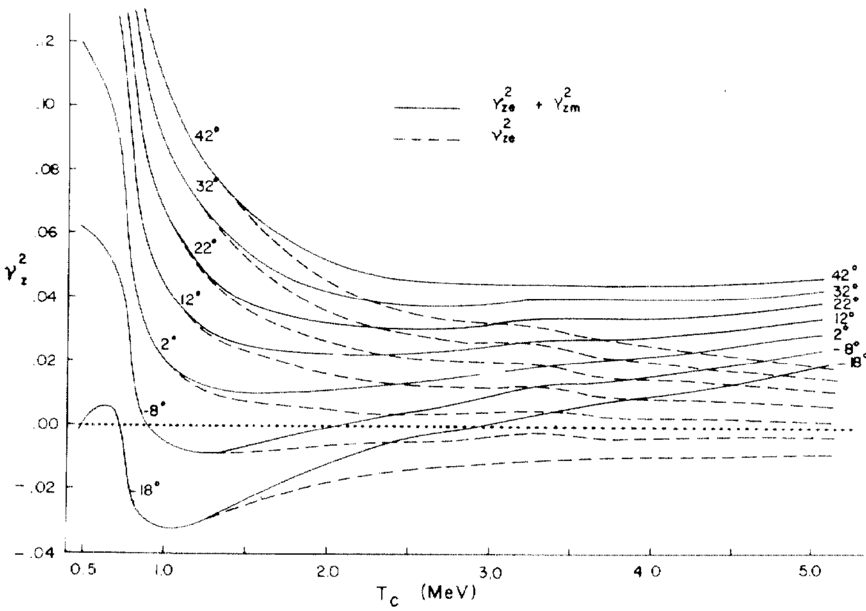

Fig. 2. Magnetic and electric axial focusing vs energy for various RF phases.

For the pure fundamental RF mode of operation the lower phase limit for the present geometry is evident from Fig. 2 where ν_{ze}^2 (produced by the electric field only) and $\nu_{ze}^2 + \nu_{zm}^2$ (magnetic focusing included) are plotted vs radius and phase. The ν_{ze}^2 values were calculated numerically using program TRIWHEEL with three-dimensional potential grid obtained by relaxation calculation. The ν_{zm}^2 values were obtained with the equilibrium code CYCLOPS. The transition phase between vertical focusing and defocusing appears to be about -5 deg, the critical energy being between 1.0 and 1.5 MeV. However, with 4 mA of peak current a space charge contribution to ν_z^2 of about -0.01 would shift this transition phase up to about +2 deg.

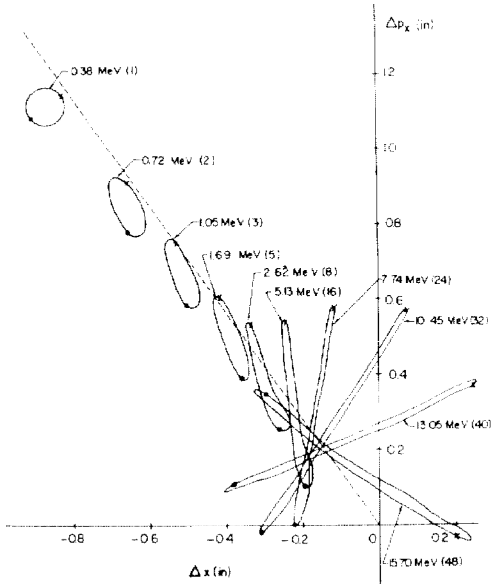

Fig. 3. Shearing of the emittance in the radial phase space at 54.5 deg to the dee gap due to r-ϕ coupling for a phase $\phi = 28$ deg. Particles on the dashed line have centre points along the dee gap.

