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PROPOSAL FOR A SECTOR-FOCUSING SYNCHRO-CYCLOTRON

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Abstract

A modification is proposed for the Uppsala 185 MeV Synchro-cyclotron to obtain a high intensity beam of different particles. By introducing sectors, a radially increasing magnetic field can be used. The reduced bandwidth makes it possible to replace the present rotating condenser by an electronically tuned broadband system. Protons will be accelerated with decreasing and heavier particles with increasing frequencies. No correction coils are required. Expected external currents are of the order of 10 μA.

Introduction

A design study for the improvement of the Uppsala 185 MeV Synchro-cyclotron has been performed. A substantially higher external current than the present 0.01 μA, multiparticle acceleration, and variable energy are demanded. Such features are obtained in the present sector-focusing cyclotrons (SFC). However, at higher energies the construction problems are considerable for those machines. They are simplified if the isochronous condition is abandoned. The combination of sector focusing and frequency modulation[1] is expected to give essential improvements of the synchro-cyclotron.

With sectors, the vertical focusing will be stronger than that presently obtained. This raises the space charge limit at all radii, which is important, when considering high currents. A reduction of the frequency range is achieved by making the field radially increasing. The rotating or vibrating capacitor might therefore be replaced by a more efficient r.f. system. The pulsed nature of the internal beam makes it possible to apply a high voltage at capture time. Before extraction the beam can be stretched by stacking close to the maximum radius.

In the following a redesign of the Uppsala synchro-cyclotron to a sector-focusing synchro-cyclotron (SFSC) is outlined. Sectors will be attached to the poles of the present magnet. A geometry with iron sectors and beam-defining slits is proposed for the central region. Broadband amplifiers will deliver the various r.f. programmes. The d.c. power is limited by an existing 300 kW supply, which makes it necessary to use two accelerating electrodes in addition to the stretching one. A linear regenerative system is anticipated for extraction of the particles.

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Magnetic Field

The mean field is chosen such that a broadband system can work efficiently for the particles of interest. In Fig. 1, isochronous field plots are shown for some particles. The B·r product at full radius corresponds to that for 185 MeV protons, and the real mean field starts midway between the proton and deuteron isochronous fields. This choice yields the same attainable voltage for the mentioned particles. It is also a compromise, which gives good capture efficiencies and sufficient tolerance margins.

When the magnet current is lowered, the hill field decreases slowly compared with the valley field. This depends on the saturated iron. The effect is more pronounced at larger radii, which makes Q_r increase there. The effect on Q_z due to this can be found from the expression

$$Q_z^2 = 1 - Q_r^2 + \frac{(B_H - \bar{B})(\bar{B} - B_V)}{\bar{B}^2} (1 + 2 \tan^2\gamma). \quad (1)$$

$\bar{B}$, B_H, B_V denote average, hill, and valley field, respectively. It is evident that an increasing Q_r is compensated for by the associated increase in the flutter term. As Q_z is allowed to vary considerably, one can expect to get energy variability, the range of which is to be determined from model magnet measurements. Once the saturation region has been penetrated, the focusing properties remain unchanged. Then other factors, such as the functioning of the extraction system and the flexibility of the r.f. system, set the limit for the energy range.

The magnetic fields should be chosen such that the present magnet excitation is not exceeded. By using the linear edge approximation,[2] it has been possible to get a design giving sufficient flutter Q_z. The pole geometry with three sectors and 55° spiral angle at full radius is shown in Fig. 2. With a hill field of 2.2 Wb/m², sufficient flutter will be achieved. Table 1 shows $Q_r(r)$ and $Q_z(r)$. The valley field is calculated in such a way as to give the desired $\bar{B}(r)$ function.

