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EXTRACTION STUDIES IN AN AVF CYCLOTRON

Invited Paper

H.L. Hagedoorn and P. Kramer
Philips Research Laboratories
N.V. Philips' Gloeilampenfabrieken
Eindhoven - Netherlands

Abstract

For the extraction of the beam in an AVF cyclotron a regenerative system or an electrostatic channel can be used.
Some factors of importance for the beam will be discussed.

Extraction with an electrostatic channel can give very high external beam quality and extraction efficiency. For such a system it is of importance whether the beam is extracted before or after the $\nu_R=1$ resonance. The beam can even pass the $\nu_R=2\nu_z$ resonance without blowing up vertically if attention is being paid to the radial oscillation amplitude and some machine parameters.

The general orbit theory can be used for an analytical study of a regenerative system in an A.V.F. cyclotron.

1. Some general remarks

The separation between successive orbits in a cyclotron is the most important quantity for the extraction of the beam. In classical cyclotrons very high Dee voltages are used. The separation is, therefore, rather large (1 cm). In these cyclotrons the beam is extracted by means of an electrostatic field applied between a septum and a high voltage electrode. With a cooled septum large external beam currents can be obtained[1].

In synchrocyclotrons the Dee voltage is rather small and the orbit separation a fraction of a millimeter. In this case an electrostaic deflection system is useless. However, there are several methods to enlarge the orbit separation. In the first place, with the regenerative method the radial oscillations are excited in such a way, that a large increase of the oscillation amplitude per revolution is obtained[2,3,4].
Secondly, the precession of the orbit centre can be used to get sufficient orbit separation[5]. This separation is equal to $2\pi(\nu_R-1)A$ (where A represents the radial oscillation amplitude and ν_R the radial oscillation frequency).

Regarding this second method it is very convenient to have a radial oscillation frequency, which differs much from unity. This occurs if the beam is accelerated far into the fringing field.

In most cyclotrons the particles cannot be accelerated beyond the n = 0,2 point. At this place the $\nu_R = 2\nu_z$ coupling resonance causes the beam to blow up. However, with careful control of the radial and axial oscillations the beam can be accelerated through this resonance to the n=1 point, where it has to leave the cyclotron almost automatically[6].

With the regenerative method large increasing radial oscillation amplitudes are introduced, the phase of the oscillations remaining constant. With the orbit precession method a fixed amplitude and a rapidly increasing phase is used. Both mechanisms give about the same results if applied in the right place.

As the quality of the internal beam in classical- or synchro-cyclotrons is very poor anyway, the external beam is relatively undisturbed by the method of extraction. If, however, the internal beam has a high quality one has to take care that the extraction system does not introduce non-linear effects, affecting the external beam quality.

In isochronous cyclotrons the quality of the internal beam can be very high as a result of careful adjustment of the magnetic field[7] and the geometric position of important components such as source and Dee[8]. In isochronous cyclotrons large oscillation amplitudes can easily give rise to nonlinearities. Extraction by means of the 3/3 resonance driving forces, which results in a nonlinear motion, is rather difficult. Therefore, extraction systems using the smallest possible radial oscillation amplitudes are preferred.

The best methods appear to be the normal electrostatic extraction channel with use of orbit precession[9] and the linear regenerative system with small amplitudes[10,11].
The first method permits the ions to be accelerated to a higher energy than the second one. Especially in small cyclotrons the difference can be relatively large.

2. The extraction by means of the orbit precession method

For a study of the orbit precession method the following points are important. The acceleration through the $\nu_R = 2\nu_z$ resonance, the aperture of the electrostatic channel and the influence of small radial oscillations on the properties of the external beam. These points together with some experimental results will be discussed in the next sections.