Fig. 3 illustrates the problems introduced by the phase dependent radial longitudinal (r-ϕ) coupling at phase $\phi = 28$ deg and for an initially circular emittance of 0.016 in.^{2} (corresponding to 0.5π in.-mrad for the 300 keV injected beam). The situation at 20 MeV is illustrated in Fig. 4a for three different phases. The observed phase dependent stretching in the radial phase space[4] is due to the fact that orbits whose centres are initially displaced from the dee gap centre-line suffer a phase oscillation at succeeding dee gaps. The phase oscillations cause different momentum gains with respect to a centred particle at alternate dee gap crossings and, while $\nu_r = 1$, this produces a net drift of the centre points parallel to the dee gap centre-line. The rate of drift depends on the magnitude of the displacement from the dee gap centre-line and the sense depends on the sign of the displacement, resulting in shearing of the ellipse. If y_c is the displacement of the orbit centre from the dee gap centre-line and δx_c the corresponding drift per half-turn parallel to this line and assuming the energy gain ΔT small with respect to the ion energy T, it can be shown that

$$\frac{\delta x_c}{y_c} = \frac{h}{2T} \frac{\partial}{\partial \phi} (\Delta T) . \qquad (1)$$

This effect is particularly important for TRIUMF because fifth harmonic acceleration is used (h=5). One can reduce the eccentricity of the phase space figures at 20 MeV for the larger positive phases by initially accepting the beam in an appropriate stretched phase space area; however, this acceptance generally does not overlap with the corresponding acceptances for other phases and, in general, the overlap becomes smaller as the phase interval gets larger. In our case, with the geometry of Fig. 1 and an isochronous magnetic field, the overlap between acceptances for ±600 keV energy resolution at extraction in the 0 → 28 deg interval has been found to be less than half the expected injected beam emittance and is also difficult to match in practice due to its large eccentricity.

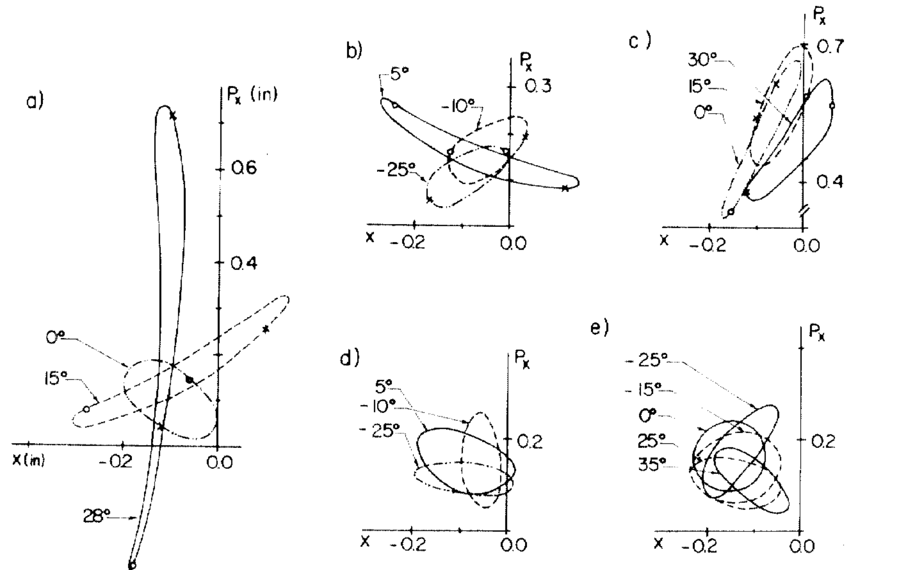

Fig. 4. Phase space figures at 20 MeV. a) Uncorrected case, b), d) non-isochronous magnetic field, c) magnetic second harmonic, e) added third harmonic RF. The initial emittance 0.005π in.^{2} was circular for a), b), c) and was matched to the acceptance overlap for d), e).

INCREASED PHASE ACCEPTANCE USING A NON-ISOCHRONOUS MAGNETIC FIELD

For the fundamental RF mode suitable phase shifts during the first turns can substantially improve the phase acceptance and beam quality. A magnetic field dip before T=1.2 MeV, producing a magnetic defocusing action and having the effect of shifting the particles towards positive phases after injection, prevents the sharp decrease of ν_z^2 in that region (see Fig. 2). For a magnetic field deviating from isochronism, as illustrated in Fig. 5a, the variations of ν_z^2 with radius for ions injected with phases of -25 deg, -10 deg, +5 deg become as shown in Fig. 5b. The phase history obtained for the particle injected at -10 deg is plotted in Fig. 5a. The 30 deg phase shift (Fig. 5a) results in the following advantages: (i) for the same beam quality an additional phase range between -5 deg and -25 deg is accepted; (ii) for this additional negative phase range the r-ϕ coupling partially compensates for the r-ϕ coupling that is to take place from the second to the fifth turn, where the phases are positive, thus giving improved radial behaviour for a 30 deg phase interval.