Resonances

In an isochronous cyclotron constructed to give a maximum energy of 185 MeV for protons, the operation is complicated by the fact that the $Q_r = 3/2$ or $Q_r = 4/3$ resonances are involved.[3] If one also wants to be able to accelerate heavier particles, the magnetic field has to be corrected, and then Q_r is decreased by a considerable amount. Without flutter coils, used to reduce the positive focusing term, Q_z will cross

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the value ½. These difficulties are overcome in the sector-focusing synchro-cyclotron proposed here, as Q_r is only 1.1 at full radius and as the change of particle is done without field correction. The comparatively small energy gain due to the time stretching acceleration system probably makes it impossible to pass the $Q_r = 1$ and $Q_r = 2\,Q_z$ resonances at full radius unless the radial oscillation amplitudes are excited prior to $Q_r = 1$. This will be done by means of a linear regenerative extraction system. The lower limit of Q_z at full radius is set by $Q_z = Q_r - 1$, where $Q_r = 1.1$ for the SFSC, compared to 1.2 for the SFC of the same energy. At small radii the fast increase of Q_z due to the central sectors will, after a few particle turns, make $Q_r - Q_z$ become low enough to avoid the resonance.

Central Region

The essential parts of the central region are:

- hooded arc ion source vertically inserted through the magnet;
- puller electrode and beam-defining slits for phase selection and electric phase focusing;[4]
- central sectors of iron to give an adequate vertical focusing;
- two axial coils to give an increased capture efficiency of heavier particles.

As variable energy and multiparticle acceleration is aimed at, a great flexibility of the central geometry is demanded if the maximum performance of the r.f. system is to be utilized. In practice, a too great complexity in the adjusting mechanism for puller and beam-defining slits should be avoided. Orbit schemes that are essentially constant are therefore suggested for each of the two acceleration modes (first and third harmonic) that will be used.

MacKenzie has shown that the ion density at small radii limits the output from a cyclotron.[5] He has found that, under assumption of abrupt onset of the magnetic focusing forces, the current is proportional to the third power of the dee voltage. Calculations by Lawson[6] with a parabolic magnet field give a $V^{5/3}$ dependence. For sector focusing, still another voltage dependence is obtained. The magnetic force F_z for a sinusoidally varying field with no spiral is approximately given by

$$F_z = -m\omega^2 \frac{f^2}{2} z \qquad (2)$$

where

z = distance from the median plane
f = flutter factor.

If the flutter increases parabolically with radius,

$$f = ar^2 \ . \qquad (3)$$

The space-charge force given by MacKenzie is

$$F_m = \frac{4m\omega I}{\epsilon_0 \varphi V} \ . \qquad (4)$$

With Eqs. (2), (3) and (4), an expression is obtained for the mean beam current which can be focused:

$$I = \frac{3}{16\sqrt{5}} \cdot \frac{h\epsilon_0\varphi a}{\pi B} \qquad (5)$$

where

h = dee aperture
φ = azimuthal extension of the beam "sausage"
B = magnetic field
V = voltage gain per turn.

It is thus very beneficial to have a high dee voltage, and our efforts have been directed towards increasing this as much as possible. The great importance of vertical focusing in the centre has also led us to investigate the possibility of obtaining a flutter increasing very rapidly with radius.

With iron ridges placed on the pole faces an appreciable flutter is obtained at a radius comparable to the pole gap. Since a considerable decrease of this gap is not possible, the only way of increasing the flutter is to place iron sectors on the dee and dummy dee electrodes. Some preliminary measurements have been performed for the following dimensions:

vertical height of sectors	2.0 cm
vertical distance between sectors	3.5 cm
vertical distance between pole faces	16 cm
angle of sectors	30°

Figure 3 shows the azimuthal field variation when the sectors start at a radius of 0.64 cm. In Fig. 4, curves of the mean field and the flutter are shown for the same case. The reason why the sectors have to be brought so close to the centre is to prevent a too strong dip in the field. The small dip which still appears is not important as the first orbit has a radius of 1.5 cm. The flutter increases approximately parabolically to a value of 0.115 at a radius of 3.5 cm. For this case, formula (5) predicts 2 mA of protons when V = 60 kV. A comparison of this value with currents obtained in SF cyclotrons shows that the figure is realistic. The magnitude of the flutter at small radii, as measured here, is 5 to 25 times more than that obtained in SF cyclotrons, although the dee voltage is generally smaller in our case. As the central sectors start only 0.64 cm from the centre of the magnet, their azimuthal positions have to be chosen so that enough space is given for the ion source. Unfortunately this means that the positions of the central sectors correspond to the valleys of the main field. Thus there will be a region at which the magnetic field has a sixfold symmetry. Measurements of this configuration have not yet been done, but it is assumed that with a proper design of the sectors an adequate focusing can be sustained. At the position of the ion source, the magnetic field gradients are rather great. Special attention has to be paid to the design of the ion source to make it work safely under these conditions. A closer examination of the central sectors-ion source problems has to be done, however, before it can be concluded what is the best arrangement.

