# VARIABLE ENERGY EXTRACTION FROM NEGATIVE ION CYCLOTRONS* 

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## Abstract

Negative hydrogen ions are accelerated at a fixed magnetic field. The energy of an extracted beam is determined by the radius of a stripping foil and its azimuth and radius determine the exit point of the beam from the cyclotron. The exit points can be adjusted so that an auxiliary magnet causes the various extracted beams to enter a common beam transport system. Since February, 1966, external beams whose energy is variable from 25 to 50 MeV have been produced by the above means at UCLA and the University of Manitoba. While this is primarily a report on the UCLA system and its performance, special features of the Manitoba system are presented.

## Introduction

The acceleration of negative ions in a cyclotron was first reported by Lofgren. 1 The possibility of premature stripping of the ions during the course of the acceleration by the relatively large stripping cross section of air molecules raised doubts as to the usefulness of $\mathrm{H}^{-}$ions for the production of external beams of reasonable energy. However, a calculation of this cross section ${ }^{2}$ led to the suggestion that useful beams could be obtained up to 50 MeV even in synchrocyclotrons. This suggestion first was confirmed for 4.4 MeV protons ${ }^{3}$ and later for 45 to 50 MeV .4

The possibility of producing external beams of different energies by stripping the electrons from the ions at various radii is clear. ${ }^{2}$ Slightly less obvious is the possibility of producing a variable energy beam along a single external beam line. This possibility was suggested first through the use of extracted neutral particles, 5 and later through the use of extracted ions. 6 It is the development of the second method which is presented in this paper.

## Variable Energy Extraction

Variable energy extraction is accomplished by stripping negative hydrogen ions with a thin foil placed in the cyclotron circulating current. The magnetic field is held constant while the radius and azimuth of the stripping foil are chosen so that the trajectories for any energy
cross at a common point outside the cyclotron vacuum chamber. A "combination" magnet placed at this crossover point bends the trajectories into the common beam transport system as shown in Fig. 2.

It should be noted that maximum excitation of the combination magnet and the maximum deflection are required for the lowest beam energy. Virtually zero excitation is needed at the maximum beam energy.

## Limitations on Energy Range

The lower energy limit of 25 MeV currently is set by a limitation on the azimuthal swing of the stripper probe, although a push to lower energies would not extend the limit much below 25 MeV since the uniform-field width of the combination magnet is only about 2 inches. Protons of less than 20 MeV are expected to curve back into the cyclotron, setting this as the absolute minimum extracted beam energy.

## The Stripper

The stripper foil must be positioned at radii between 14 and 21 inches in order to extract protons in the energy range 25 to 48 MeV . The azimuthal location of the foil is chosen so that the trajectory of a particle originating on an equilibrium orbit will pass through the combination magnet. These criteria require that the stripping foil cover the dashed area indicated in Fig. 2.

The stripper foil is carried on a $3 / 4$ inch probe, 42 inches in length, driven radially by a lead screw. This probe and lead screw are mounted inside a $3 \times 5$ inch rectangular vacuum box, adjustable in elevation, which can be moved about a vertical axis located at the face plate of the cyclotron. The stripper foil location is readout by a digital servo system accurate to 0.04 de grees and 0.015 inches. These "swing" and "longation' readings can be converted to machine radius and azimuth by reference to a conversion table. Ordinarily a 0.1 mil beryllium stripping foil is used.

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## The Combination Magnet

The combination magnet was designed to the following constraints: 1) the magnet should fit into the extremely limited space between the cyclotron and the first quadrupole, 2) the magnet should operate with an available 4 kilowatt supply capable of supplying 100 amperes at 40 volts and 3) the magnet should have sufficient field to deflect a 25 MeV proton at least 20 degrees.

In order to make the magnet as small as possible, it was decided to use a 3 inch aperture, although the other external beam magnets all have 4 inch apertures. The 3 inch gap was reasonable since the beam height was not expected to be larger than 1 inch at the location of the combination magnet. The vacuum tank lids are $1 / 2$ inch thick. They limit the useful beam aperture to 2 inches.