After about 2 MeV, where the magnetic vertical focusing begins to be adequate, a field bump of about 80 G/in. brings back the 30 deg phase interval toward negative values, thereby partially compensating again for the stretching which occurred from the second to the fifth turn.

From Fig. 4b one sees that the maximum stretching due to r-ϕ coupling anywhere in the 30 deg phase interval has been reduced by a factor larger than 2. Small centring errors introduced by the non-isochronous field have been corrected using the first harmonic coil method.[6] The acceptance overlap for this 30 deg phase width now turns out to be 0.020 in.^{2}, larger than the expected injected beam emittance. By optimizing the shape of the injected emittance (0.016 in.^{2}) to correspond to the acceptance overlap, the size of the phase space figures at 20 MeV and therefore the amplitudes of the radial oscillation are further reduced (Fig. 4d).

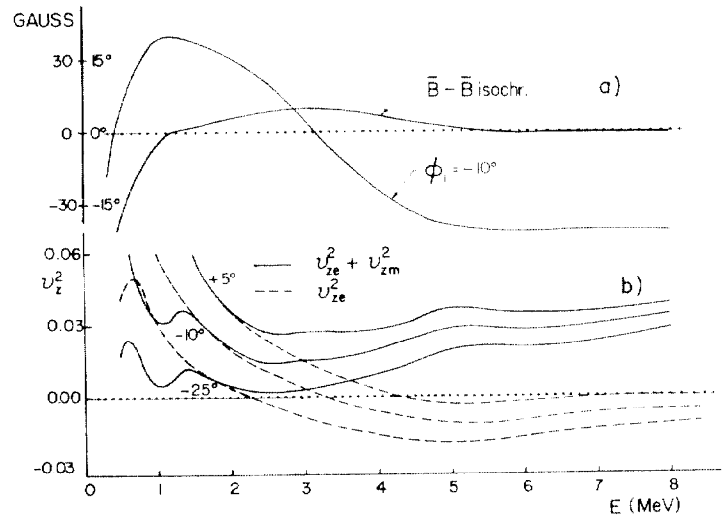

Fig. 5. Variation of ν_z^2 and phase with energy for the non-isochronous field illustrated

INCREASED PHASE ACCEPTANCE USING ADDED THIRD HARMONIC RF

In previous work[2,4] studying the effect of adding third harmonic RF, analytical theories were used to determine the electric vertical focusing, and the program GOBLIN, assuming an instantaneous energy gain at the dee gap, was used to determine the r-ϕ coupling effect. Recent numerical studies have shown that the short dee gap transit time approximation on which the previous studies were based, although successful in predicting results for acceleration with the fundamental RF mode, is not adequate when the third harmonic is added. Due to the large transit time on the first turns (about 60 deg up to 30 in. radius) the rate of variation of the energy gain with phase, on which the vertical electric focusing and r-ϕ coupling largely depend, can no longer be considered as proportional to the derivative of the RF wave form for the case where the latter is no longer a simple cosine wave.

A simply theory, assuming a constant electric E_y field component through the gap (constant gradient approximation), shows improved agreement with numerical results for the added third harmonic mode of operation. Including the contributions due to the phase change, the energy gain, and the z-change through the gap, we obtain for the electric focusing the formula:

$$\nu_z^2 = -\frac{\alpha R_1}{\pi g_e}\left\{\frac{\left[1 - \alpha f(\tau_1)\right] f(\tau_2)}{1 + \frac{\alpha}{\tau_2-\tau_1}\left[g(\tau_2) - g(\tau_1)\right]} - f(\tau_1)\right\} \qquad (2)$$

where $\alpha = qV_d/T_1$, q is the ion charge, V_d the dee voltage, and T_1 and R_1 the initial energy and radius of curvature of the particle before the gap crossing.

Also $f(\tau) \equiv \cos(\tau+\phi) + \varepsilon\cos3(\tau+\phi+\delta)$
$g(\tau) \equiv \sin(\tau+\phi) + \frac{1}{3}\varepsilon\sin3(\tau+\phi+\delta)$.