Capture Efficiency

Dee voltage and magnetic field parabolicity in the centre

To get a good vertical focusing, it is desirable to have a high dee voltage during capture time and a high value of the parabolicity at small radii. These parameters do not affect the acceleration time, if it is assumed that the magnetic field, the voltage and the frequency time functions are not changed outside the central region. In this case, the following expression for the capture efficiency can be derived with the aid of formulae (3), (9), and (10) in the paper by Bohm and Foldy[7]

$$\epsilon = \frac{4}{\tau}\left[\frac{\pi\, mc^2\; F_1(\varphi_0,\varphi_s)}{eV_c\,\sin^2\varphi_s\,(\omega_s^2 + h_c c^2)}\right]^{1/2} \qquad (6)$$

where eV_c is the energy gain per turn at capture time and h_c is the parabolicity at small radii. No loss of particles is supposed when the magnetic field and the voltage assume the normal values. If $\sin\varphi_s$ does not change, the ratio of the bucket areas is

$$[V(\omega_s^2 + hc^2)]\Big/[V_c(\omega_s^2 + h_c c^2)] \; .$$

In such a case a spill of particles will occur when $V < V_c$ and $h < h_c$. With constant phase-space density, ϵ will then be inversely proportional to $V_c(\omega_s^2 + h_c c^2)$.

Heavier particles

The optimum capture efficiency is obtained for a starting phase φ_0 near the equilibrium phase φ_s. For electrical phase focusing, the ion source and puller angle should be such that φ_0 is greater than 90°, suitably between 90° and 140°. In Fig. 5 are plotted separatrices for protons and deuterons. The starting angles indicated in the figure should give a well-optimized capture efficiency for protons. Deuterons will be captured only in a range between 90° and 120° and with a much lower efficiency. As the extension in the $\dot{\varphi}$-direction of the stable phase space at a certain φ_0 is proportional to the capture time, an improvement can be obtained if the phases after a few turns can be moved to values around φ_s. A bump in the magnetic field at the centre can be used for that purpose. Furthermore, if the phases are brought to small values outside the normal stable phase space, an accumulation can be expected for heavier particles. This is due to the fact that $d\omega/dt$ is positive. A particle starting earlier in time is therefore slipped to a smaller phase by the bump field than a later starting. The subsequent difference in energy gain between them will give a spatial accumulation. After that, the particles can be brought into the normal phase stability region if the magnetic field is slightly below that normally used.[8]

The shaping of the magnetic field needed for heavier particles will be done with two axial coils. Figure 6 shows a theoretically calculated field, which for a numerical study of the particle motion was superimposed on the main magnetic field. Figure 7 shows the capture efficiency obtained for deuterons of 100 MeV final energy, V_c = 60 kV and V = 40 kV. A current of 75 amps corresponds to the already mentioned motion of the phase from φ_0 = 95° to a value around 60° giving an increase in ϵ of 1.5 times. The further increase when the current is raised to 150 amps is due to the accumulation effect.

Different frequency swings

The choice of mean magnetic field configuration and the associated frequency swing Δf has a great effect on the capture efficiency ϵ. A simple formula has been derived,[8] when $d\omega_s/dt$ and φ_s are constant:

$$\epsilon = 2\cdot\frac{\tau_a}{\tau}\left[-\frac{eVf}{\pi E_f \Delta f}\cdot F_1(\varphi_0,\varphi_s)\right]^{1/2} . \qquad (7)$$

where
f = mean revolution frequency
E_f = final kinetic energy
τ_a = average acceleration time.

