EFFECTS OF AXIAL MISALIGNMENT OF THE DEES AND THEIR CORRECTION

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ABSTRACT

Axial misalignment of the dees near the centre of a cyclotron can lead to the build-up of large coherent axial oscillations. Numerical computations made for the TRIUMF cyclotron are shown to be in agreement with an analytic treatment of the problem. General formulae are given, applicable to all cyclotrons. In the case of TRIUMF the tolerance allowed on the vertical positions of the dee lips in order to maintain beam quality is ±0.25 mm, quite tight considering the size of the cyclotron.

Various methods of reducing the effects of dee misalignment are considered. For TRIUMF a system of specially shaped electrostatic deflecting plates acting over the first few turns has been proposed; these could relax the misalignment tolerance by a factor four. Other advantages and disadvantages of this scheme are discussed, including its effects on the radial motion.

INTRODUCTION

Axial misalignment of the dees can disturb the axial motion of the beam through two mechanisms. Firstly, the dee-dee capacitance may be altered, resulting in a difference in RF voltage between the upper and lower dee arms and hence an axial electric field across the beam. This effect has been treated elsewhere;[1] in the case of TRIUMF the resulting tolerance on the voltage difference is as small as 0.5% near the centre. The second mechanism, which will be the subject of this paper, is distortion of the electric field. Fig. 1 illustrates the equipotential pattern for axially misaligned dees, obtained by a two-dimensional relaxation calculation; the paths of two ions which either enter or leave in a dee symmetry plane are also shown, with arrows indicating the direction of the electric forces acting on them at various stages. Paradoxically, we see that both ions will suffer a deflection in the opposite sense to the dee misalignment. For other ion paths there is, in addition, the differential focusing effect, but the deflection due to the misalignment turns out to be the same, independent of the axial displacement z.

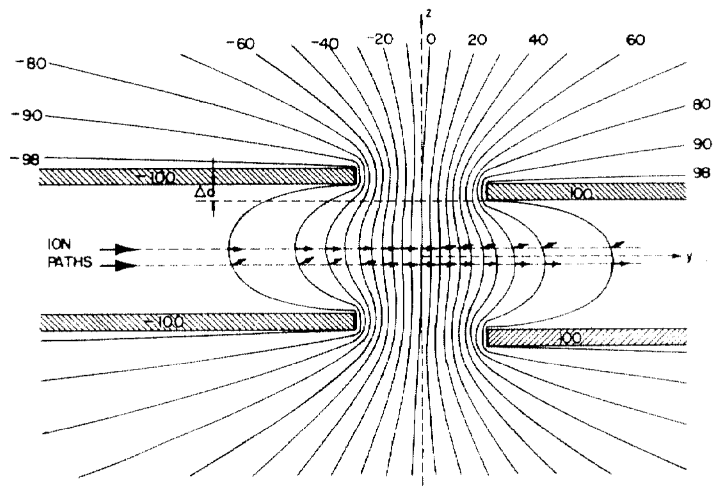

Fig. 1. Effect of the misaligned dees electric field on ions crossing the dee gap.

DEFLECTION AT A SINGLE ACCELERATING GAP

The change in the momentum $\vec{p}$ of an ion of charge q at the dee gap may be written

$$\Delta\vec{p} = q \int_{gap} \vec{E}(x,y,z) \cos \omega t \, dt \tag{1}$$

where $\vec{E}(x,y,z)$ describes the space dependence of the RF electric field (of angular frequency ω); and t is time relative to a moment of peak RF voltage.

To make a theoretical estimate of the deflection, some quantitative knowledge of the effect of the dee misalignment on the electric field $\vec{E}$ is needed. The parallel nature of the central equipotentials near the dee mid-plane (y=0) in Fig. 1 suggests a model in which misalignment of the dees simply shifts the field patterns for the entrance and exit halves of an aligned dee gap along with the dees, the mid-plane y=0 being the slip-plane. Comparison of the true E_z field component with that predicted by the model shows only small differences near the mid-plane. For the TRIUMF dee geometry (see Fig. 2a) the differences integrated across the gap prove to be less than 2%. However, for different dee gap geometries these differences can amount to 20-30% (Fig. 2b).