$\tau_1 = -hg_e/2R_1$ and $\tau_2 = hg_e/2R_2$ are the transit times (in RF degrees) for the first and second halves of the dee gap, ϕ is the particle phase at the centre of the gap, R_2 the radius of curvature after the gap, ε and δ are the fraction and phase shift of the third harmonic component, respectively. If the constant electric field component E_y is set equal to zero outside, then the effective gap width g_e is defined by $g_eE_y(x,0,0) = 2V_d$.

The ν_z^2 values obtained with this simple approach and those obtained with the analytical formulas[2,10] previously used are compared with the values obtained numerically using TRIWHEEL in Figs. 6 and 7. We see that the agreement is reasonably good for both theories with only fundamental RF but that (2) above is superior in the presence of third harmonic.

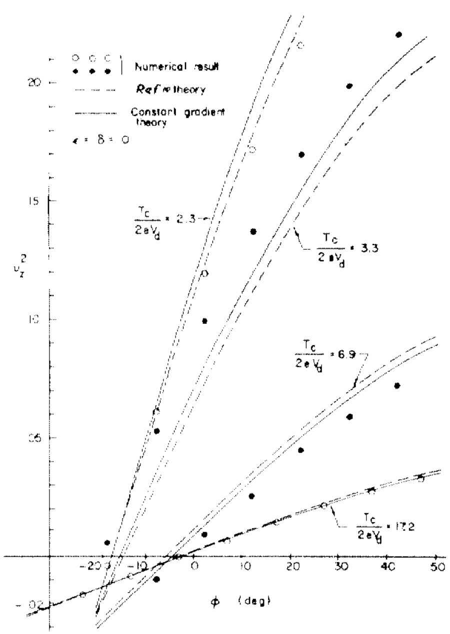

Fig. 6. Phase variation of ν_z^2 at various energies for RF fundamental only

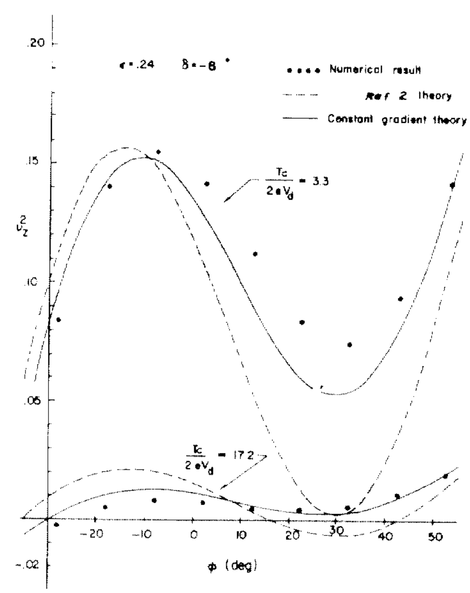

Fig. 7. Phase variation of ν_z^2 at various energies with third harmonic $\varepsilon = 0.24$ $\delta = -8$ deg

In order to evaluate phase acceptance limits for vertical focusing one needs to evaluate ν_z^2 in our case only in the critical low focusing region around R ≈ 20 in. In that region (∿1 MeV) the contribution of the energy change and z-change focusing terms can be neglected relative to the contribution of the phase change term. This phase change term can be shown to be proportional in the first approximation to $T^{-1}(\partial/\partial\phi)\Delta T$ which is also proportional to the r-ϕ coupling effect per turn [see (1) above]. From studies with fundamental RF only, it was found that particles with $\nu_z^2 > 0.02$ are vertically accepted even with a 4 mA peak beam current and that particles with phases below 13 deg (corresponding to $\nu_z^2 \leq 0.04$) would experience an acceptable r-ϕ effect. The optimization of the fraction ε and phase shift δ of the third harmonic to be added to the fundamental was consequently based on the criterion of obtaining $0.02 < \nu_z^2 < 0.04$ around 1 MeV for the widest possible phase interval. The search indicated that $\varepsilon = 0.24$ and $\delta = -8$ deg gave the best solution. Results for ν_z^2 are plotted in Fig. 8 and are compared with results for $\varepsilon = 0$ and $\varepsilon = 0.15$ showing the considerable improvement in phase acceptance with larger values of ε.