The present frequency range for 185 MeV protons is 33 to 26 MHz, achieved by means of a rotating condenser. In the redesign, the frequency range is 24.5 to 22.9 MHz. This gives an improvement of ϵ by a factor 2. Another factor 2 is gained as $\tau_a = \tau$ for the proposed r.f. system. One can assume φ_0 = 120° to be a good average value of the capture angles. With φ_s = 150°, eV_c = 60 keV and eV = 30 keV, 2% capture efficiency is obtained. Thus 40 μA internal protons should be anticipated. To get a good beam quality it is desirable to traverse small radii rapidly. This requires large values of $\sin\varphi_s$ with a subsequent reduction of the capture efficiency. If φ_s = 120°, the internal current is reduced to 24 μA. The final mode of operation must be determined experimentally to give a good compromise between internal current and beam quality.

r.f. System

A broadband amplifier system is more flexible than mechanically tuned systems. The great problem, however, is to limit the power consumption. Various methods of doing this are suggested. The change of the magnetic field to radially increasing makes Q (centre frequency over bandwidth) larger and hence the required power smaller. For the present synchro-cyclotron, Q = 4.2, and for the proposed SFSC, Q = 14.0. The power reduction is 3.4 due to this. A split of the single dee into one dee and one cee[9] reduces further the required power four times. In the hill region the pole gap is 16 cm, and in the valley region 40 cm. This fact makes it feasible to minimize the capacitance by a suitable positioning of the electrodes (see Fig. 2). The capacitance will also be reduced by using a small cee angle.

In order to feedthe power from the final amplifier to the λ/4 resonator, of which the electrode forms a part, a quadrupole coupling with two peaks in the passband[10] will be used. A quarter wave line connects the amplifier to the resonator near the short. Low-level measurements have been made on this arrangement for conditions corresponding to the acceleration of 185 MeV protons. The frequency range from 24.5 to 22.9 MHz is split into 600, 800 and 200 kHz bandwidths for the dee, cee, and stochastic cee, respectively. The results of the measurements for the cee system are given in Fig. 8. The peak-to-valley ratio of the plate impedance Z_3 has been chosen to give a reasonable phase variation. The dashed curve shows the gap voltage V_1 for constant plate voltage V_3 which gives optimum working conditions for the final tube at all frequencies. The required d.c. power amounts to 125 kW when class C operation is considered. As a high voltage V_1 is needed at the start of the acceleration cycle it has been found necessary to sidetune the dee resonator. By sacrificing bandwidth, a naturally high dee voltage can be obtained for the starting frequencies (see Fig. 9). The dotted line shows the desired gap voltage V_1. A slight modulation of the grid voltage to the final tube is necessary for these frequencies, but for the remaining frequencies the r.f. plate voltage can be held constant. The voltage and frequency programmes are shown in Fig. 10. It is to be noted that the dee is on for one-third and the cee for two-thirds of the total time. Totally, 200 kW d.c.-power will therefore be required. The calculated voltages for other particles at two different B.r.-products are shown in Table 2.

Tolerances

The tolerances on the magnetic field as a function of radius are determined mainly by the degree to which the electric frequency time curve can be fitted to the mean field. A relation between these quantities is given by the Bohm and Foldy formulae (1) and (3). The magnetic field is contained in K, which, in order to avoid a phase transition, must have the same sign at all radii. Due to this, certain restrictions are imposed on the desired mean field $\bar{B}(r)$ in Fig. 1. The $^3He^{2+}$ and the H^+ isochronous fields are seen to limit the mean field. The condition for avoiding phase transition can be stated as:

$$\frac{d\bar{B}_1(r,B)}{dr} \le \frac{d\bar{B}(r,B)}{dr} \le \frac{d\bar{B}_2(r,B)}{dr} . \qquad (8)$$

Index 1 stands for the $^3He^{2+}$ and index 2 for the H^+ isochronous fields.

