

ACCELERATING SYSTEM OF ISOCHRONOUS CYCLOTRON  
WITH A NON-SINUSOIDAL ELECTRIC FIELD

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ABSTRACT

A non-sinusoidal accelerating voltage was considered which may be formed with a dee system consisting of two dees. The fundamental harmonic voltage is applied to the first dee and the third harmonic is fed to the other dee.

It has been found by the study of the ion motion in the cyclotron median plane that there exists a correlation between the growth of radial oscillation amplitude and the azimuthal distribution of accelerating gaps.

A criterion has been formulated for eliminating the growth of amplitude of radial oscillations due to electric field.

INTRODUCTION

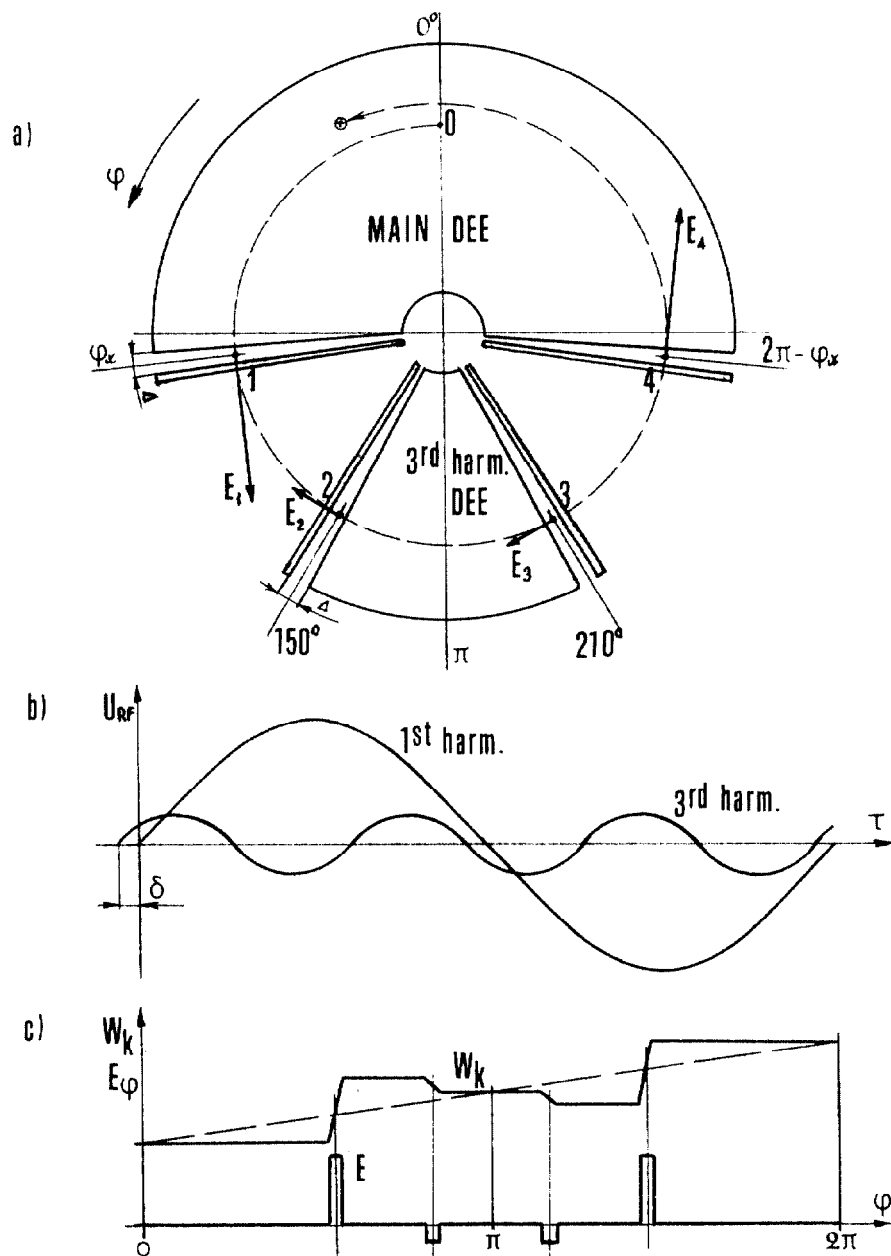
A number of papers appeared recently, which related to an improvement of the energy spread of the particle beam outlet from the cyclotron <sup>1, 2</sup>.

The variant considered of the dee system is characterized by a simplicity also with respect to the RF energy supply of the dee system. The asymmetrical azimuthal distribution of accelerating gaps is compensated by a convenient arrangement of the main dee (fig. 1.).