Making use of the axial shift model for the electric field, replacing E_z by the first term in a Taylor expansion about the symmetry plane for each dee, and changing the independent variable from t to y, the change in z-component of momentum may be written

$$\Delta p_z = q\int_{-\infty}^{0}(z-\tfrac{1}{2}\Delta d)\,\frac{\partial E_z}{\partial z}\cos\omega t\,\frac{dy}{v} + q\int_{0}^{\infty}(z+\tfrac{1}{2}\Delta d)\,\frac{\partial E_z}{\partial z}\cos\omega t\,\frac{dy}{v} \qquad (2)$$

Here we write $dy/dt = v$, the instantaneous speed, neglecting the contribution of transverse components, and extend the integrals to $y = \pm\infty$, neglecting the curvature of the orbit. This expression may be rearranged into two terms, one involving the displacement z and the other the misalignment Δd. Rose and others[2] have evaluated the first of these terms:

$$q\int_{-\infty}^{+\infty} z\,\frac{\partial E_z}{\partial z}\cos\omega t\,\frac{dy}{v} = \frac{zp}{f}\,, \qquad (3a)$$

to obtain the effective thin lens focal length f for the dee gap assuming any change in z within the gap is negligible. The form of the second term:

$$-\frac{q}{2}\,\Delta d\left[\int_{-\infty}^{0} - \int_{0}^{\infty}\right]\frac{\partial E_z}{\partial z}\cos\omega t\,\frac{dy}{v} \qquad (3b)$$

shows that a dee misalignment Δd results in a change in p_z proportional

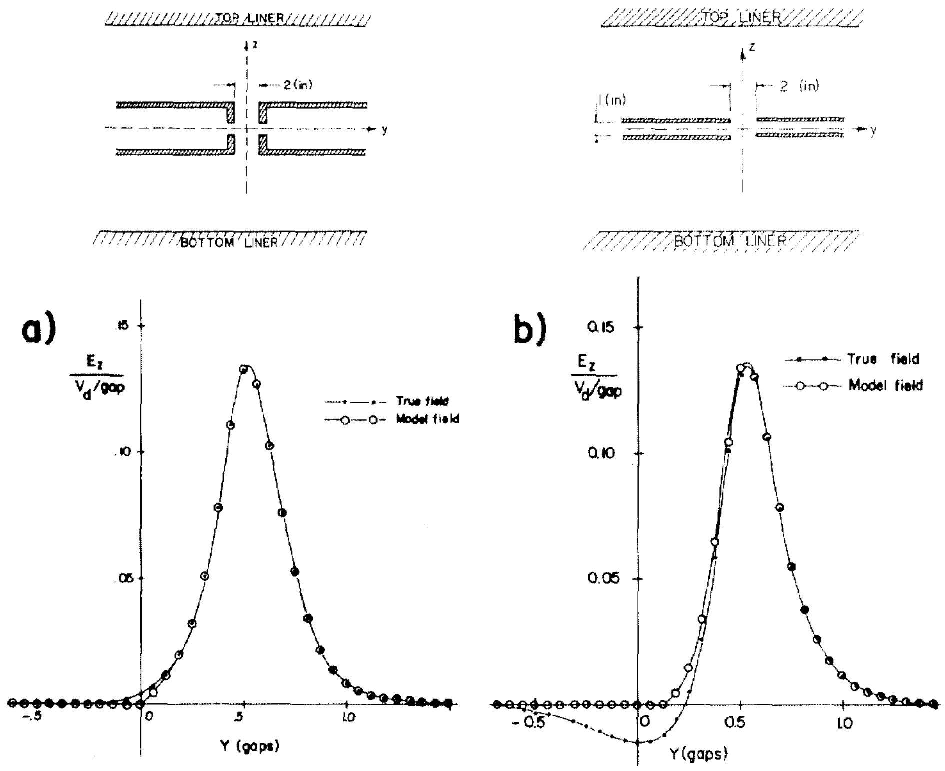

Fig. 2. Electric field component E_z across a misaligned dee gap: a) TRIUMF geometry, b) another geometry