Starting with the central magnetic field, which is determined by the frequency ranges chosen, Eq. (8) can be integrated to yield the absolute tolerance limits shown as dashed lines.

The required voltage modulation and the magnetic field determine the $\omega(t)$ function. The tolerances on the $V(t)$ and $\omega(t)$ functions can be found by differentiating Eq. (3) in the Bohm and Foldy paper. The relation between synchronous energy and frequency is used, and the time fluctuations of the magnetic field are neglected. This yields

$$\frac{\Delta V}{V} + \frac{\Delta \sin \varphi_s}{\sin \varphi_s} + \frac{2+K}{K} \frac{\Delta\omega_s}{\omega_s} = \frac{\Delta\dot{\omega}_s}{\dot{\omega}_s} \qquad (9)$$

When approaching isochronism, K becomes small and special attention has to be paid to the term containing K. For reasonably large K, however, the error in ω_s can be neglected. By allowing a 2% fractional change of $\sin \varphi_s$ we get a tolerated error of 1% each for $V(t)$ and $\dot{\omega}_s(t)$.

The radial overlap of the electrodes is determined from the penetration of the electric fringing fields, the bucket size at the transition radius, and the precision with which the switching can take place. In the median plane the potential can be allowed to drop to 90% of its full value. This figure is attained 1 cm inside an electrode of 4 cm aperture. The bucket size is 1.1 cm radially at the transition radius. A switching error of 1 μsec or equivalently 8 kHz implies a radial positional error of 0.5 cm. A total overlap of 3.6 cm is therefore required.

Conclusion

The estimated internal currents are between 20 and 40 μA. Large fractions of these currents can be extracted if the beam quality is good. This is found from numerical calculations on the regenerative extraction system for the present machine.[11] Until similar calculations have been carried out for the SFSC, it might be justified to use the results from the mentioned investigation and assume that the required beam quality can be obtained. Hence it should be possible to obtain external currents of the order of 10 μA, with 1.5‰ energy spread and 30% microscopic duty factor. The macroscopic time structure of the internal beam will be removed by means of storage at the extraction radius. For certain particles charge exchange effects and ion source, output will limit the intensities to smaller values than those given here.

Acknowledgements

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References

1) L.B. Mullet, AERE GP/R 2069, Harwell (1956).
N.M. King and W. Walkinshaw, AERE GP/R 2095, Harwell (1957).
J.R. Richardson, Int.Conf. on SF Cyclotrons and Meson Factories, CERN Report 63-19 (1963), p. 317.
K.R. MacKenzie, CERN Report 63-19 (1963), p.405.

2) B.L. Cohen, H.G. Blosser, E.D. Hudson, R.S. Lord and R.S. Bender, Nucl.Instr. and Methods 6, 105 (1960).

3) W. Walkinshaw and N.M. King, AERE GP/R 2050, Harwell (1957).

4) M. Reiser, Michigan State University Report MSUCP-20 (1964).

5) K.R. MacKenzie, Technical Report UCLA, May 1964.

6) J.D. Lawson, Nucl.Instr. and Methods 34, 173 (1965).

7) D. Bohm and L.L. Foldy, Phys.Rev. 72, 649 (1947).

8) S. Kullander, S. Holm, B. Johansson and A. Tidriks, Synchro-cyclotron Development Progress Report II, Uppsala 1965.

9) E. Molthen, Conference on High-Energy Cyclotron Improvement, Williamsburg (1964), p. 128.

10) A. Susini, CERN Report 64-48 (1964).

11) S. Kullander, S. Lindbäck and Å. Svanheden, to be published in the proceedings of this conference.

Table 1.
Magnetic field parameters.

m	Wb/m²	Wb/m²		
r	$\bar{B}$	B_v	Q_r	Q_z
0.30	1.619	1.162	1.008	0.26
0.40	1.629	1.137	1.014	0.27
0.50	1.643	1.140	1.022	0.27
0.60	1.659	1.158	1.031	0.26
0.70	1.678	1.186	1.041	0.24
0.80	1.700	1.222	1.054	0.22
0.90	1.725	1.264	1.067	0.19
1.00	1.753	1.312	1.081	0.17
1.15	1.800	1.391	1.103	0.20

Table 2.
Frequency ranges and voltages for some particles at two B · r products.

