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FILLING OF THE PHASE SPACE IN THE CENTRAL REGION OF A SYNCHROCYCLOTRON

D. THOUROUDE
University of Rennes, France

ABSTRACT

Numerical methods have been used to study the filling of the energy-phase plane and the evolution of the beam during capture time. Beam shapes and distribution of particle densities can thus be obtained at various instants in phase and median planes. The results are meant to be used in studying space charge effects during injection.

INTRODUCTION

Although the equations of motion are well known for the particles in a synchrocyclotron, they have rarely been used to study anything else than the motion of single particles. Consequently the beam shape in the central region does not seem to have been determined with accuracy until now. In this paper, we study numerically the acceleration of protons during capture time and analyse the manner in which the phase space is filled as well as the geometrical features of the beam.

The results will be used in subsequent work for studying beam motion in the presence of space charge. The calculations, which have been done for the CERN synchrocyclotron, can easily be extended to other synchrocyclotrons.

1 METHOD OF CALCULATION

We draw at random some of the initial phases of the extracted particles at each period of the R.F field, and integrate the equations of motion of these particles with a step of half a turn. At each turn, new protons are injected and the useful parameters of all the particles in the beam are stored, i.e. a) the phase at the first gap b) the phase at the kth gap c) the kinetic energy at this gap d) the radius of curvature of the trajectory e) the coordinates of the curvature centre f) the vertical coordinate z.

At any instant of time, by means of projections, beam shapes and distribution of particle densities can be obtained in phase, median and meridian planes.

2 PARAMETERS OF THE CENTRAL REGION

The central geometry of the synchrocyclotron has been considered in detail by Comiti and Giannini.[1] They pointed out that, with cut cones where the accelerating electric field is tangent to the orbit, the energy gain per turn is maximum and axial losses are minimised. We consider such a geometry and we start by giving some parameters of the CERN improvement programme.[2]

The magnetic field at the centre B_o is set at 1,97 Tesla. The parameter K_o, which defines the magnetic field shape, is nearly constant and equal to 2. The cosine of the synchronous phase is chosen so that the capture time should be optimum, i.e $\cos \Phi_s = 0,133$. The R.F voltage has a value of 30 kV and the frequency versus time programme is linear, at least in the considered region. One has

$$\omega_e = \omega_{so} + \dot{\omega}_e t$$

$$\dot{\omega}_e = - \frac{V\omega_o^2 K_o}{\pi U_o} \cos\Phi_s$$

$$\omega_o = \frac{B_o c^2}{U_o}$$

U_o being the rest energy of the proton (in eV) and $\frac{\omega_{so}}{2\pi}$ the initial frequency, set at 30,05 MHz.

3 RESULTS
3.1 Filling of the phase space

Figures 1 and 2 represent the particle density in phase space at times $4,8\mu s, 7,8\mu s$ (t = 0 being the start of the injection). These figures were obtained by projecting on the plane $\Phi, \dot{\Phi}$ the "coordinates" of all the particles making up the beam at the considered instants of time. The plane $\Phi, \dot{\Phi}$ has been divided in elementary rectangles, the size of a rectangle being given by the space line and the space column of the printing machine ; a symbol indicates the number of particles which are projected in each elementary rectangle of this plane[*].

We can see on the figures that the phase space fills itself with a uniform density except at the centre where there is a "hole". This may be explained in the following way : the phases exceeding 60° are not accepted for their kinetic energy at the exit of the slit is not large enough to make them go round the source. They are lost from the start by striking the source. We know that the orbits in phase space are curves centered around Φ_s. The "hole" that we observe corresponds to the interior part of the curve followed by the particle for which $\Phi_o = 60°$ and $\dot{\Phi}_o = 0$. The motion in the phase space obeying the equation

$$\dot{\Phi}^2 - \dot{\Phi}_o^2 = \frac{2V\omega_o^2 K_o}{\pi U_o} \left[(\Phi_o - \Phi) \cos\Phi_s + (\sin\Phi - \sin\Phi_o) \right] ,$$

[*] If the number of particles which are projected in an elementary rectangle is lower or equal to 9, the symbol is the corresponding digit. If this number is greater than 9, the following convention is used:A=10, B=11,..., Z=35, $ for numbers greater than 35.

we may determine theoretically the maximum values $\Delta\Phi$ and $\Delta\dot\Phi$ of the limiting trajectory ($\Phi_0 = 60°, \phi_0 = 0°$). We find $\Delta\Phi = 44°$ and $\Delta\dot\Phi = 0,5$ (normalised). These values are in good agreement with our results.