2.1 Acceleration through the $\nu_R = 2\nu_z$ resonance

If the particles are to be accelerated

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as far as possible into the fringing field (see chapter 2,3) they have to pass the $\nu_R = 2\nu_z$ resonance. This resonance has already been studied by several authors[12,13].
Following the analytical studies one observes that blowing up of the beam due to this resonance can be avoided if the radial oscillations, as well as the second derivative of the mean magnetic field, are small and if there are not too many revolutions lying inside the resonance region. If the Dee aperture is large enough and a sufficient number of revolutions are made a total exchange of energy between the two oscillators is possible without loss of beam intensity (if the amplitudes are too large an instable motion can occur[12]).
Smallest possible energy exchange is required for maximum extraction efficiency and beam quality.
In fig. 1 the effect of the resonance is shown in a well adjusted and a maladjusted magnetic field. In the latter case there is no decrease of the total internal beam current, but a lower external beam intensity, due to the aperture of the electrostatic channel.

(For the investigation of resonances it is very convenient to use action and angle variables, see ref. 14). We will apply these here very briefly to the $\nu_R = 2\nu_z$ resonance. These variables are defined by

$$\pi_x = (2 I_x \nu_x)^{\frac{1}{2}} \sin(\varphi_x - \nu_x \vartheta)$$

$$x = \left(\frac{2 I_x}{\nu_x}\right)^{\frac{1}{2}} \cos(\varphi_x - \nu_x \vartheta)$$

$$\pi_z = (2 I_z \nu_z)^{\frac{1}{2}} \sin(\varphi_z - \tfrac{1}{2}\nu_x \vartheta)$$

$$z = \left(\frac{2 I_z}{\nu_z}\right)^{\frac{1}{2}} \cos(\varphi_z - \tfrac{1}{2}\nu_x \vartheta)$$

(I_x, φ_x and I_z, φ_z are the new action and angle variables; π_x, x and π_z, z are the relative radial and axial moments and coordinates; ν_x and ν_z are the radial and axial frequencies; ϑ is the azimuth).

This canonical transformation is applied to the Hamiltonian of the coupled motion (eq. 42 , ref. 12).
We neglected orders higher than the third degree but we take into account terms with first and second derivatives of the mean magnetic field ($\mu' = \frac{r}{B}\frac{dB}{dr}$ and $\mu'' = \frac{r^2}{B}\frac{d^2B}{dr^2}$).
A new simple time independent Hamiltonian is found which reveals all the properties of the resonance:

$$H = I_z \Delta\nu - g'' I_x^{\frac{1}{2}} I_z \cos(\varphi_x - 2\varphi_z)$$

($\Delta\nu = \nu_z - \frac{1}{2}\nu_x$; $g'' = \frac{1}{\sqrt{2}}(\mu' + \mu'') + \frac{1}{4\sqrt{2}}$; if $|\mu''| \gg |\mu'|$ and $|\mu''| \gg 1$ one finds $g'' = \frac{1}{\sqrt{2}}\mu''$).

H and $I_o = I_x + \frac{1}{2} I_z$ are constants of motion. Therefore, the above equation completely described the relation between the variables I_z and $\Phi = \varphi_x - 2\varphi_z$.)

Numerical calculations in the Philips prototype cyclotron[15] gave, as a very pessimistic estimate a 40% increase in height. The observed increase of the height is well within this value.

Heavy ions (with a very low e/m ration) wich are accelerated in a variable energy AVF cyclotron for 150 MeV protons will pass the $\nu_R = 2\nu_z$ resonance at a radius equal to 85% of the maximum orbit radius (then $g'' \approx \frac{1}{4\sqrt{2}}$). Though the number of revolutions in the resonance region can be very large in this case, it can be shown that the increase of the height of the beam is negligible.

2.2 The aperture of the electrostatic channel

For the calculation of the aperture of a channel it is convenient to assume a central trajectory, and to consider small linear deviations with respect to this trajectory. Using matrix calculation the channel walls at arbitrary azimuths can be transformed to the entrance. In this way the aperture of an electrostatic channel of the prototype cyclotron has been calculated and also the region in phase space representing the particles striking the cyclotron centre side of the septum[9].
The size of the aperture in phase space can be about 15 mm mrad before intensity loss of the external beam becomes observable.
If the shape of the particle orbits changes with energy due to the influence of saturation effects in the pole the size of the aperture will accordingly change.
For this reason an electrostatic channel must be mechanically adjustable in order to keep the aperture as large as possible and furthermore very high magnetic fields must be avoided.

