Some peculiarities of phase motion in the phasotron with spatial variation of magnetic field

V. P. Dmitrievsky, B. I. Zamolodchikov, N. I. Polumordvinova, and V. V. Kolga
Joint Institute for Nuclear Research, Dubna, U.S.S.R.

Presented by V. P. Dmitrievsky

Accelerator designs envisaging a considerable increase of beam intensity (20–50 times) will be of great significance for the development of nuclear physics in the region below 1 GeV.

The conversion of present day synchrocyclotrons into phasotrons with spatial variation of the magnetic field is the most promising direction. Designs based on this method have been developed at Dubna (U.S.S.R.),[1] and Columbia University (U.S.A.),[2] and a similar project has been started at the Carnegie Institute of Technology (U.S.A.).[3]

A radially increasing magnetic field results in considerable reduction of the required frequency sweep which makes it possible to increase the accelerating voltage amplitude several times and increase the capture time by decreasing the parameter of the 'K' phase equation.[4]

The choice of the main parameters of such a phasotron is determined to a considerable extent by the investigation of particle motion in the acceleration process.

Here the criteria determining the choice of the law of the average magnetic field along the radius as well as field tolerances along the radius are considered. The tolerances for this field in the area of the first phase oscillation are also treated.

In determining the required law of the average magnetic field we assume $\sin\varphi_s = \text{const.}$, considering that the proper choice of the (frequency-time) dependence can be easily achieved. Under this assumption it is necessary to find the condition excluding phase loss in acceleration. For this purpose, it is sufficient that the vertical size of the separatrix ($\dot{\varphi}_{max}$) be larger or equal to $A_\varphi\Omega$, where A is the amplitude of the phase oscillations and Ω is the frequency. Since $\dot{\varphi}_{max}$ and Ω are equally dependent upon the phase equation parameters, it is sufficient that the phase oscillation amplitude should not increase in acceleration. In the adiabatic variation of the parameters the phase oscillation amplitude is determined by the invariant

CYC69H04

$$\left[\frac{eVE_s}{\pi\omega_s^2 K_s}\right]^{1/2} I(A_\varphi, \sin\varphi_s) = \text{const},$$

where
$$I = \int_{\varphi_{\min}}^{\varphi_{\max}} [C + \cos\varphi - (\pi - \varphi)\sin\varphi_s]^{1/2}\, d\varphi \qquad (1)$$

The function $I(A_\varphi)$ is increased monotonically in increasing A_φ,[5] therefore, in order to keep the phase oscillation amplitude from increasing during acceleration, it is necessary to fulfil two conditions:

$$K \leqslant K_g = K_0 \frac{E}{E_0} \frac{V}{V_0} \left(\frac{\omega_0}{\omega}\right)^2 \qquad (2)$$

where K_g is the maximum tolerable value of the parameter K, and the subscript '0' describes the values of the parameters at the beginning of acceleration. Since $K = 1 - n/(1+n)\beta^2$, then the condition $K = K_g$ defines the law of the average magnetic field variation along the radius given the value of K_0, the energy of particle acceleration and the total change of the magnetic field from the centre to the full radius.

Further, we shall assume that there is no deep amplitude modulation in acceleration, i.e. V= const. Then, substituting the appropriate average magnetic field instead of energy we reduce expression (2), to:

$$1 - \frac{n}{(1+n)\beta^2} = K_0 h \left(\frac{\omega_0}{\omega}\right)^3 \qquad (3)$$

where $n = \xi/h . \mathrm{d}h/\mathrm{d}\xi$, $h = H(\xi)/H_0$ is the relative value of the average magnetic field, $\xi = r/r_k$ is radius divided by the full radius in the cylindrical co-ordinate system.