to Δd and independent of z. Assuming that the energy gain across the gap ΔT is small compared to the ion's kinetic energy T_c (at the gap centre), so that its speed v is essentially constant, we can write

$$\omega t = \frac{\omega y}{v} + \phi = \frac{hy}{R} + \phi, \qquad (4)$$

where ϕ is the RF phase when the ion is at y=0, h=5 is the RF harmonic, and R is the mean radius of the equilibrium orbit for kinetic energy T_c. Then with div $\vec{E}$=0, the angular deflection due to axial dee misalignment can be written

$$\Delta\alpha = \frac{q\Delta d}{4T_c}\left[\int_{-\infty}^{0} - \int_{0}^{\infty}\right]\left(\frac{\partial E_y}{\partial y} + \frac{\partial E_x}{\partial x}\right)\cos\left(\frac{hy}{R} + \phi\right)\, dy. \qquad (5)$$

Integrating the $\partial E_y/\partial y$ term by parts and assuming E_y is symmetric and E_x antisymmetric about y=0 (recall that the field values here are to be evaluated on the dee symmetry planes $z=\pm\frac{1}{2}\Delta d$), (5) can be written

$$\Delta\alpha = \frac{q\Delta d}{2T_c}\cos\phi\left[E_y(x,0,0) - \frac{h}{R}\int_0^{\infty} E_y \sin\frac{hy}{R}dy - \int_0^{\infty}\frac{\partial E_x}{\partial x}\cos\frac{hy}{R}dy\right]. \qquad (6)$$

$E_y(x,0,0)$ can be taken from the median plane electric field data. For E_y we make the constant gradient approximation which assumes that E_y is equal to the value at the dee gap centre $[E_y(x,0,0)]$ inside the gap and zero outside. The effective gap width g_e is defined by $g_e E_y(x,0,0) = 2V_d$, V_d being the dee voltage. Then we can write:

$$\Delta\alpha = \frac{q\Delta d}{2T_c}\cos\phi\left[E_y(x,0,0)\cos\frac{hg_e}{2R} - \int_0^{\infty}\frac{\partial E_x}{\partial x}\cos\frac{hy}{R}\,dy\right]. \qquad (7)$$

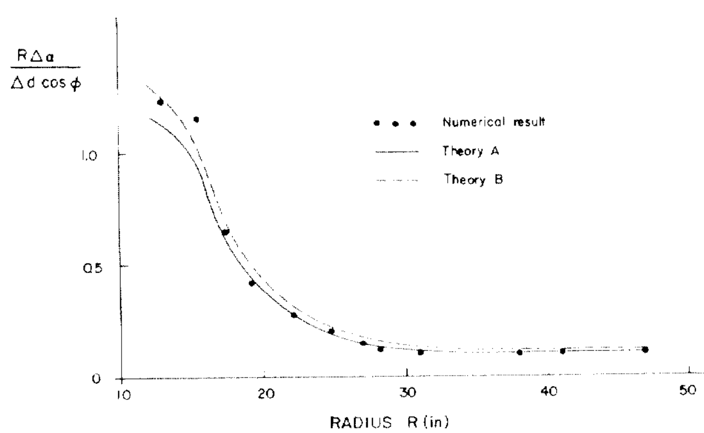

Fig. 3. Vertical impulse as a function of radius for a constant misalignment.

In Fig. 3 curve B gives the behaviour versus radius of the first term in (6) and curve A that of the first term in (7), which differs by the transit time factor $\cos(hg_e/2R)$. A comparison of the curves with the points, which were obtained by numerical integration through the dee gap field, shows good agreement for the first term in (7), demonstrating the importance of the transit time effect. The final term in (6) and (7), associated with non-uniformity of the dee gap, has been evaluated numerically for the TRIUMF tapered gap (see Fig. 9). Its contribution turns out to be <5% everywhere except near 16 in. radius where there is a strong curvature of the equipotentials; there it increases the predicted deflection by up to 30%, in agreement with Fig. 3.