The beam leaving the extraction channel has a large horizontal divergence. In order to refocus this beam, either a special shape must be given to the electrode and septum of the channel[16], or a magnetic channel with a positive field gradient must be used. The latter method is very attractive as the deflecting power of the electrostatic channel can be decreased, that is to say, a lower high voltage on the electrode.

The ion optics of the beam in the fringing field is a determining factor for the quality of the external beam[8].

2.3 The influence of small radial oscillations

The maximum separation between two successive orbits is given by the following relation:

$$\Delta R \approx \left(\frac{V/E}{1+\mu'} + 2\pi\,|\nu_R - 1|\,\frac{A}{R}\right) R$$

(where 2V is the energy gain per revolution, E the energy of the particles, ν_R the radial frequency, A the radial oscillation amplitude, R the radius and $\mu' = \frac{r}{B}\frac{dB}{dr}$).

The first part on the right hand side is the increase in radius due to the increase in energy. The second part arises from the orbit precession. For typical numbers $\nu_R = 0{,}8$, $\mu' \approx -0{,}5$, $R \approx 500$ mm, $V/E \approx 1/500$ and $A \approx 3$ mm we find a separation $R \approx 5{,}6$ mm.

For a good extraction efficiency the cohaerent radial oscillation amplitude is about equal to the incohaerent oscillation amplitude. If the cohaerent oscillations are allowed to be made much larger, loss of intensity occurs at the $\nu_R = 2\nu_z$ resonance. Due to the radial oscillations only a small fraction of the beam will hit the front of the septum. As the septum is rather thin (e.g. 0,5 mm) the above estimated numerical value for the orbit separation ΔR is sufficient.

In fig. 2 we have represented the motion of a grid of particles with the same energy during the last revolutions before extraction. Line a shows a particle which practically moves on an "equilibrium orbit". Line b is the motion of a particle with a radial oscillation amplitude of about 5 mm. One can observe clearly the two effects which cause orbit separation and which together result in a cycloidal motion.

If the septum is placed at 53,4 cm (see fig. 2) particles of two successive revolution numbers can be extracted.

A more detailed analysis shows that the full width half peak value of the energy spectrum is about equal to the energy difference between the two drawn grids (for this calculation we started from a continuous distribution in energy before $\nu_R=1$).

For a good energy resolution it is important that all particles with the same energy have the same radial oscillation phase, in order to be extracted at the same moment. Particles which start before the $\nu_R=1$ resonance at the same position in phase space and with the same H.F. phase but having different energies, after a number of revolutions, will have got a radial oscillation phase difference. The particle with less energy will get a retarded phase. This particle will be accelerated further until its phase has the value required for extraction. Due to this effect the energy spread of the extracted beam is diminished.

The particles get their radial oscillation phase at the $\nu_R=1$ resonance. (A simple calculation illustrates this effect. We assume two particles with an energy difference ΔE. These particles start at the $\nu_R=1$ resonance with the same radial oscillation phase: $\varphi(E_{10},\nu_R=1) = \varphi(E_{20},\nu_R=1)$; $E_2 > E_1$.

The coordinates are transformed in phase space in such a way that $\nu_R - 1 = \omega$ is the new frequency. After n revolutions the two particles will have oscillation phases equal to

$$\varphi_1 = \int_a^b \omega(E_1)\,dn \qquad \varphi_2 = \int_a^b \omega(E_2)\,dn$$

The difference thus becomes in phase

$$\Delta\varphi = \int_a^b \omega(E_2) - \omega(E_1)\,dn$$

The integration extends from $\nu_R=1$ (a) to extraction (b).