The magnet yoke and pole pieces weigh 3200 pounds. They were fabricated from 1020 steel. The coils (copper tape) produce a maximum of $6 \times 10^{4}$ ampere-turns and weigh 600 pounds. Cooling is accomplished by two water cooled copper plates on each of the coils. Fig. 3 shows the magnetic field along the $Z$ axis of the combination magnet while Fig. 4 shows the field normal to the $Z$ axis at $2.5,5.0$, and 8.0 kilogauss. Note that the rectangular field equivalent length remains essentially a constant 13.5 inches for all excitations.

Since a magnet of such small pole area and large aperture will have a significant fringing field, we made "floating wire" measurements prior to its installation. The results are shown in Fig. 5. Here the experimental points are indicated by (.) and the calculated bend for these field values and a pole length of 13.5 inches are indicated by ( $x$ ).

The angle of bend for 25 MeV protons is 21.5 degrees at a field strength of 8 kilogauss. Since the required bend is only 15 degrees the combination magnet does not limit the lower energy of the system. The field is sufficiently strong to allow the combination of particles of 20 MeV if they could be extracted from the cyclotron. The operating conditions for the combination magnet are shown in Tables 1 and 2. The combination magnet is shown in Fig. 6.

## Variable Energy External Beam System

The combination magnet is located as close to the cyclotron as possible. Their yokes are separated by the 2 inches necessary to clear the cyclotron power leads. As a result of this close proximity the field in the combination magnet is 0.56 kG with no excitation. The cyclotron fringe field produces a field of 10 kG in the return yoke of the combination magnet. A magnetic field
plot extending from the combination magnet well into the cyclotron will be made for various excitations of the combination magnet during the next scheduled shutdown. The transformation matrix obtained from the field plot will allow better ion optic calculations to be made.

The "vault" quadrupole is located 24 inches from the combination magnet. This quadrupole produces a beam waist at the slit box just beyond the vault shielding wall, some 100 inches downstream. This waist serves as the effective source for the energy analyzing part of the system formed by the 45 degree bending magnet and second slit box. The bending magnet produces a waist in the horizontal plane 46 inches beyond its exit. The calculated resolution at full energy is 0.20 MeV FW . The second quadrupole produces an image of the second slits at the center of a scattering stand 80 inches downstream. The measured energy spread at this target is $<0.20 \mathrm{MeV}$ FWHM. A 35 MeV beam picture taken at this point is shown in Fig. 7. It is essentially an image of the momentum defining slits. Fig. 8 shows the 35 MeV beam picture taken at the straight-through target area. There are no defining slits in this beam.

Fig. 9 shows the 45 MeV beam photographed at the center of the combination magnet. The height is 0.3 inches. The series of seven photographs must be properly overlapped to obtain the horizontal beam size. It is $7 / 8$ inches.

The beam characteristics and magnet parameters for 25,35 , and 45 MeV beams in the straight through beam area are given in Table 3. A similar set of parameters for the 45 degree target area is given in Table 4.

## Interpolation Tables

For the initial runs at $45,40,35,30$, and 25 MeV the location of the stripping foil and the combination magnet excitation current were based on a knowledge of the equilibrium orbits in the cyclotron and on trajectory calculations through the fringing field. After the experimental determination of these parameters, a computer code was written to compute the values expected at other energies. This interpolation table gives the combination magnet power supply helipot readings and the stripper location necessary for the extraction of all energies from 20 to 50 MeV in 0.20 MeV steps. These values serve as initial settings for a new energy.

A similar procedure is used to establish the settings for the 45 degree bending magnet. Its field is adjusted by setting the power supply helipot to values found from the interpolation tables. The value can be verified by an $N M R$ monitor whose frequency is also tabulated. The
energy inferred from the deflection by the 45 degree momentum analyzing magnet has been verified by measurements using the crossover technique. ${ }^{3}$

It is interesting to note that beam energies based upon use of the stripper location alone have been within $1 \%$ of the values determined by the bending magnet field required for transmission to target. Also the bending magnet field can be set to within $0.1 \%$ of the desired field by use of the interpolated power supply helipot values.

We have found that a change in energy within the 25 to 48 MeV range can be accomplished in about 10 minutes using the above procedures. This is reduced to 2 minutes if one uses only the stripper location as an indication of the energy.