AXIAL PERTURBATIONS OF THE ORBITS

The overall effect of the misalignment deflections occuring at each accelerating gap can only be determined precisely by numerical integration. However, as an aid to understanding, it is helpful to study the effect of the deflections on the unaccelerated equilibrium orbit (EO); this would of course normally be flat in the magnetic median plane (MMP) z=0. We shall restrict ourselves to the case of 180 deg dees with accelerating gaps at $\theta=0$ deg and $\theta=180$ deg on the positive and negative x-axes, respectively. In the case of TRIUMF each dee consists of twenty sections 30 in. wide placed symmetrically on either side of the y-axis. We shall consider only orbits inside a radius of 30 in.; in this region the dee system consists of four independent quadrants, with axial misalignment from the MMP, which we shall label $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ (see Fig. 4a). These may be expressed in terms of four basic symmetry patterns (Fig. 4b-e) with relative misalignments given by

$$\Delta d_s = \tfrac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3 - \Delta_4) \quad \text{"symmetric"}$$
$$\Delta d_a = \tfrac{1}{2}(\Delta_1 - \Delta_2 + \Delta_3 - \Delta_4) \quad \text{"antisymmetric"}$$
$$\Delta d_t = \tfrac{1}{2}(\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4) \quad \text{"transverse"}$$
$$\Delta d_u = \tfrac{1}{4}(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4) \quad \text{"uniform"} \qquad (8)$$

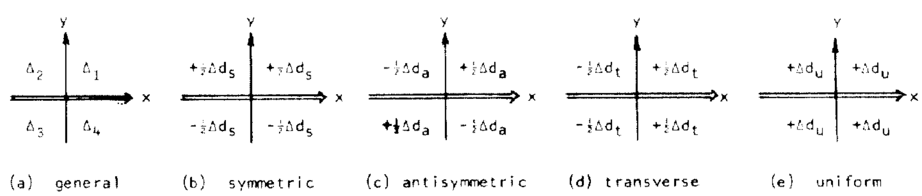

Fig. 4. Schematic representation of misalignments of the four dee quadrants

Symmetric misalignment: In this case ions are deflected in opposite senses at the two accelerating gaps. As a result the EO follows a zigzag vertical path (Fig. 5), crossing the downward deflecting gap at $z=+z_s$ and the upward deflecting gap at $z=-z_s$. It is straightforward to show that

$$z_s = \Delta\alpha \left(\frac{2\nu_{zm}}{R}\cot(\tfrac{1}{2}\pi\nu_{zm}) - \frac{1}{f(\phi)}\right)^{-1} \qquad (9)$$

where ν_{zm} is the vertical tune in the absence of electric focusing at the dee gaps. If $\nu_{zm} \ll 2/\pi$, as is the case for most cyclotron central regions, and if the dee gap focal length $f \gg R$, as is the case for the phase range of interest, then (9) can be written

$$z_s = \tfrac{1}{4}\pi R\Delta\alpha \left[1 - \tfrac{1}{4}\pi^2\left\{\nu_{ze}^2(\phi) + \tfrac{1}{3}\nu_{zm}^2\right\}\right]^{-1}. \qquad (10)$$

Referring back to (7) we see that for the weak focusing conditions holding near $\phi=0$, z_s is relatively independent of phase but drops off sharply with energy or radius; the latter variation is the same as in Fig. 3, where the ordinate gives the approximate value of $z_s/\tfrac{1}{2}\pi\Delta d_s \cos\phi$.