The beam which has in phase space the shape of a portion of a ring at the start of the injection, takes later on a spiral shape. This may be explained by the fact that particles are decelerated when the phase is larger than 90°. When the phase reaches 90° for the second time, the minimum radius is reached and if it is not large enough, particles strike the source. This explains the important loss of particles at this instant (Fig. 2). Phases between 0° and 40° are lost first, negative phases being defocused from the start of the injection. After capture time (equal to 16µs), the spiral shape is less definite and the density becomes uniform (except around Φ_s).

3.2 Beam shape in the median plane

For each particle, we know the curvature radius of the orbit, the position of the curvature centre and the phase at the gap. It is then possible to represent, except for a rotation, the beam shape in the median plane. Fig. 3 gives an example of beam shape at t = 4,8µs. At the start of the injection the beam has a sector-like form, the radius of which is about 7 cm and the azimuthal width 90°. Very quickly (Fig. 3) the "saussage" appears. The particle density is nearly constant. After the capture time, there are no more accepted particles, the sector has disappeared.
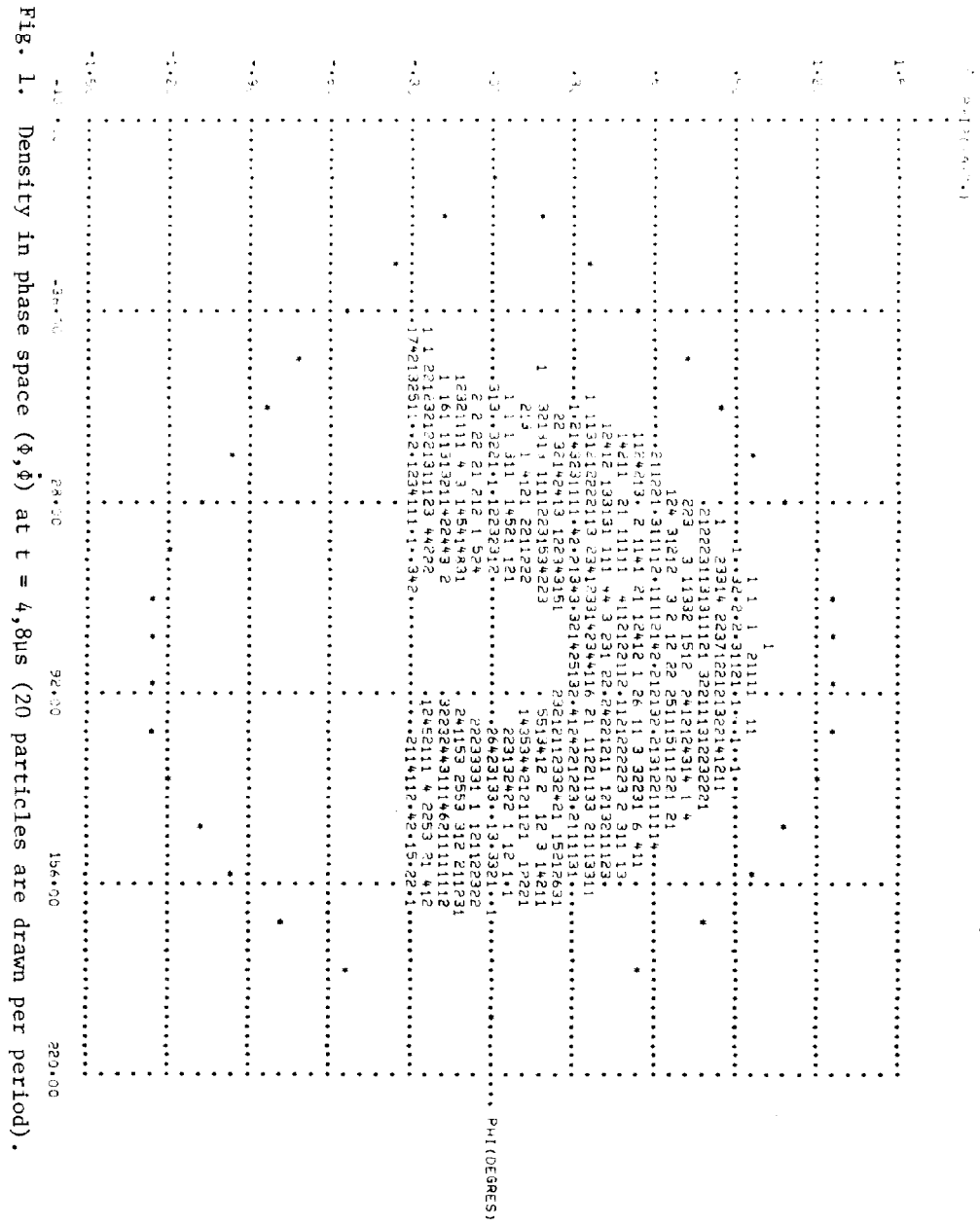
4 CONCLUSION