The results given were obtained by a numerical solution of motion equations <sup>3</sup> of the proton in the median plane of an isochronous cyclotron.

DESIGN OF THE RF SYSTEM

First the variant of the dee system with  $\varphi_x = \pi/2$  was investigated. The ion is accelerated by the voltage of the fundamental harmonics in the points 1 and 4 and in the points 2 and 3 it is deaccelerated by the voltage of the third harmonics of the field.



The phase of ion with respect to the phase of the fundamental harmonics in the moment when the ion is situated in the point 0 is considered as the initial phase of the ion  $\tau_p$ . During the ion acceleration ( $\tau_p \in (-5^\circ, 5^\circ)$ ) the amplitude of radial oscillations increases in this dee system, as is shown on fig.2. ( curve 1 ).

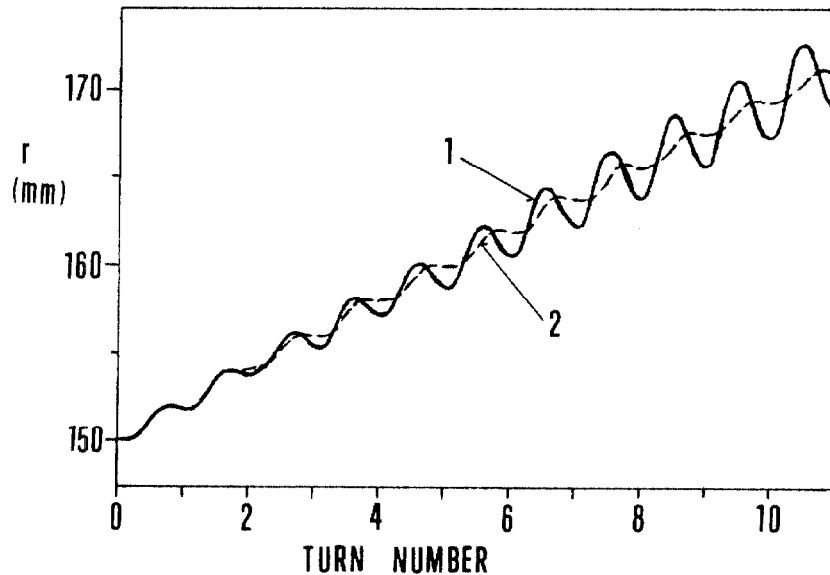


Fig. 2. Radius  $r$  vs azimuth  $\varphi$  for ten turns. Line 1 is for the system with  $\varphi_x = \pi/2$ , line 2 is for the modified system.

This increase of the radial oscillation amplitude is due to shifting forces to which the ion is subjected during each revolution and it may be suppressed by an angular extension of the main dee. The magnitude of this extension was determined from the study of azimuthal distribution of the intensity of electric field and the change of kinetic energy of ion per one turn.

The Fourier analysis of the azimuthal electric field values may be written in the form

$$E_\varphi(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\varphi + b_n \sin n\varphi) \quad (1)$$

The kinetic energy of the particle may be then expressed by means of the relation

$$\begin{aligned} W_k(\varphi) &= e \int r E_\varphi(\varphi) d\varphi = W_0 + e \int_0^\varphi r E_\varphi(\psi) d\psi = \\ &= W_0 + \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \sin n\varphi + B_n \cos n\varphi) \end{aligned} \quad (2)$$

where

$$\begin{aligned} A_n &= \frac{2U_0 e \sin n\Delta}{\pi n^2 \Delta} (\sin \varphi_x \cos \tau_p \cos n\varphi_x - q \cos 3(\tau_p + \delta) \cos n\pi/6) \\ B_n &= -\frac{2U_0 e \sin n\Delta}{\pi n^2 \Delta} (\sin \tau_p \cos \varphi_x \sin n\varphi_x) \end{aligned} \quad (3)$$

$U_0$  is the dee voltage,  $\Delta$  is the radial width of accelerating gaps,  $\varphi_x$  and  $2\pi - \varphi_x$  are the positions of the accelerating gaps for the fundamental harmonics,  $q$  is the amplitude ratio of the third and fundamental harmonics,  $\delta$  is the phase shift of the third harmonics. For  $\tau_p=0$ ,  $\delta=0$ , the values  $B_n=0$  and

$$A_n = \frac{2U_0 e \sin n\Delta}{\pi n^2 \Delta} (\sin \varphi_x \cos n\varphi_x - q \cos n\pi/6) \quad (4)$$

The condition for the suppression of the growth of the radial oscillation amplitude is in this case

$$A_1 = 0 \quad (5)$$

which may be interpreted as the zero resultant of the shifting forces. For  $\varphi_x$  we obtain the relation

$$2 \sin \varphi_x \cos \varphi_x = q \sqrt{3} \quad (6)$$

For  $q=1/9 \rightarrow \varphi_x=95.55^\circ$  with the use of (5).