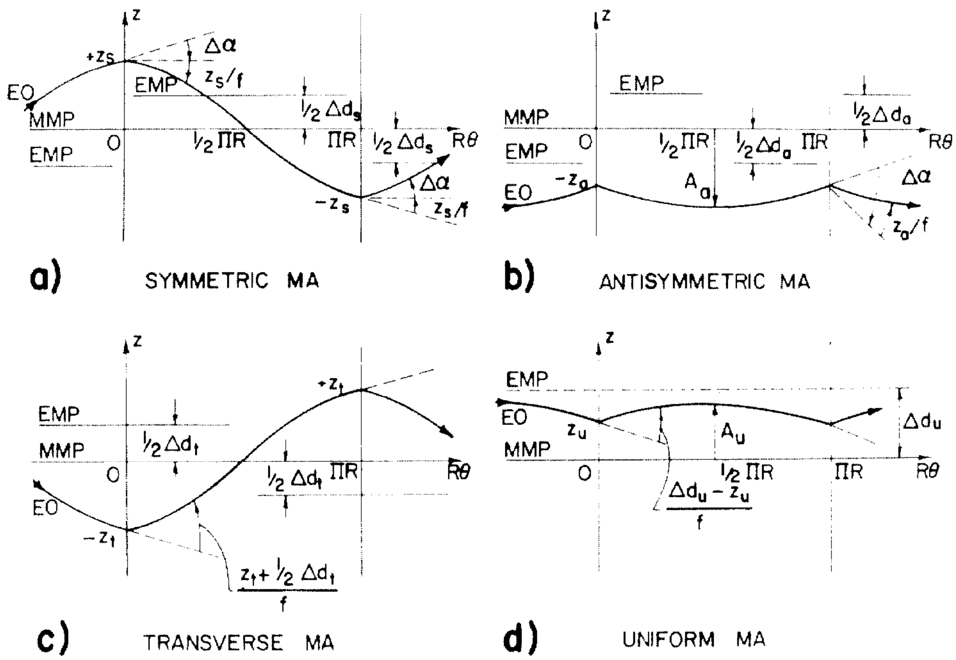

Fig. 5. Vertical equilibrium orbits (EO) for the misalignment patterns of Fig. 4.

Ions off the EO will of course oscillate about it $\nu_z = \sqrt{\nu_{zm}^2 + \nu_{ze}^2}$ times per turn with an amplitude depending on the starting conditions. For instance, an ion injected in the MMP where $\nu_{zm} \simeq 0$ and $\nu_{zi} \equiv \nu_z$ will develop an oscillation amplitude $A_i \approx z_{si}/\tfrac{1}{2}\pi\nu_{zi}$, where z_{si} is the initial value of z_s. Assuming that the axial motion proceeds adiabatically as ν_z changes during acceleration, the amplitude later becomes (non-relativistically)

$$A = z_{si}/\tfrac{1}{2}\pi\sqrt{\nu_{zi}\nu_z}\,. \qquad (11)$$

For a 0 deg phase ion $\nu_{zi}=0.33$ while $\nu_z=0.15$ at 4 MeV, giving $A=4.0\Delta d$. For phases up to 30 deg the equation predicts similar values for A (Fig. 6). Thus the rather weak axial focusing near the centre gives rise to oscillations large compared to the misalignment. Numerical tracking of the ion paths through the measured magnetic and calculated electric grids, using the TRIWHEEL code, confirms the magnitude of this offset (Fig. 6). However, the variation with phase is quite different, presumably indicating some breakdown in the adiabatic assumption.

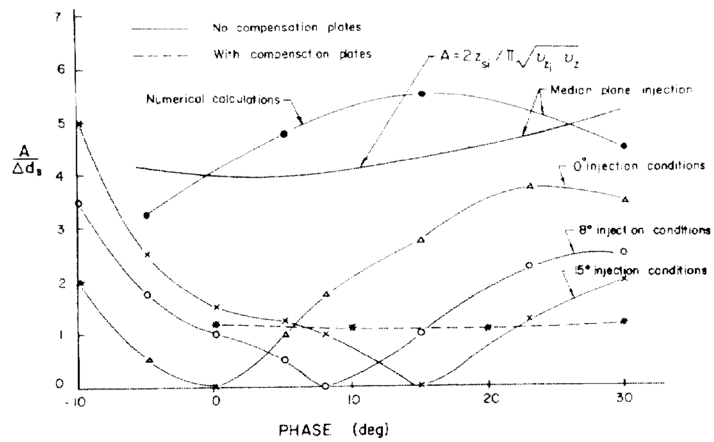

Fig. 6. Coherent vertical amplitude developed at 4 MeV under various injection conditions for a symmetric misalignment Δd_s.