Particle	E_k (MeV)	f_0 (MHz)	f_1 (MHz)	V (kV peak)
p	186.7	24.5	22.9	15.4
	101.6	17.7	17.9	37.6
d	100.0	12.3	13.0	16.0
	52.8	8.8	9.6	13.6
$^{3}He^{+}$	67.8	8.2	9.0	13.3
	35.5	5.9	6.5	12.5
$^{3}He^{2+}$	262.1	16.4	16.8	25.4
	139.6	11.8	12.6	15.6
$^{4}He^{+}$	51.3	6.2	6.8	12.6
	26.9	4.5	5.0	12.3
α	201.3	12.3	13.1	16.1
	106.3	8.9	9.7	13.5
$^{12}C^{4+}$	272.5	8.2	9.0	13.3
	142.9	5.9	6.6	12.5
$^{16}O^{6+}$	458.5	9.3	10.1	13.8
	240.9	6.7	7.4	12.8

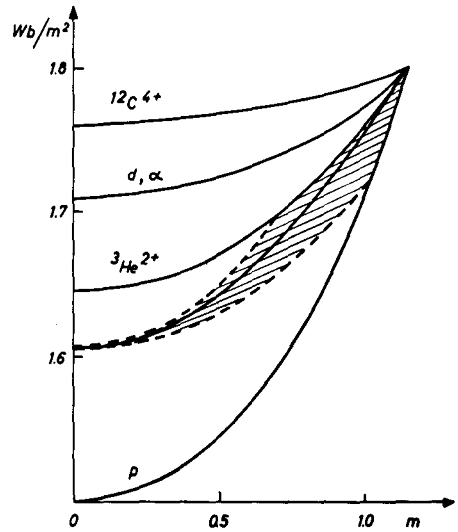

Fig. 1. Isochronous fields for some particles and mean field with tolerance limits.

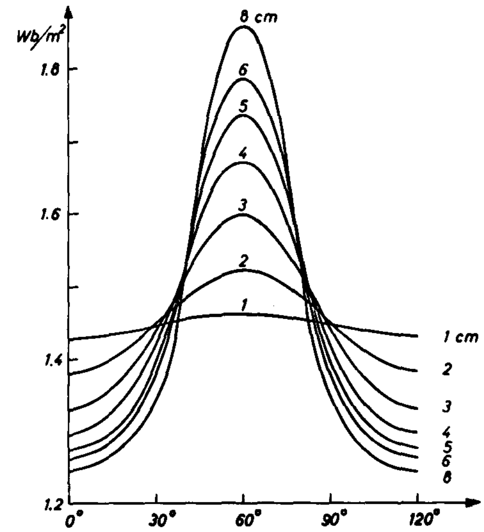

Fig. 3. Azimuthal field variation at different radii for sectors starting at r = 0.64 cm.

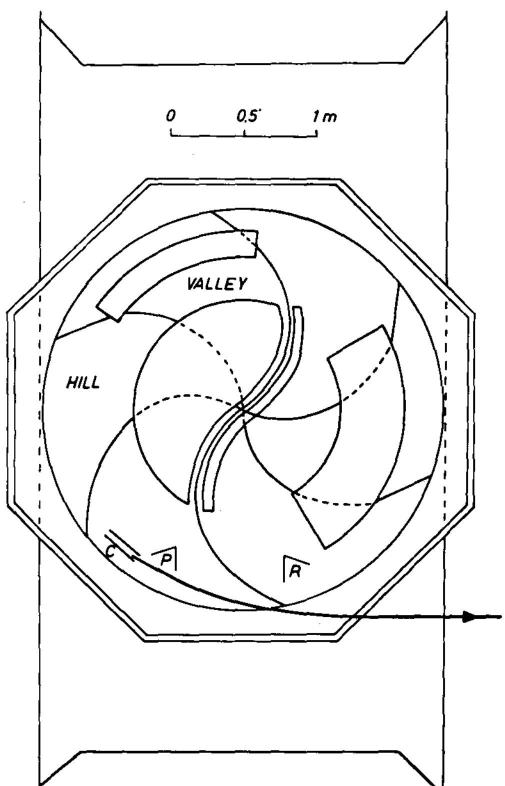

Fig. 2. Horizontal view of poles and electrodes.