Radial coordinate of ion with  $\tau_p=0$  vs azimuth ( or vs turn number ) is also represented on fig. 2. ( curve 2 ) for the modified system.

## RESULTS OF CALCULATIONS

For the calculations following values were selected: initial conditions for proton acceleration:  $\varphi_p=0$ ,  $W_p=3.21$  MeV,  $r_p=15$  cm,  $(dr/d\varphi)_p=0$  and  $U_0=50$  kV. The value  $q=1/9$  yields the most perfect behaviour of a voltage for the range of initial phases  $\tau_p \in \langle -5^\circ, 5^\circ \rangle$  considered. The value  $\delta=0.35^\circ$  was estimated for minimum energy spread in first fifteen turns. It has been found that the energy spread  $\Delta W/W$ , which is  $\approx 4$  in this range, depends strongly on an accurate determination of  $\delta$ .

The choice of initial conditions has the result that a stable condition in the acceleration process is attained at a substantially higher turn number  $\nu \geq 100$ . In this way the phase of the ion during the flight through the gaps is also changed and the energy spread is deteriorated, as may be seen on fig. 3.

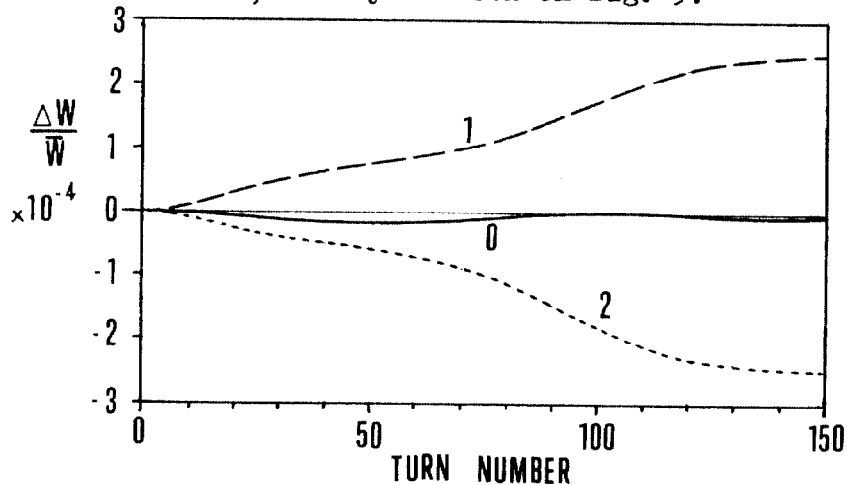


Fig. 3. Energy spread  $\Delta W/W$  of the beam with the phase width  $10^\circ$  vs turn number. Lines 0, 1, 2 are for ions with the starting phases  $\tau_p=0^\circ$ ,  $5^\circ$  and  $-5^\circ$  respectively.

On fig. 4. the ion phases with  $\tau_p=0^\circ$ ,  $5^\circ$  and  $-5^\circ$  at the azimuth  $\varphi = \pi/2$  and  $3\pi/2$  vs turn number are shown for  $\delta=0.35$ . On fig. 5. the development of the radius vector of the amplitude of radial oscillations for the ions having the same starting phases  $\tau_p$  is represented.

It may be seen from the two latter figures that both phase and orbital stabilities of the accelerated particles are provided in the system proposed.