## Transmission to Target

Beam transport calculations have been made, based on the assumption that the beam at the point of stripping can be represented by an upright phase ellipse whose projections on the $x$, $\theta, y$, and $\phi$ axes are 0.1 inches, 5.4 millirad, 0.4 inches and 5.4 millirad respectively. The effect of the cyclotron fringing field at the various energies has been taken into account by calculating its transformation matrix along the beam trajectory. In this calculation the trajectory was divided into 24 one inch segments and the non-uniform field matrix calculated for each segment. The product of all such matrices is assumed to represent the fringe field. The beam envelope was then calculated through the use of the computer code TRANSPORT. ${ }^{8}$

It can be seen from Fig. 10 that the calculated dispersion at the first slit is small, while the calculated dispersion at the second slit is 0.41 inches per percent change in momentum. When the second slit horizontal aperture is 0.08 inches, only particles with $\mathrm{dp} / \mathrm{p}$ less than $\mathbf{~} 0.1 \%$ (dE less than $\pm 100 \mathrm{keV}$ ) are transmitted to the target. With this resolution $14 \%$ of the initially extracted beam should reach the target for an assumed initial energy spread of 1 MeV . Eighty percent of the extracted beam is calculated to be lost at the second slit while $6 \%$ is calculated to be lost at the first slits.

In these calculations it has been assumed that all magnets are properly aligned with respect to the optic axis. Currently the vault quadrupole is misaligned. It produces a vertical walk in the beam. This results in a loss of transmission to the target and possibly an increase in beam spot size. The measured transmission for the 45 degree target area at full energy is $10 \%$. It drops to $7 \%$ at 35 MeV and $5 \%$ at 25 MeV . of course, variation in transmission is expected as the energy is changed since the emittance of the beam is unlikely to be the same for all energies. The transmission for the straightthrough target area at full energy is $70 \%$. It
drops to $39 \%$ at 35 MeV and $40 \%$ at 25 MeV . Shortly we will undertake a magnet alignment program which should result in a better transmission than is reported above.

The calculated beam envelopes for 50,40 , 35, 30 , and 25 MeV are shown in Fig. 11 for the $45^{\circ}$ target area. The 48 MeV envelope for the straight-through target area is shown in Fig. 12.

## The University of Manitoba System ${ }^{9}$

In the extraction system in operation at the University of Manitoba, the cyclotron magnet vertical yoke is used as the flux return path for the combination magnet. Without excitation the field in the combination magnet is 4.8 kG , about the correct value for a 35 MeV beam. For minimum energy the field must be increased to 8.3 kG and for maximum energy it must be reduced to -800 Gauss.

At maximum energy, no beam loss is observed at the stripping foil or along the beam pipe ( 10 cm . I.D. with 7.6 cm . aperture). At lower energies losses occur at the exit hole and in the beam pipe. The losses reach $50 \%$ at 25 MeV .

The axial emittance is -30 mm mrad for all energies. The radial emittance increases from -120 to $>240 \mathrm{~mm} \mathrm{mrad}$ as the energy is varied from maximum to minimum. Beam energy spread is likely responsible for the rather large radial emittances.

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Table 1

Combination Magnet Parameters

| Field <br> Kilogauss | 8.0 | 5.0 | 2.5 |
| :--- | :--- | :--- | :--- |
| Rectangular equivalent magnet <br> pole length <br> Inches | 13.45 | 13.57 | 13.67 |
| Field variation over 4 <br> inch width, Gauss | 40 | 21 | 10 |
| Field variation over 6 <br> inch width, Gauss | 316 | 178 | 84 |
| Current <br> Amperes | 96 | 53 | 25 |

Table 2
Variable Energy Extraction Parameters

| Energy <br> MeV | 45 | 40 | 35 | 30 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Longation <br> Synchro units | 20.30 | 19.02 | 17.93 | 16.42 | 14.91 |
| Swing <br> Synchro units <br> Combination magnet field <br> Kilogauss | 00 | 50 | 33 | 15 | -15 |
| Radius of curvature <br> Inches | 0.5 | 1.8 | 2.7 | 4.5 | 6.2 |
| Angle of bend <br> Degrees <br> Maximum width of magnet <br> pole used | 1.0 | 200 | 125 | 70 | 46 |