We have assumed that ω only depends on the energy. This is true as long as the motion remains linear, i.e. if the oscillation amplitudes are not too large. It follows that

$$\Delta\varphi = \frac{\Delta E}{dE/dn}\int_a^b d\omega = \frac{\Delta E}{dE/dn}\,(\omega_b - \omega_a)$$

As $\omega_a = 0$ $(\nu_R=1)$, we find

$$\Delta\varphi = \frac{\Delta E}{dE/dn}\cdot\omega_b$$

ω_b is the oscillation frequency at extraction, dE/dn is the energy gain per revolution. Thus for $\Delta E = dE/dn$, particle 1 will be accelerated one revolution more to get the right phase at extraction. At the same time particle 1 will then have the same energy as particle 2. This effect of phase retardation only appears if the radial frequency changes.)

If not too many revolutions are made after the $\nu_R=1$ resonance the cohaerence of the excited radial oscillations will persist and HF phase differences between the particles will not disturb the effect of the retarded phase.

If we consider fig. 2 again we can give some criteria for improvement of the energy resolution. If the two grids are farther separated than in this figure, the energy resolution will be better. This separation will be large if the rate of change of oscillation phase is large (i.e. if ν_R differs much from unity). The same effect will be caused by a large oscillation amplitude.

Thus, small energy spread requires: a large (ν_R-1) value, as large as possible an oscillation amplitude and, of course, a good beam quality. In fig. 3 a grid of particles is shown having a smaller amplitude than in fig. 2. For this case a worse energy resolution is to be expected. In fig. 4 the calculated energy spectra are given corresponding to figures 2 and 3.

The energy spectrum depends in a peculiar way on the Dee voltage. If the Dee voltage is higher than the one used in fig. 2, the energy difference between the two grids becomes larger, but the separation will not increase appreciably as the main contribution to the orbit separation originates from the precessional motion. One cannot expect a better energy resolution. It may even be worse, unless larger oscillation amplitudes are used, or unless the beam is accelerated still further into the fringing field, so that the (V_R-1) value increases. (This can be done owing to the higher Dee voltage. For very large orbit separations the extraction channel becomes energy selective and the extraction efficiency will be decreased.)

If the Dee voltage is lower than the one used in fig. 2 it may occur that the distance between the two turnpoints P_1 and P_2 of the cycloid becomes smaller than the radial width of the beam. Due to this a part of the beam will be accelerated $\frac{1}{\nu_R - 1}$ revolutions more. Separate peaks in the energy spectrum of the external beam will appear and at the same time the width of the peaks is increased strongly. This very point will be explained in more detail in section 2.7.

One thus can conclude that there is an optimal value, or at least a minimum value of the Dee voltage, for maximum energy resolution.

The orbit separation in a cyclotron with k times larger dimensions will remain the same if the ν_R-1 value and the oscillation amplitude remain constant and if the Dee voltage is increased k times. This means that the energy resolution improves inversely proportionally to $\sqrt{E} = k$.

The energy resolution which can be obtained in small cyclotrons (30 MeV) is about 0,3%. This value will be less than 0,2 % in cyclotrons in the 70-100 MeV region.

2.4 The excitation of small radial oscillations

By means of first harmonics in the magnetic field at the place of the ν_R=1 resonance the phase and amplitude of the radial oscillations can be controlled. The energy of the external beam can be varied by changing the phase at the ν_R=1 resonance or by changing the number of revolutions by means of the Dee voltage and the main magnetic field.

2.5 Some experimental results

The energy spread of the external beam in our small 30 MeV cyclotron is about 0,3%.

Extraction efficiencies above 80% can be reached for variable energy. Due to this high efficiency large external currents are possible (100 μA).
The quality of the external beam was found to be about 10 to 15 mm mrad for 25 MeV protons (axial and radial[5]).

2.6 Observation of the radial oscillations before extraction

With a differential probe the radial oscillations can be observed. The intensity measured on the differential probe is a minimum when the orbit centre moves in the direction of the probe and a maximum when the centre moves in opposite direction. Thus a smaller fraction of the beam will hit the front of the septum as the centre moves in the direction of the entrance of the extraction channel. On a target 180° away from this entrance maximum intensity can be expected on the differential probe if the cyclotron is adjusted for the best extraction efficiency.

In fig. 5 the intensity on the differential probe is represented as a function of radius. The azimuthal position of this probe is about 150° upstream with respect to the extraction channel entrance. The increase in intensity just before extraction is clearly visible. This indicates that the radial oscillation phase has the value required for extraction.