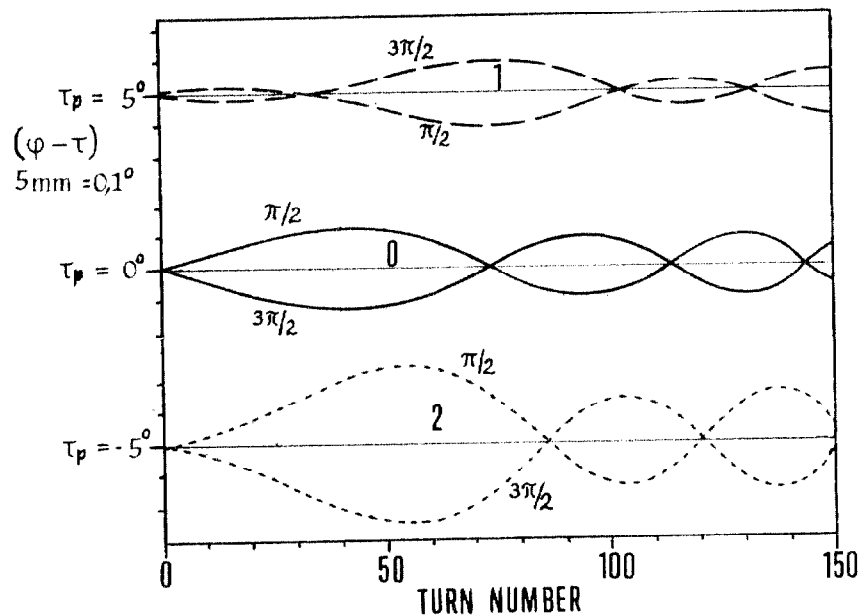


Fig. 4. Phase of ion, at  $\varphi = \pi/2$  and  $3\pi/2$  vs turn number. Lines 0, 1, 2 are for the ions with the same  $\tau_p$  as in fig. 3.

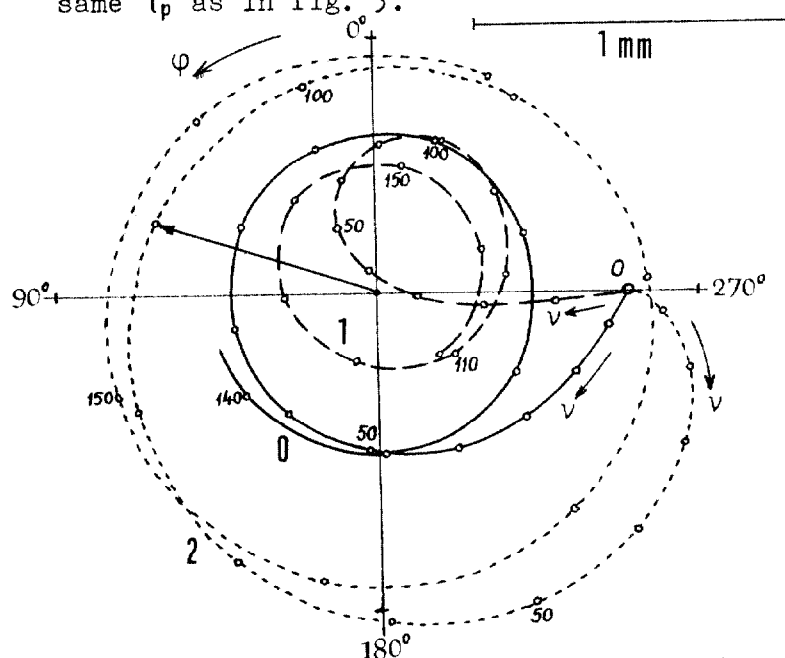


Fig. 5. Development of the amplitude of the radial oscillations as the radius-vector.

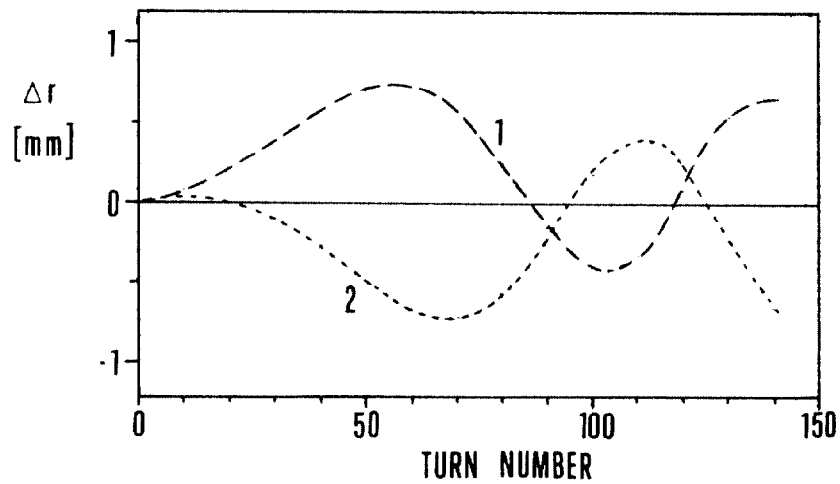


Fig. 6. Radius difference  $r(\tau_p) - r(0)$ , at  $\varphi = 2\pi$  vs turn number. Lines 1, 2 are for  $\tau_p = 5^\circ$  and  $-5^\circ$  respectively.

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