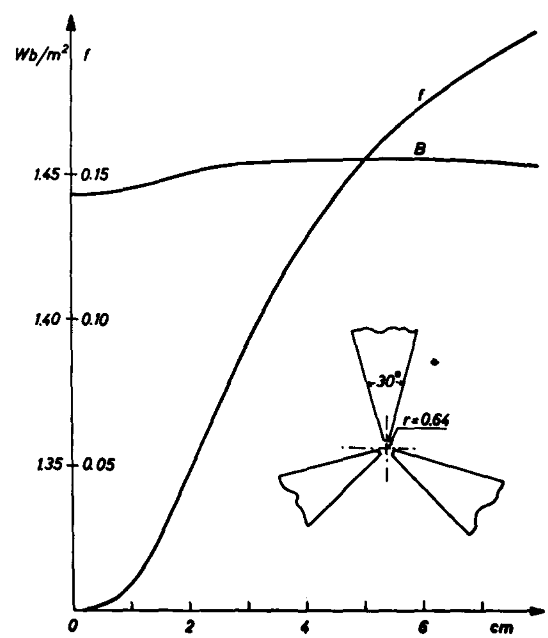

Fig. 4. Mean magnetic field B and flutter f at small radii.

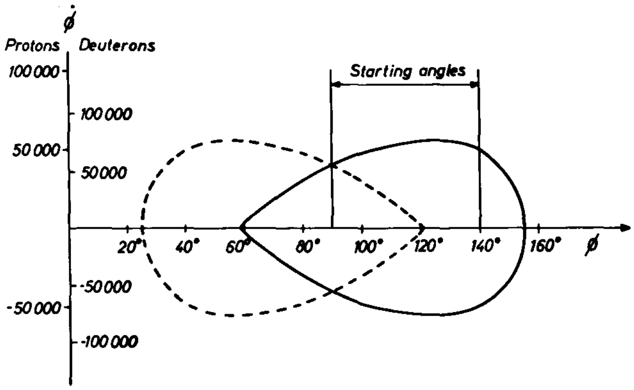

Fig. 5. Phase space for protons (solid line) and deuterons (dashed line) with $\sin\varphi_s = .0.85$, eV = 40 keV.

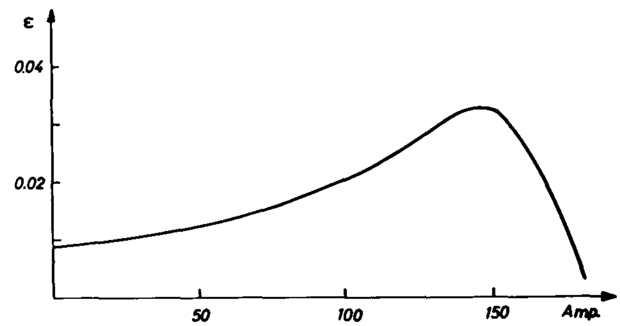

Fig. 7. Capture efficiency for deuterons as a function of coil currents.

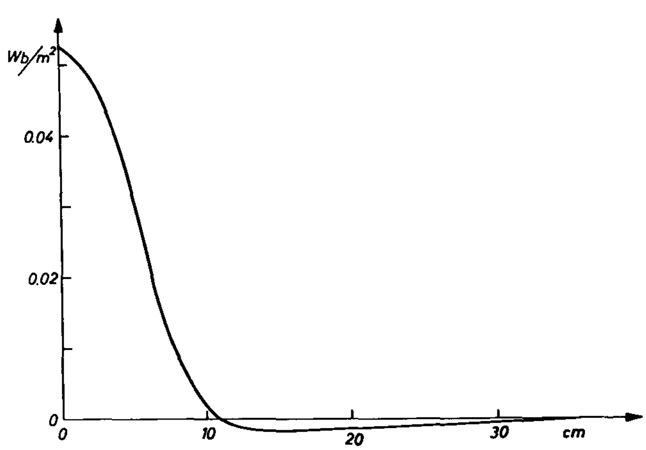

Fig. 6. Central coil magnetic field as a function of radius.