Table 3
Beam Characteristics in Straight-Through Area

| Beam energy |  |  |  |
| :---: | :---: | :---: | :---: |
| MeV | 45 | 35 | 25 |
| Stripper current Microamperes/2 | 0.043 | . 12 | 0.20 |
| Faraday cup current Microamperes | 0.030 | . 046 | 0.09 |
| Transmission Percent | 70 | 39 | 40 |
| Longation | 20.23 | 17.93 | 14.58 |
| Swing | 62.4 | 32.4 | -21.6 |
| Combination setting | 0.39 | 3.08 | 6.25 |
| Quadrupole A | 4.10 | 3.74 | 4.39 |
| Quadrupole B | 4.70 | 2.68 | 3.10 |
| Quadrupole 1 | 2.00 | 1.33 | 0.11 |
| Quadrupole 3 | 3.70 | 2.87 | 2.00 |
| Full image size | $1 / 4 \times 7 / 16$ | $3 / 8 \times 1 / 2$ | 5/16x9/16 |

Table 4
Beam Characteristics in the $45^{\circ}$ Area

| Beam energy MeV | 45 | 40 | 35 | 25 |
| :---: | :---: | :---: | :---: | :---: |
| Stripper current Microamperes/2 | . 120 | . 22 | 0.40 | 0.25 |
| Faraday cup current Microamperes | 0.016 | 0.015 | 0.026 | 0.013 |
| Transmission Percent | 13.3 | 6.7 | 6.5 | 5.2 |
| Longation | 20.34 | 19.02 | 17.93 | 14.84 |
| Swing | 59.4 | 45 | 31.1 | -15.6 |
| Combination setting | 0.31 | 1.92 | 2.95 | 6.98 |
| Quadrupole A | 4.30 | 4.20 | 3.63 | 2.70 |
| Quadrupole B | 3.75 | 3.65 | 3.22 | 2.48 |
| $45^{\circ}$ bending magnet setting | 465.00 | 415.49 | 379.44 | 306.25 |
| Quadrupole 1 | 2.50 | 212 | 2.30 | 1.62 |
| Quadrupole 3 | 4.00 | 402 | 3.71 | 2.85 |
| Target size Inches | 5/32x7/16 | 7/32×3/8 | $1 / 8 \times 5 / 8$ | 5/16x1/2 |



Fig. 1. Layout showing developed variable energy beams in the south experimental area.

Fig. 3. Magnetic field along the z-axis of the combination magnet.


Fig. 2. Variable energy extraction scheme using negative ion stripping technique.

Fig. 4. Magnetic field along the x-axis of the combination magnet.


Fig. 5. Angular deflection produced in the combination magnet for various energies and magnetic fields. (.) are the points measured by wire orbiting and (x) are the points calculated for an assumed field length of 13.5 inches.


Fig. 6. Photograph of the combination magnet.


Fig. 7. 35 MeV beam photograph taken at the 45 degree target area.


Fig. 8. 35 MeV beam photograph taken at the straight-through target area.


Fig. 9. Beam photograph taken at the center of the combination magnet at 45 MeV . The beam spot height is 0.3 inches.

Fig. 11. Variable energy beam trace of 50, 40, 35,
and 25 MeV for the 45 degree target area.


Fig. 10. Calculated beam trace of the 45 degree
beam, showing the effect of $0.05 \%$ and

## PANEL DISCUSSION

TENG: Before we open the general discussion, we will have a short panel discussion. I believe we can summarize the first three papers as all belonging to the resonant extraction scheme; the last paper is, of course, in a category of its own. Can we get some opinion from the panelists as to the resonant extraction first? Many different resonances have been used: integral, halfintegral, third-integral. As a guidance for future designers for extraction systems, which resonance is best in relation to energy spread, oneturn extraction, and the energy compression effects you talked about?