Solving Eqn (3) with respect to $\mathrm{d}h/\mathrm{d}\xi$, one finds the differential equation determining the variation of h and n along the radius with the given initial and boundary conditions

$$\frac{\mathrm{d}h}{\mathrm{d}\xi} = \frac{Y^2 h\xi[h^2 - K_0(1 + Y^2h^2\xi^2)^{3/2}]}{1 + K_0 Y^2 \xi^2 (1 + Y^2 h^2 \xi^2)^{3/2}} \qquad (4)$$

where
$$Y = \frac{\beta_k \gamma_k}{h_k}, \quad \gamma_k = \frac{E_k}{E_0}$$

Eqn (4) was calculated using the computer. The parameter Y and boundary conditions $h_0 = 1$, h_k were taken. As a result of the solution, K_0, $h(\xi)$, $n(\xi)$ were found.

Fig. 1 shows the functions $n(\xi)$, $h(\xi)$ (curves I) with W_k = 680 MeV, h_k = 1·3675 which have been taken for the Dubna project. In this case K_0 = 0·256.

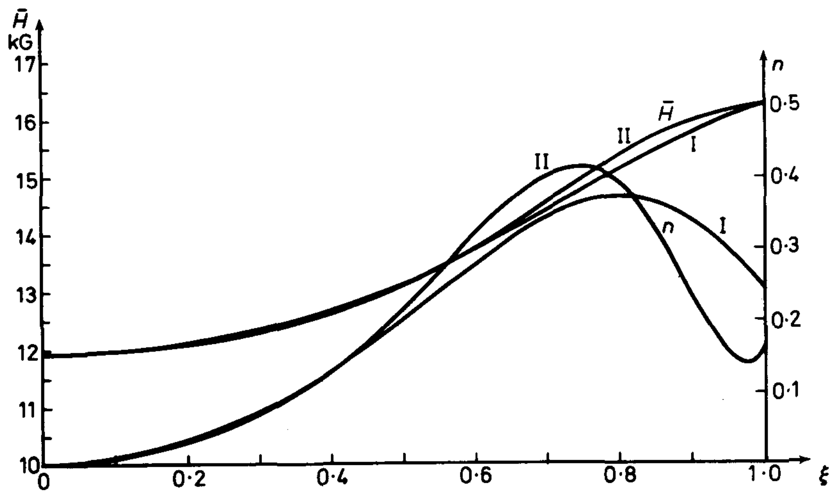

Fig. 1. Dependence of the average magnetic field and the quantity n *upon radius*

As is seen from Fig. 1, at the full radius $n_k = 0{\cdot}246$, which corresponds to free oscillation frequencies $\nu_r = 1{\cdot}13$ ($\nu_z \approx 0{\cdot}2$) and noticeably varies along the radius (at a 13 cm distance from full radius, $\nu_z = 1{\cdot}16$). However, in using one of the versions of the regenerative method for particle extraction from the phasotron it is necessary for the radial oscillation frequency within 10–13 cm from the full radius to be $\nu_r = 1{\cdot}08–1{\cdot}09$, which corresponds to $n_k = 0{\cdot}14–0{\cdot}16$.

Therefore, in order to investigate various possibilities of obtaining the necessary variation of the magnetic field with the radius it is reasonable to give the average magnetic field analytically. The dependence should provide a possibility of varying the field and the gradient near the full radius without changing the law of magnetic field growth in the central region, where this law is dependent on minimising phase loss.

These requirements can be satisfied if one chooses the analytical dependence of the average field, e.g. in such a form

$$h = \exp \Sigma_m C_{m-1} \xi^{2m} \tag{5}$$

The first coefficient C_0 ($m = 1$) determines the field shape in the central region and the value of the parameter $K = K_0$. The next value should be taken so that the coefficient corresponding to it does not significantly affect the value of K_0. The total number of terms is determined by the given change of the magnetic field and the conditions which are imposed on the field gradient in the vicinity of the phasotron full radius. Consider the effect of various coefficients on the K parameter in the central region. For this purpose the analytical dependence Eqn (5) can be given as follows:

$$h = \exp (C_0\xi^2 + C_1\xi^4 + C_2\xi^6 + C_3\xi^8 + C_4\xi^{10})$$

$$n = 2C_0\xi^2 + 4C_1\xi^4 + 6C_2\xi^6 + 8C_3\xi^8 + 10C_4\xi^{10} \tag{6}$$

In this case the parameter K in the central region ($\xi = 0$) can be expressed by the formula

$$K_0 = 1 - \frac{2C_0 h_k^2}{\beta_k^2 \gamma_k^2} \tag{7}$$

The presence of strict tolerance for the magnetic field in the central region of the accelerator (see below) as well as the requirement to the system of beam extraction from the accelerator chamber, resulted in the following parameter values, which define phase motion in the central region: $K_0 = 0{\cdot}32$; $C_0 = 350$, $C_1 = 0$ with $h_k = 1{\cdot}3675$.

The coefficients C_2, C_3, C_4 are determined by the given drop h_k and also by the two given values of n with $\xi = 1$, $\xi = 1 - \Delta r/r_k$.

At the same time the choice of coefficients should not destroy the condition $K < K_g$. In order to satisfy the condition of particle beam extraction from the chamber the following values have been taken in the machine 'ph' ($\Delta r = 13$ cm, $n_k = n_{\Delta r} = 0{\cdot}16$). In this case $C_2 = 0{\cdot}83$; $C_3 = -1{\cdot}611$; $C_4 = 0{\cdot}735$. Fig. 1 shows the obtained dependence of $\bar{H}$ and n (curve 2). It is seen that nearly up to $r = \frac{1}{2} r_k$ the dependence $n(\xi)$ is very close to the function $n(\xi)$ obtained using the computer with minimally possible K_0, whereas in the region of the extraction radius $n(\xi)$ satisfies the given conditions. Fig. 2 shows the dependence of K_g and K obtained using Eqn (6).

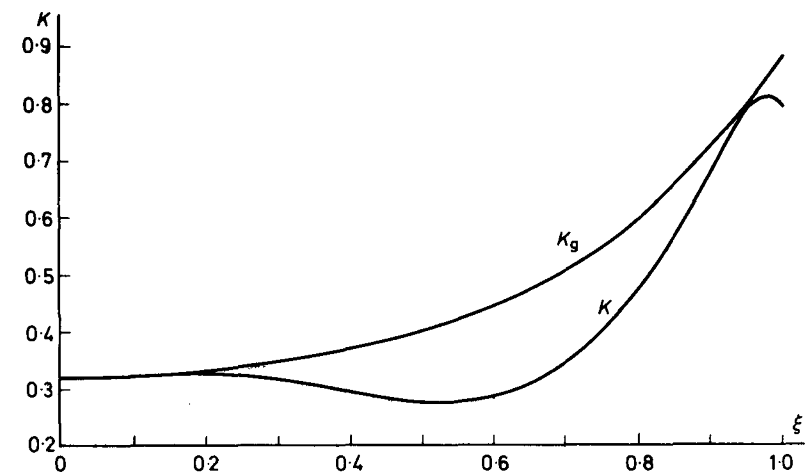

Fig. 2. 'K' as a function of radius

The reduction of the value 'K' in the central region of the accelerator makes it necessary to carefully consider the tolerance for the magnetic field in the area of the first phase oscillation. If the changes of the parameter of the phase equation during phase oscillation are not small, the equation can be written as follows:

$$\frac{\sin\varphi_s}{|\dot{\omega}_s|}\ddot{\varphi} + \frac{\frac{\mathrm{d}}{\mathrm{d}t}(\sin\varphi_s)}{|\dot{\omega}_s|}\dot{\varphi} + \sin\varphi = \sin\varphi_s \tag{8}$$

where $$\sin \varphi_s = \frac{2\pi E_s}{eV\omega_s^2 K_s} |\dot{\omega}_s|,$$

ω_s is the angular frequency of accelerating voltage, eV is the maximum energy gain per turn.