Large oscillations may be avoided through the use of suitable injection conditions. These have been determined numerically by making backward runs from a high energy EO. However, the starting conditions required are strongly phase dependent (because of $\nu_{ze}^2)^3$ and, as can be seen from Fig. 6, the oscillation amplitudes as large as $2\Delta d$ are produced 15 deg to either side of the optimum phase. The incoherent axial oscillation amplitude arising from the injected beam emittance of 0.5π in.mrad at 300 keV amounts to 0.16 in. for $\nu_z=0.2$. Remembering that other imperfections can also lead to axial oscillations, we require the coherent amplitude due to dee misalignment not to exceed 0.1 in.; to satisfy this over a 35 deg phase range requires $\Delta d_s < 0.040$ in.

Antisymmetric misalignment: In this case (Fig. 4c) ions are deflected in the same sense at the two accelerating gaps. This results in a vertical shift of the EO (which remains flat when $\nu_{zm}=0$) so that it crosses the dee gaps (see Fig. 5b) at

$$z_a = \Delta\alpha \left(\frac{2\nu_{zm}}{R}\tan(\tfrac{1}{2}\pi\nu_{zm}) + \frac{1}{f(\phi)}\right)^{-1}. \qquad (12)$$

If $\nu_{zm} << 2/\pi$ and we write $f = R/\pi\nu_{ze}^2$, this becomes

$$z_a = R\Delta\alpha/\pi \left[\nu_{zm}^2 + \nu_{ze}^2(\phi)\right]. \tag{13}$$

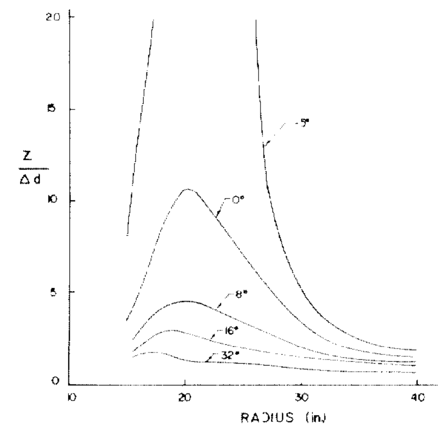

Fig. 7. Displacement of the equilibrium orbit vs radius for an antisymmetric misalignment.

Fig. 7 illustrates how z_a varies with radius for various phases in the case of TRIUMF. For each phase there is a peak displacement of the EO near R=20in. where ν_z^2 has a minimum, while at each radius there is a strong phase dependence, also through ν_z^2.

Ions injected on the MMP will begin an oscillation about the EO with amplitude z_{ai} $\sec(\frac{1}{2}\pi\nu_{zm}) \approx z_{ai}$ (since $\nu_{zm} \approx 0$ at injection) where z_{ai} is the initial value of z_a. Assuming that the axial motion proceeds adiabatically, the amplitude some turns later will be given by $A = z_{ai}\sqrt{\nu_{zi}/\nu_z}$. The values of A obtained from this equation for 4 MeV are plotted versus phase in Fig. 8.

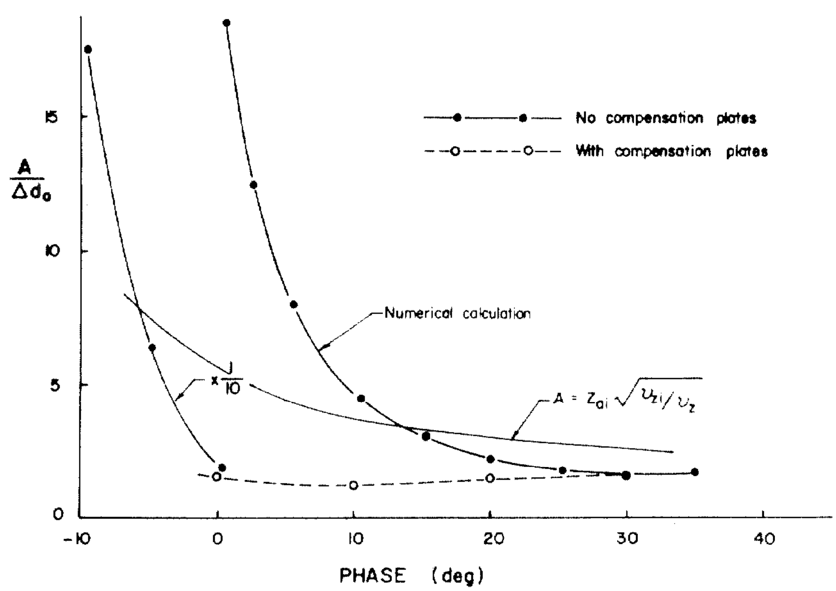

Fig. 8. Coherent vertical amplitude developed at 4 MeV for an antisymmetric misalignment Δd_a.