HAGEDOORN: The regenerative action of peeler and regenerator arises from the radial derivatives of the mean magnetic field and the second harmonic field component. Generally, large oscillation amplitudes are excited due to the halfintegral resonance, which appears in the second degree part of the Hamiltonian. Often the firstdegree part in the Hamiltonian is forgotten. This part arises from the first harmonic components of the field perturbation, which are energy dependent components. The energy dependent components give the effect of energy. compression, and thus better energy resolution. If the oscillation amplitudes increase the energy independent parts of the Hamiltonian become more important, and the effect of energy compression becomes relatively less important. In the case of precessional extraction, the derivative of the mean magnetic field is energy dependent. The quality of an external beam will remain good as long as the extraction systems remain linear. Therefore, I do not like the use of nonlinear extraction systems.

GORDON: I think an essential difference is that the integral resonance drives the beam as a whole off center, develops a coherent oscillation in the beam. The half-integral resonance is a higher order effect and, I think, requires that the beam be initially off center. As for higherorder resonances, I think this would be most difficult; the non-linearity would really distort the phase space very badly. In the half-integral resonance, because of a stop band, there is some energy within which to operate and to develop turn separation. In the integral resonance, just passing through the resonance itself has the nice effect of driving the beam off center.

KIM: In the beam extraction for the sectorfocused cyclotron, I would like to talk about two methods, using $\nu_{r}=1$ resonances, namely $2 / 2$ stop band as regenerative beam extraction, and 3/3 non-linear resonance. I prefer to use regenerative beam extraction rather than existing non-linear force from the machine itself, because we can control the strength of the driving force easily in the regenerative extraction by changing the additional gradient harmonics, but we cannot control the driving force in 3/3 resonance.

TENG: In the case of non-linear resonances, the statement people usually make is that there is this precession; therefore, the so-called turn separation could be larger. It is really not turn separation any more. One uses third integral resonance as every third turn. That's your $\nu_{r}-1$ term. Is that a significant advantage of going ${ }^{r}$ to non-linear resonances? To have larger turn separations?

HAGEDOORN: Turn separation is an important parameter; but it is not necessary to have too much separation. The linear extraction systems already give sufficient separation for beam extraction.

It is possible, however, to use the $3 / 3$ resonance for extraction. But we all have seen the strangely distorted phase-space figures which arise from originally nice well-concentrated spots. Of course, the phase-space area remains constant. But the physicist will observe the distorted, banana-shaped figures as rather large areas, thus as beams with a bad quality. So, I must conclude that at this moment the use of $3 / 3$ resonance does not seem to be favorable.

TENG: I believe that one of the main advertised advantages of the negative-ion acceleration is the ease of extraction at various energies. Dr. Wright, would you like to say something in that regard?

WRIGHT: As most people probably know, for the UCLA cyclotron it's really the only possibility for getting variable energy. Since the cyclotron is a very high-field machine, we cannot vary the shape of the field for producing variable energy. We were very thankful that negative ions are available.

We do have under construction a special probe with which we hope to measure the linear dimensions of the source at the stripping foil. We use a foil that strips about $99.9 \%$ of the ions from negative to positive, but leaves maybe a tenth of $1 \%$ as neutrals. By measurements on those neutrals inside the machine we hope to get some idea of the angular width of the source, and with that information as input data to follow the beam from that point down to the various target areas, in a quantitative fashion. That's our program. We have only a very short period of operation in testing the variable energy negative-ion setup.

GORDON: We have been very careful in magnet and rf design to maintain a constant-turn pattern. Also, with the trim coils we have found it not difficult to maintain the variation of $\nu_{r}$ with energy in the extraction region independent of excitation. We have found that the extraction proceeds at all energies with about the same conditions.

HAGEDOORN: Negative ions are extracted by foils. Thus, the external beam consists of positive ions. For the energy resolution of analyzing magnets, slit-scattering is a serious problem. This problem can be decreased by the use of negative ions in the external beam. For
this reason the extraction of negative ions by the conventional extraction systems would be nice.

KIM: Precessional methods and single-turn extraction are essentially the same, except it requires narrow rf phase and an accurate instrumentation to work the single-turn extraction. In the low-energy high-dee-voltage machine the regenerative extraction may give the same efficiency and quality if the beam has the narrow rf phase and the good internal beam quality required for single-turn extraction. For the higher energy machine, over $100-\mathrm{MeV}$ protons, the rf phase slip near the extraction radius would introduce difficulties of operation for the precessional method and single-turn extraction. The regenerative extraction may operate very well for the higher energy machine. Consequently, a combination of the three methods may be best for the future variable-energy multi-particle machine.