The deviation from the equilibrium phase value during phase oscillation which is determined by errors in the magnetic field law (via the deviation from the value 'K') is found from the expression

$$\sin \varphi_s = \sin \varphi_0 \left(1 - \frac{\Delta K}{K}\right) \tag{9}$$

where φ_0 is the initial value of the equilibrium phase φ_s.

Under these assumptions the phase equation for the parameters of the machine 'ph' can be written as follows:

$$46{\cdot}2 \times 10^{-2} \, [1 + \delta \sin (1{\cdot}47 \times 10^{-2} m\sqrt{\cos \varphi_0}], \frac{d^2\varphi}{dm^2}$$

$$+ \; 68\sqrt{\cos \varphi_0} \; \delta \cos (1{\cdot}47 \times 10^{-2} m \sqrt{\cos \varphi_0}) \frac{d\varphi}{dm} + \sin \varphi$$

$$= \; \sin \varphi_0 \, [1 + \delta \sin (1{\cdot}47 \times 10^{-2} m\sqrt{\cos \varphi_0}] \tag{10}$$

where $m = \omega_0 t/2\pi$ corresponds approximately to the number of ion rotations, and ω_0 is the ion frequency in the centre of the accelerator. The deviation of the magnetic field in the central region of the accelerator from the given law was approximated by the formula

$$\Delta H = \alpha \cos \frac{\pi}{R_b^2} r^2 \tag{11}$$

R_b—radius of the perturbation area of the magnetic field.

In this case

$$\alpha = -\frac{H_0}{2\pi} K_0 \frac{R_b^2}{r_\infty^2} \delta \tag{12}$$

where $r_\infty = c/\omega_0$, and H_0 is the magnetic field strength in the accelerator centre. The results of the numerical calculations of Eqn (8) are shown in Figs. 3 and 4 for $\sin \varphi_s = 0{\cdot}4$, $\sin \varphi_s = 0{\cdot}5$, respectively. The number of ion rotations, where the effect of perturbation stops in the equation was found from the expression

$$1{\cdot}47 \times 10^{-2} m\sqrt{\cos \varphi_0} = \pi \tag{13}$$

The abscissa in Figs 3 and 4 is the value which is proportional to the amplitude of magnetic field perturbation (δ), the ordinate is the maximum phase of the deviated separatrix of Eqn (10) with $d\varphi/dm = 0$.

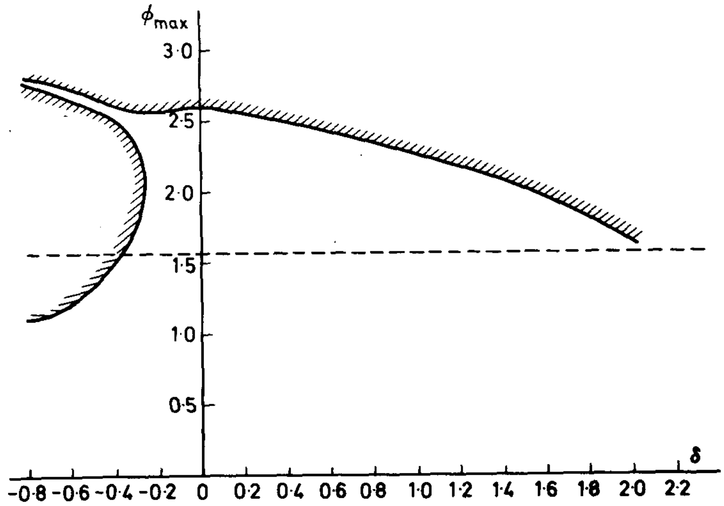

Fig. 3. The limit phase of the separatrix as a function of perturbation with sin $\varphi_s = 0{\cdot}4$