A comparison with the values obtained by tracking accelerated orbits numerically shows that the equation underestimates the growth of A as negative phases are approached and the focusing grows weaker. The numerical results show that while $A \leqslant 2\Delta d_a$ for $\phi \geqslant 20$ deg it is as large as $10\Delta d_a$ for $\phi=4$ deg and $60\Delta d_a$ for -5 deg. The deviation from the formula can be attributed to breakdown of the adiabatic assumption under rapid changes in ν_{ze}^2 and z_a, and also, for small phases, to large oscillations taking the ions very close to the dees (see Fig.10, reference 4) where the restoring forces are non-linear. To keep the coherent amplitude $A<0.1$ in. for $\phi>10$ deg requires a tolerance $\Delta d_a<0.020$ in. It was not found possible to steer the beam initially (numerically) into a trajectory ending up on the MMP, mainly because of the prohibitively large and strong phase-dependent initial displacement required.

Transverse and uniform misalignments: These cases (Figs. 4d,e) do not involve any axial misalignment at the accelerating gaps. However, the electric and magnetic median planes are separated at the gaps, causing the equilibrium orbit to deviate from the MMP. In the "transverse" case (Fig. 5c) the new EO is of the same form as in the symmetric case (Fig. 5a), and the displacement z_t can be computed from (9) or (10) by substituting $\Delta d_t/2f=\pi\nu_{ze}^2\,\Delta d_t/2R$ for $\Delta\alpha$. Similarly, in the "uniform" case (Fig. 5d) the new EO is of the same form as in the antisymmetric case (Fig. 5b), and the displacement z_u can be computed from (12) or (13) by substituting $\Delta d_u/f=\pi\nu_{ze}^2\,\Delta d_u/R$ for $\Delta\alpha$. For the same Δd, z_t and z_u are generally much smaller than z_s and z_a; this is mainly because $\nu_{ze}^2 \propto \sin\phi$ while $\Delta\alpha \propto \cos\phi$. Numerical orbit tracking has confirmed the relative unimportance of misalignments of these two types in the phase range of interest.

CORRECTIVE METHODS

Wider accelerating gap: Eq.(7) shows that the deflection $\Delta\alpha$ is proportional to E_y and would therefore be reduced by widening the dee gap For TRIUMF, however, this is ruled out by the effects of longer transi time, in particular the lower energy gain on the first turn, which reduces the phase acceptance because of the tighter centre post clearanc

Other dee gap geometry changes: Eq.(7) is based on the "axial slip" field model described above. This is quite accurate for the present TRIUMF dee shape (Fig. 2a), but could lead to an overestimate of the deflection for different geometries (Fig. 2b) or more extreme gap:heig ratios. However, with gap ratios of even 4:1 (which would in any case be impractically narrow) the difference does not amount to more than 30%.

Magnetic fields: Some compensation can be obtained by introducing a radial component of magnetic field B_r in the z=0 plane (i.e. shifting the magnetic surface where $B_r=0$). This could best be achieved by powering the circular or harmonic trim coils asymmetrically, or by using special coils. Uniform or antisymmetric misalignments would seem particularly amenable to this method of correction, although matching the rapid radial variation of z_a and z_u would be difficult. For symmetric or transverse misalignments the method would be more complicated.

Electrostatic steering plates: For symmetric (though not antisymmetric) misalignments it has been shown (Fig. 6) that suitable initial steering of the beam is a very effective method of correction. This can be obtained by the use of electrostatic steering plates early in the first turn, together with some means of shifting the point of injection axially (in TRIUMF this can be obtained by moving the spiral electrostatic inflector vertically).