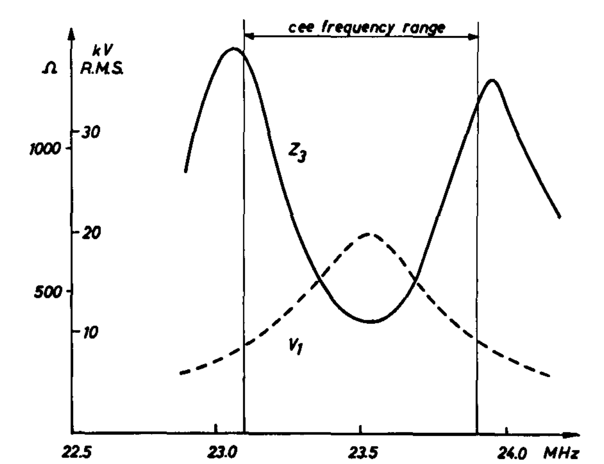

Fig. 8. Gap voltage V_1 and power tube plate load Z_3 for dee system; d.c. power = 125 kW.

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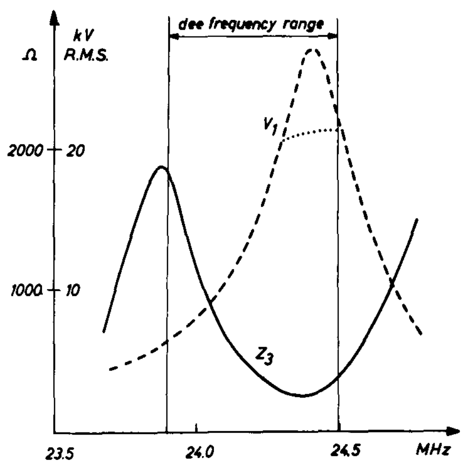

Fig. 9. Gap voltage V_1 and power tube plate load Z_3 for the dee system; d.c. power = 350 kW.

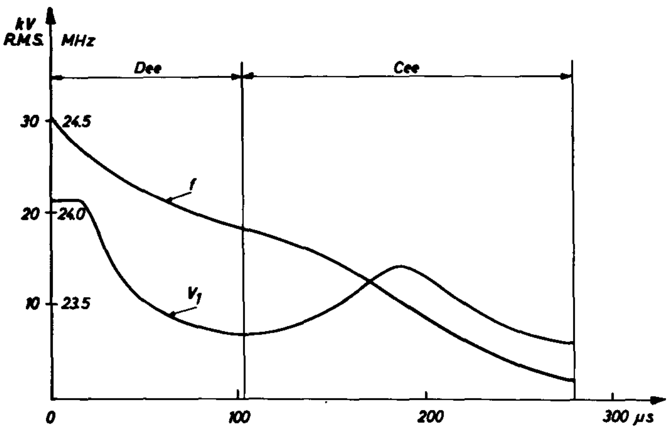

Fig. 10. Frequency f and accelerating voltage V_A ; d.c. power = 200 kW, $\sin\varphi_s$ = 0.85.

DISCUSSION

KHOE: Did you not cross the nonlinear coupling resonance $2\nu_x + 2\nu_y = 3$?

SVANHEDEN: I don't think there will be any special difficulties with these coupling resonances. As you see there, ν_r would be about 1.12, or so at full radius, and we haven't investigated all these resonances as yet; just lower modes of couplings.

KHOE: Did you study it statically or dynamically? When you consider that the momentum is constant with the synchrotron orbit, then the resonance may not be serious, but it can be serious if you cross this resonance many times, due to the phase oscillation, and because the energy gain per turn is rather small.

SVANHEDEN: I think we have to study this by computers.