We have studied the motion of the beam during injection by means of numerical procedures. This study allows one to obtain the beam geometry and its evolution in time. The results are meant to be used in subsequent study on space charge problems.

ACKNOWLEDGEMENTS

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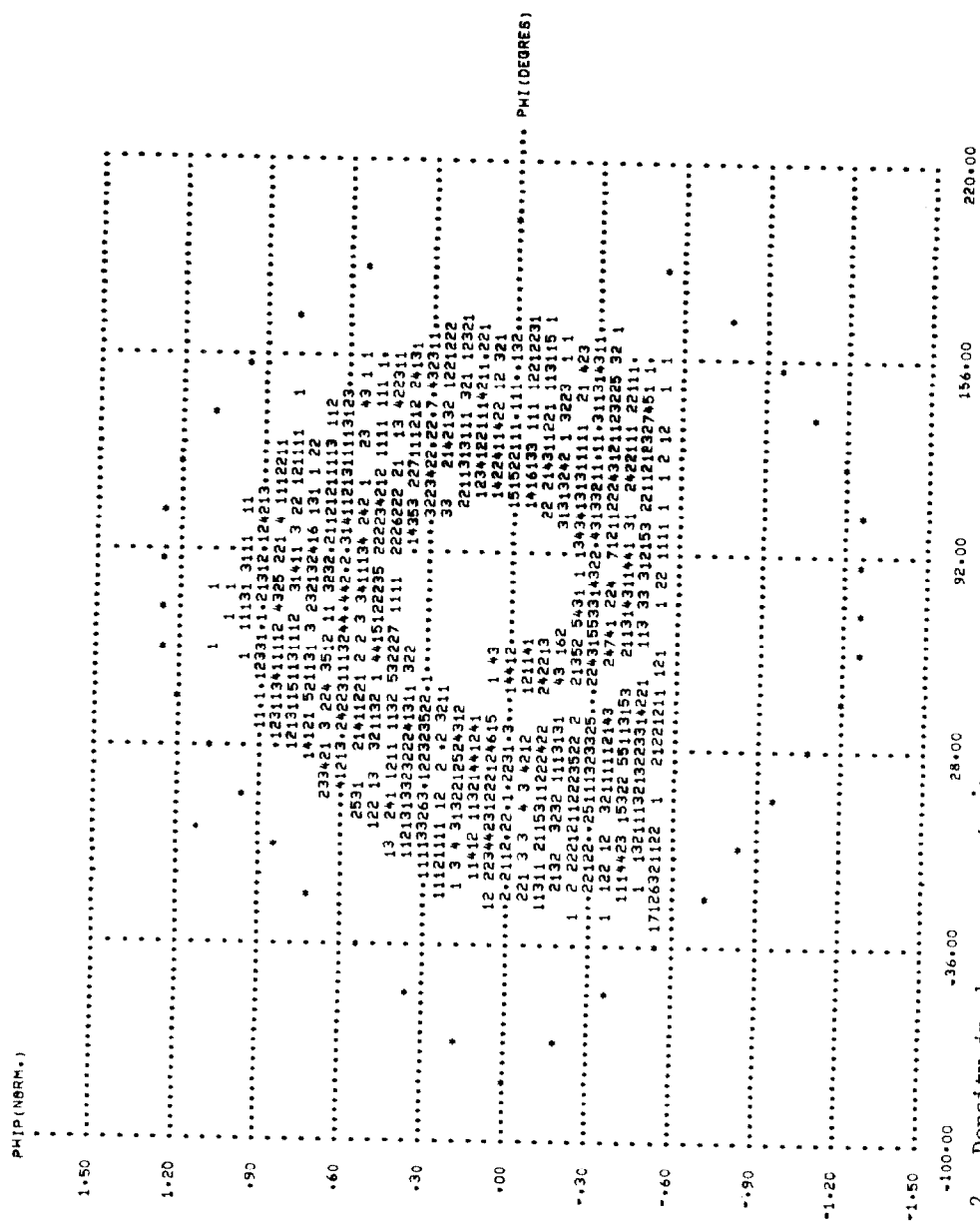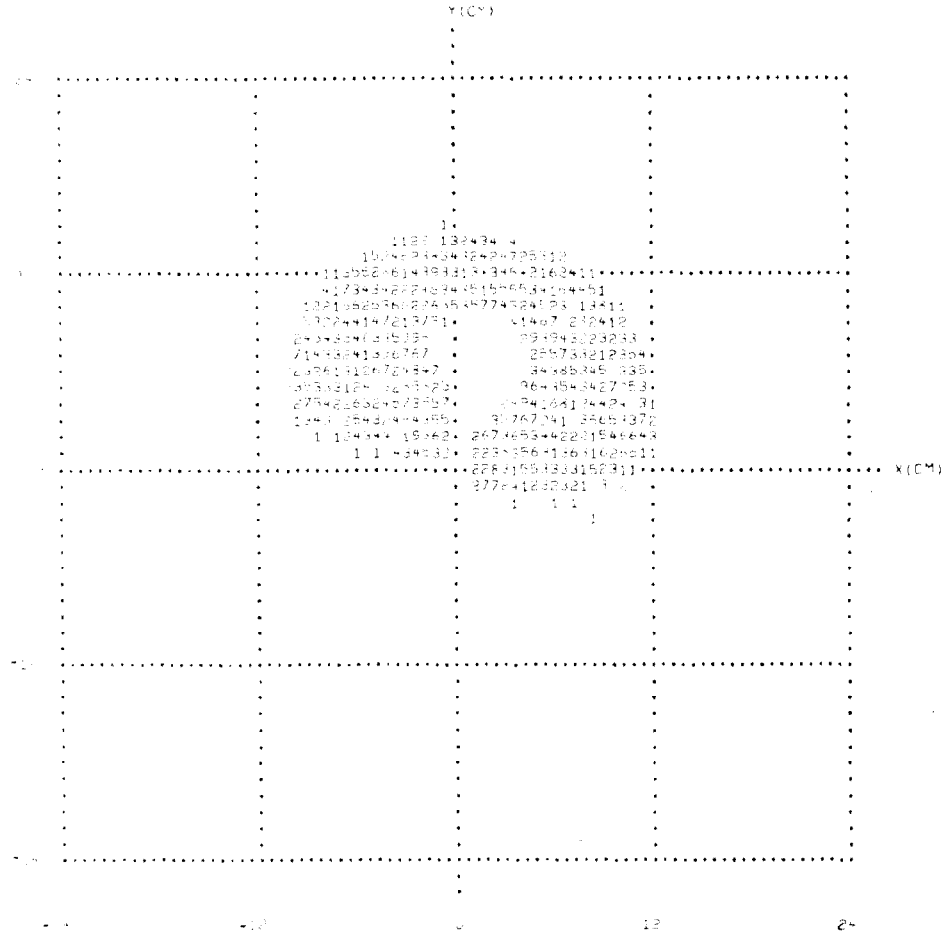
Fig. 1. Density in phase space $(\Phi,\dot{\Phi})$ at t = 4,8μs (20 particles are drawn per period).

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Fig. 2. Density in phase space $(\Phi, \dot{\Phi})$ at $t = 7,8\mu s$ (20 particles are drawn per period).

Fig. 3.   Density in the median plane at t = 4,8μs.