Perfect compensation for dee gap misalignments could be obtained by means of a series of independently-controlled RF steering plates, one at each gap crossing. While this may be impractical it is possible to obtain good compensation using DC steering plates extending over several turns, provided they are suitably shaped. DC voltages are adequate for the phase range of interest ($|\phi|<30$ deg) since the misalignment deflection $\Delta\alpha \sim \cos\phi$. The azimuthal width is chosen to make the correcting deflection $\Delta\alpha_c$ equal and opposite to $\Delta\alpha$. For TRIUMF four pairs of steering plates have been designed (Fig. 9) to permit correction of misalignments $\Delta d = \Delta d_o[1+a(R-R_1)]$ varying linearly with

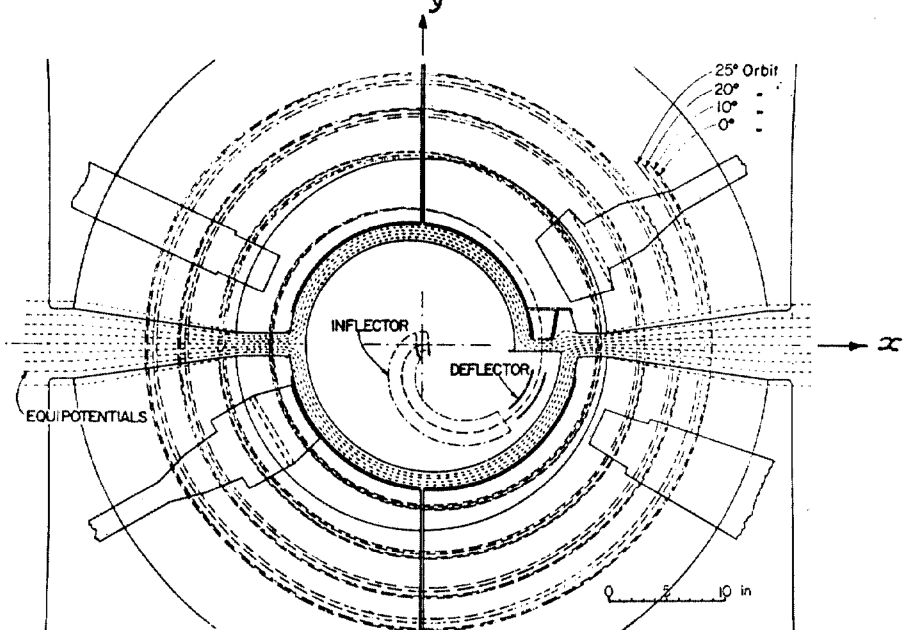

Fig. 9. Central region layout in the median plane, showing the radial shape of the four pairs of steering plates.

radius (Δd_o, a and R_1 are constants). Two pairs are placed 30 deg after the two dee gaps and are shaped radially to cancel the deflections caused by Δd_o. The other two pairs are placed 30 deg before the dee gaps and are shaped to correct the sloping misalignment $a\Delta d_o(R-R_1)$.

R_1 has been chosen as the radius of the first turn so that the voltages on the first pairs of plates can be selected experimentally to produce a horizontal beam entering the second turn. The voltages on the second pairs can then be selected to level the beam on later turns. Since the RF fields prevent the plates being placed right at the dee gap, the compensation is not perfect. However, a very effective compensation is achieved, both for symmetric and antisymmetric misalignments; the numerical results plotted in Figs. 6 and 8 show that the coherent amplitudes at 4 MeV can be reduced to $A \lesssim \Delta d$ over the whole phase range. This enables the tolerances on both Δd_s and Δd_a to be raised to 0.080 in., a figure which is more readily achievable for horizontal resonator sections 120 in. long supported only at their roots.

The complicated shapes of the steering plates lead to the possibility of there being radial fringing field components E_r for $z \neq 0$, which could distort the radial motion. In linear approximation it may be shown that the net radial impulse $\Delta p_r \approx z(\partial/\partial r)\Delta p_z$. For the most unfavourable particle we find that the radial amplitude induced at 20 MeV in correcting for $\Delta d_s = 0.080$ in. amounts to no more than 0.02 in., corresponding to ±100 keV energy dispersion at extraction. For the majority of ions the dispersion will be much smaller.

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