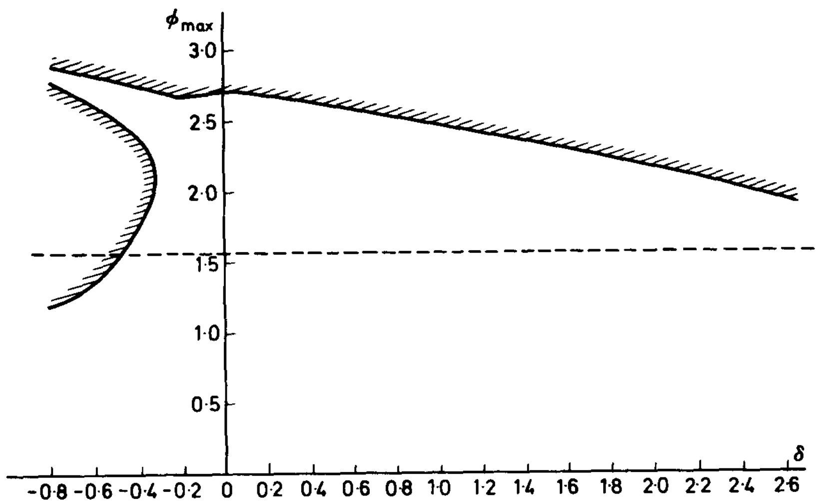

Fig. 4. The limit phase of the separatrix as a function of perturbation with sin $\varphi_s = 0{\cdot}5$