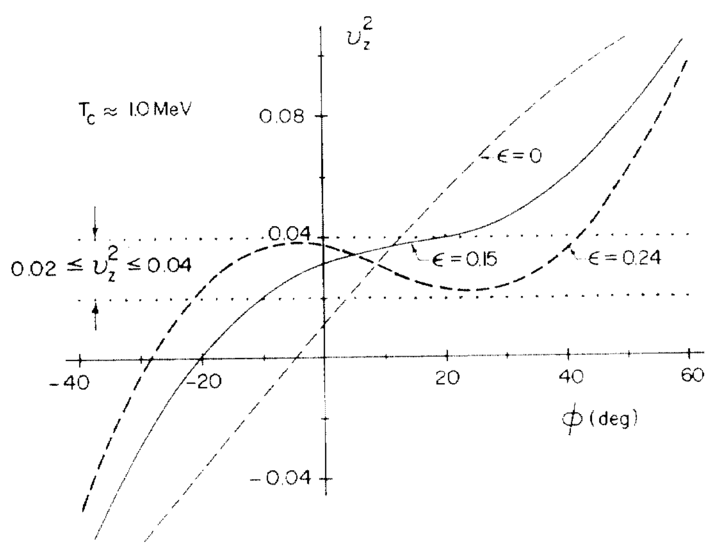

Fig. 8. ν_z^2 obtained numerically for various fractions of third harmonic RF voltage with optimum phase shift δ

Numerically we have checked that particles with phases above -28 deg are vertically accepted. The radial beam behaviour at 20 MeV for an injected beam emittance of 0.5π in.-mrad, stretched in such a way as to minimize the overall amplitude at 20 MeV (see Fig. 4e) is quite satisfactory. The radial acceptance overlap for ±600 keV energy resolution at extraction and a 60 deg wide phase interval is twice the injected beam emittance.

INCREASED PHASE ACCEPTANCE USING A MAGNETIC SECOND HARMONIC

Another method which can usefully reduce the r-ϕ coupling ellipse stretching is the use of a suitable second harmonic in the central magnetic field configuration. It can be shown from Hagedoorn and Verster[11] that the particle's centre point, with co-ordinate (ρ,α), rotates about the origin at a rate

$$\frac{d\alpha}{d\theta} = -\left\{(\nu_r - 1) + \left(\frac{A_2}{2} + \frac{A_2'}{4}\right)\cos 2\alpha + \left(\frac{B_2}{2} + \frac{B_2'}{4}\right)\sin 2\alpha\right\}$$

where θ is the particle's azimuthal angle, A_2 and B_2 are the cosine and sine coefficients of the second harmonic, and the primes denote Rd/dR. It can be seen that a choice of second harmonic phase angle ϕ_2 = 90 deg with respect to the dee gap gives a static flow diagram such as that shown in Fig. 9 where the x-axis lies along the dee gap. For the choice ϕ_2 = 90 deg, B_2 and B_2' are both zero.

The ellipse shown in Fig. 9 has a length $2\Delta x$ along the x-axis and has its centre at $-x_o$ for one half-turn (full ellipse P,Q) and $+x_o$ for the next half-turn (dashed ellipse P',Q'). During the first half-turn the second harmonic will cause the extreme point P to move a distance $\pi(x_o + \Delta x)d\alpha/d\theta$ in the direction of positive p_x. On the next half-turn the same point P' will move $\pi(x_o - \Delta x)d\alpha/d\theta$ in the direction