2.7 Occurrence of separate peaks in the energy spectrum of the external beam

When particles with the right radial oscillation phase for extraction just miss the entrance they must make one complete radial oscillation in phase space (with frequency ν_R-1) before getting again the right phase for extraction.

To illustrate this effect (see fig. 6) the radial position of the septum in the phase space of the $(n-k)^{th}$ revolution is transformed to the phase space of the n^{th} revolution and represented by the line n-k (k is the number of revolutions before the n^{th} revolution).
All particles to the right of line n-k (direction of the arrow) go into the extraction system after n-k revolutions. They may go through the channel or hit some parts of it but they will not be accelerated any further.
Particles to the left of the line n-k will be accelerated further. In this way regions in phase space are determined representing particles extracted at a certain revolution number, i.e. with a certain energy.

Therefore, a beam represented by a shaded area in fig. 6a will give discrete energies. For large ν_R-1 values these discrete energies are close to each other, for small ν_R-1 values the discrete energies will be widely separated.

The size of the discrete energy regions depends linearly on the radial separation between two successive equilibrium orbits. This separation narrows at lower Dee voltages or at larger values of $\frac{r}{B}\frac{dB}{dr}$, which are small or positive before ν_R=1 and negative after this resonance (see section 2.1).

If these energy regions become smaller than the area occupied by the beam, several different revolution numbers will be extracted and the energy spread of the beam increases.

2.8 Extraction before or after the $\nu_R=1$ resonance?

Extraction before $\nu_R=1$ has many disadvantages compared with extraction after this resonance. The orbit separation before $\nu_R=1$ is small (see section 2.3), resulting in a lower extraction efficiency and, as more particles will hit the septum, in a strong decrease in intensity of the external beam.

The retardation effect (see sect. 2.3) of the radial oscillation phase only appears when ν_R changes rapidly. Before $\nu_R=1$ the radial frequency remains practically constant. The energy spread will, therefore, increase. The discrete energy regions (see sect. 2.7) can be rather small, resulting again in a larger energy spread. At the same time separate peaks in the energy spectrum can easily appear. Finally, the bram is extracted at a rather small radius. This requires a higher voltage on the extraction electrode and limits the maximum energy of the cyclotron.

Most of these arguments against extraction before $\nu_R=1$ are not effective if the quality of the internal beam is very high and if the high frequency phase width is small (~ 5°).

3. Application of the general orbit theory to a study on regenerative extraction

In the general orbit theory[14] we have shown the influence of first and second harmonics perturbations in the magnetic field on the radial motion of the particles. The perturbed Hamiltonian has a first degree part consisting of two parts. The first part arises from a first harmonic component in the magnetic field. The second part is e.g. the result of an interference between second and fourth harmonic components with third harmonic components if the main magnetic field has threefold symmetry. In this theory it is assumed that the components of the perturbation fields are small compared to the components of the main magnetic field. But their derivatives may be very large and can not be neglected. These assumptions are generally true for peeler and regenerator fields. For a description of the particle motion in these fields we will only use here, as an example, the linear theory. The method can, however, easily be extended to the study of the influence of non-linear terms (in ref. 14 eq. 11.2 the Hamiltonian is given up to the fourth degree). The Hamiltonian of the radial motion in action and angle variables is represented by

$$H = \frac{1}{\sqrt{2}} I^{\frac{1}{2}} (\tilde{A}_1 \cos\varphi + \tilde{B}_1 \sin\varphi) + \left\{(\nu_R-1)+ \tfrac{1}{2}A'_0 + (\tfrac{1}{2}A_2 + \tfrac{1}{4}A'_2) \cos 2\varphi + (\tfrac{1}{2}B_2 + \tfrac{1}{4}B'_2) \sin 2\varphi\right\} I .$$

The first harmonic components $\tilde{A}_1$ and $\tilde{B}_1$ are defined in equation 11.2 of ref. 14. A_0, A_1,....... are the relative harmonic components of the magnetic field. A'_0, A'_2, etc., are the relative derivatives

$$(A'_2 = r \frac{dA_2}{dr} \text{ etc.})$$

It is convenient to transform the action and angle variables to X,Y coordinates, defined by

$$X = (2\ I)^{\frac{1}{2}}\ R_0 \cos\varphi$$
$$Y = (2\ I)^{\frac{1}{2}}\ R_0 \sin\varphi$$

(see ref. 17). R_0 is the mean radius of the orbit. X and Y are now the Cartesian coordinates of the orbit centre.