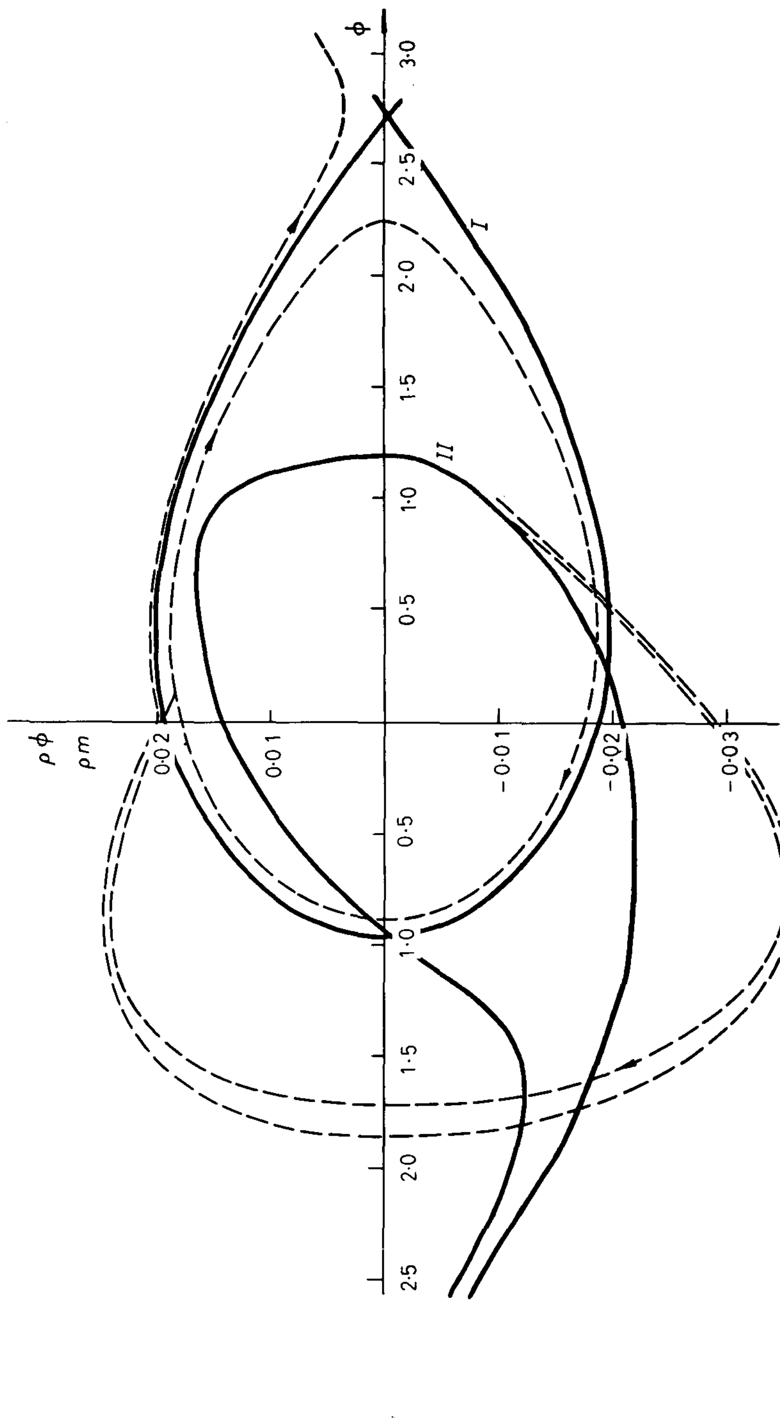

Fig. 5. The separatrix with the perturbation value δ = − 0·8

As is seen from the figures, the character of separatrix change in the first oscillation phase is strongly dependent on the sign of magnetic field perturbation. In order to avoid considerable particle loss by phase excursion with negative values of δ, it is necessary to fulfil the inequality for the perturbation field amplitude, Eqn (11).

$$\alpha \leqslant \frac{0 \cdot 3}{2\pi} H_0 K \frac{R_b^2}{r_\infty^2} \tag{14}$$

For the design of the 'ph' machine when $\sin \varphi_s = 0{\cdot}4$, $eV = 100$ keV, $R_b \approx 40$ cm,

$$\alpha \leqslant 4{\cdot}2 \text{ G} \tag{15}$$

Fig. 5 shows, as an example of calculation, the separatrix with $\delta = -0{\cdot}8$ and two boundary phase trajectories: a stable and an unstable one (dashed).

CONCLUSION

Phasotrons with the average magnetic field increasing along the radius are characterised by some peculiarities of ion phase motion in the process of acceleration.

With a constant value of $\sin \varphi_s$, it is possible to tolerate a considerable (2–3 times) increase of $K = -\omega/E.\ \mathrm{d}\omega/\mathrm{d}E$ without phase loss in acceleration. It is reasonable to take the law of the average magnetic field, keeping in mind the above peculiarity.

When 'K' in the central region is taken to be considerably smaller than unity, it is necessary to have some additional tolerance for the magnetic field in this region.

DISCUSSION

Speaker addressed: V. P. Dmitrievsky (Dubna)

Question by H. A. Willax (SIN): What are the essential parts of your extraction system?
Answer: Our extraction system is based on the resonance method at frequencies ranging from $\nu_r = 1{\cdot}08$ to $1{\cdot}09$. The system utilises two excitations of the magnetic field. We anticipate an extraction efficiency of ~50%.

REFERENCES

1. Glazov, A. A., Denisov, Yu. N., and Dzhelepov, V. P. *et al*, Proceedings of the Sixth Conference on High Energy Accelerators, Cambridge, Mass., 303, (1967).
2. Cohen, R. and Rainwater, S., *I.E.E.E. Trans. Nucl. Sci.* **NS-3-13, 4,** 522, (1966).
3. Foss, M. H., *I.E.E.E. Trans. Nucl. Sci.* **NS-5-13, 4,** 521, (1966).
4. Glazov, A. A. *et al, Preprint JINR*, 9-3951, (1968).
5. Kolomensky, A. A. and Lebedev, A. I., 'Theory of Cyclic Accelerators', *Fizmatizdat*, Moscow, (1962).