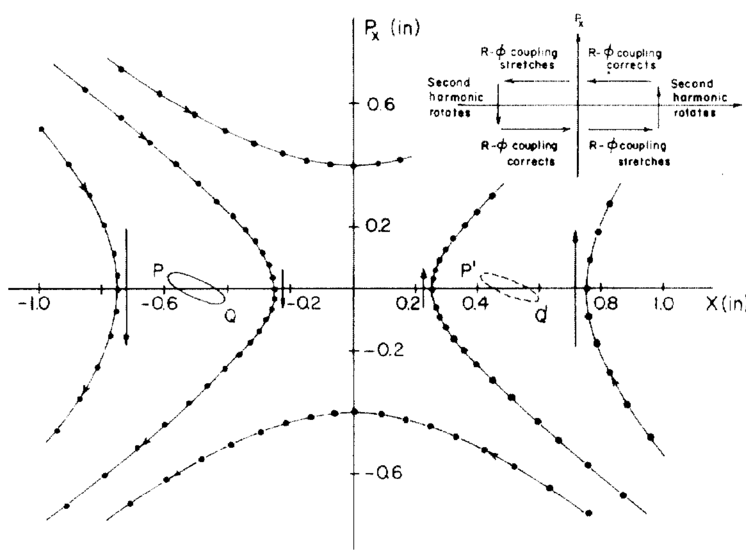

Fig. 9. A static phase plot at 2 MeV with a second harmonic field component. The x-axis lies along the dee gap and ϕ_2 = 90 deg. The inset shows the action of the second harmonic together with the r-ϕ coupling.

of positive p_x. The converse is true for the point Q,Q', and the net effect is to rotate the particles about the centre of the ellipse by an amount $2\pi\Delta x\, d\alpha/d\theta$ ($\Delta x, x_o$ and $d\alpha/d\theta$ are not constant in practice but are functions of half-turn number). The action of the second harmonic, together with r-ϕ coupling. is illustrated in Fig. 9. The r-ϕ coupling tends to stretch the ellipse along the x-axis, while the second harmonic tends to rotate the ellipse so that on later turns the r-ϕ coupling can correct its initial stretching. The processes are not discrete but occur continuously.

These effects balance each other at only one phase, while at other phases one or the other dominates and some stretching occurs; however, the overall effect is a reduction of stretching. This second harmonic phase angle (ϕ_2 = 90 deg to the dee gap) was chosen to improve positive phases. The flow lines are such that the stretching of negative phases is made worse. Fig. 9c shows the improvement in the incoherent oscillation amplitudes made by a second harmonic bump with a gradient of 4 G/in. between 15 in. and 30 in. rising to a peak of 100 G at 30 in. and falling to zero amplitude as $1/R^2$ at larger radii. This latter behaviour keeps $A/2 = -A_2/4$ and $d\alpha/d\theta = -(\nu_r-1)$, producing circular precession paths.

The gradient was discontinued at R $\approx$ 30 in. since here we have sufficient magnetic focusing that the phase slip technique could be applied. The acceptance overlap for a 30 deg phase width at injection is 0.013 in.^{2}, corresponding to 80% of the injected emittance.

INCREASED PHASE ACCEPTANCE USING CURVED DEES

It is worth while to mention briefly a method of increasing the phase acceptance whose study is still in progress. If a particle crosses a region where the electric field equipotential lines are curved, it can be shown that there is an additional focusing term associated with this curvature whose "electrostatic" focal power, provided $\Delta T \ll T$, can be expressed as

$$\frac{1}{f_z} = -\frac{1}{f_x} = \frac{q}{T}\int_{V_1}^{V_2} \frac{d^2y}{dx^2}\bigg|_V dV$$

where y(x) represents the equipotential in the median plane and V_1 and V_2 are the initial and final potential values. By shaping the dee gaps to give equipotentials of 10-12 in. radius of curvature in the horizontal plane near R = 20 in. (where the vertical focusing is critical), we have succeeded in increasing ν_z^2 by 0.015, corresponding to a phase acceptance increase on the negative side of about 10 deg. In addition, an effect similar to that produced by the second harmonic field bump reduces the r-ϕ coupling effect by a factor of two for the positive phases. Studies using this method are continuing as we feel that its full capabilities have not yet been realized.

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DISCUSSION

REGENSTREIF: You said you used a three-dimensional relaxation procedure. What was the mesh width?

CRADDOCK: 1/4 in.

REGENSTREIF: How many points did you consider altogether?

CRADDOCK: Something like a million.

REGENSTREIF: And the computing time?

DUTTO: On the order of 20 min depending on how many times you want to iterate and on the accuracy you need.

CRADDOCK: Right, the program uses a reducing procedure where the calculation is first done on a much larger grid spacing for which many iterations are made. The grid is then divided in two and then in two again until the final spacing is reached.

REGENSTREIF: When you have your final configuration do you use linear equations or non-linear equations computing your orbits and trajectories?

CRADDOCK: In most cases, linear equations.