TENG: Thank you. Now, let me ask for open discussion.

KHOE: How critical is the phase of the field bump?

GORDON: The first-harmonic field bump is significant only in the neighborhood of the resonance. We have very carefully measured the field there and find the intrinsic first harmonic to be about one gauss; we know what its Fourier components are. With our harmonic coils we can easily control the amplitude of the field bump to within a twentieth of a gauss, and the azimuth of the field bump to 3 degrees. This is quite adequate.

BLOSSER: With respect to the business of stripping extraction, if the beam is first directed into the fringe field, before the stripping, there is a very nice dividend in that where stripping occurs the fringe field shifts from being radially defocusing to being radially focusing, and axially, just vice versa. This is the equivalent of a very strong quadrupole pair at exactly the point of extraction; thus, an extremely compact beam comes out of the machine. As a result of this phenomenon, with our setup we are now able to put our first set of real quadrupoles something like 15 feet away from the machine.

I believe in high voltage, and I would simply like to point out that Dr. Hagedoorn did not remark that the efficiency goes down as you lower the voltage. If you put maximum emphasis on the energy spread, a way of getting it is of course always via phase selection. If we limit the phase adequately, we can make the energy spread anything we want.

HAGEDOORN: The voltage can be lowered still more; of course, you will have a smaller orbit separation. But the orbit separations may be made by precessional extraction. So I think the lower limit of the voltage in most cyclotrons will be just enough to clear the ion source. Most cyclotrons will even not reach the best voltage for the beam.

GORDON: The sensitivity of the operation of the machine to voltage errors varies inversely as the number of turns, but the sensitivity to field errors varies inversely as the square of the number of turns. I think that if the number of turns gets too large, the problem of regulation will just simply become unmanageable. I think it a great advantage to have a large energy gain per turn so that we can accelerate an extra inch, at least, into the fringe field. That is, we get the beam out to where $\nu_{r}=0.8$, and the turn separation generated by the precession is really substantial. Furthermore, there is that much less fringe field to pass the beam through; this makes the actual deflection of the beam that much easier.

REISER: High voltage is also an advantage, from the point of view of phase-space density. It is clear that the larger your dee voltage, the better you can define your beam in the center. There is more space for ion optical shaping of the electric fields, for example. It is also clear that the minimum voltage, the voltage that barely clears the ion source, does not give an ion optically well defined beam because of the electric field configuration. So, I think this is very important to look at this in terms of phase-space density, in addition to the problem of extraction efficiency.

May I ask one question of Dr. Gordon? We have discussed the difference between the resonance extraction and the regenerative extraction. In the resonance extraction you are forced to make use of the resonance. The $\nu_{r}=1$ resonance occurs in a highly non-linear ${ }^{r}$ region in your magnetic field, so that you get non-linear terms, which lead to filmentation of your phase-space areas, whereas in the regenerative extraction system you have a choice of placing it in a favorable region where non-linear terms, say prior to the $\nu_{r}=1$ resonance, do not lead to such distortion. ${ }^{r}$ I was amazed to see what very nich phasespace areas you have shown for this $\nu_{r}=1$ resonance; I remember former areas which were to some extent highly distorted. How did you get rid of the non-linearities in this region?

GORDON: First of all, the extraction takes place over a relatively few turns. It was something between 15 and 20 turns. In the regenerative case, where you are using a half-interval resonance, it requires a large number of turns. The beam is generally just sitting in a region of highly non-linear field. I suspect that the non-linearities are worse where you have to wait a long time for the resonance to take effect than they are in the case of the integral resonance where it is straight orbit displacement, 1, 2, 3, and the beam is out.

As far as the shape of those nice "flying saucers," (my Fig. 7), we have recently come to realize that we can get much more beam into a smaller radial interval than we had previously ever hoped for. Those spots corresponded to a beam which, not considering energy spread, was a tenth of an inch in radial width in the middle of the machine, and this apparently is quite adequate.


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