The new Hamiltonian is:

$$H = \tfrac{1}{2}\tilde{A}_1 R_0 X + \tfrac{1}{2}B_1 R_0 Y + \tfrac{1}{2}(\nu_R-1+\tfrac{1}{2}A'_0)(X^2+Y^2) + (\tfrac{1}{4}A_2 + \tfrac{1}{8}A'_2)(X^2-Y^2) + (\tfrac{1}{2}B_2 + \tfrac{1}{4}B'_2)XY$$

We now choose our coordinate system in such a way that B_2 and B'_2 are zero (this is permitted only if the phase of the second harmonic does not depend on the radius).

The resulting equations of motion become very simple:

$$\dot{X} = \tfrac{1}{2}\tilde{B}_1 r_0 + \left[(\nu_R-1) + \tfrac{1}{2}A'_0 - \tfrac{1}{2}A_2 - \tfrac{1}{4}A'_2\right] Y$$
$$\dot{Y} = -\tfrac{1}{2}A_1 r_0 - \left[(\nu_R-1) + \tfrac{1}{2}A'_0 + \tfrac{1}{2}A_2 + \tfrac{1}{4}A'_2\right] X$$

The solution for X is

$$X = a + be^{-\omega\vartheta} + ee^{\omega\vartheta}$$

ω is defined by

$$\omega^2 = (\tfrac{1}{2}A_2 + \tfrac{1}{4}A'_2)^2 - (\nu_R-1 + \tfrac{1}{2}A'_0)^2$$

Instability only occurs for $\omega^2 > 0$. In fact $e^{\omega\vartheta}$ and $e^{-\omega\vartheta}$ are the eigenvalues in matrix theory.[3]

For the study in the motion in a regenerative extraction system the regenerator and peeler fields are expanded in Fourier series. The Fourier components can be substituted in the above equations and the solution is easily found.

Several solutions of the system have been investigated, using an analogue computer, which is a most useful supplement to analytical and digital methods (ref. 17).

As an example we have used linear regenerator and peeler fields of the shape

$$J(R_0) + (\frac{R-R_0}{R_0})\ J'(R_0)$$

$J(R_0)$ has been changed slowly to take into account the influence of the acceleration. At the same time A_1, B_2 and A_2 vary slowly, as they originate from $J(R_0)$.

During acceleration the derivatives remain constant. In fig. 7 a grid in phase space is followed during several revolutions through the regenerator and peeler. The motion is considered in a region where $\nu_R > 1$ without perturbations. The positions of regenerator and peeler are choosen in such a way that the best position of the channel (electric or magnetic) is in a valley (see fig. 8).

From these studies some conclusions can be drawn resembling very much those of the precessional extraction case. In the first place particles with relatively low energy will have a small amplitude and will be accelerated until they have the right amplitude for extraction, the action of the regenerator system being retarded. This effect improves the energy resolution and was called energy compression by Kim[18].

A second effect is that the energy resolution depends on the size of the cyclotron roughly in the same way as with precessional extraction. Thus a k times larger cyclotron will have a resolution which is about $k = \sqrt{E}$ times better.

The retardation effect of the linear regenerative system originates mainly from the first harmonic components of the field. The influence of this effect decreases if the amplitudes become too large. In that case the time independent parts in the Hamiltonian become very important for the increase of the amplitudes. With this system orbit separations of about 1 cm with 3 cm amplitudes can be obtained.

The strength of the regenerator and peeler fields necessary to get a certain effect depends slightly on the azimuthal position[10,11]. This is shown in fig. 9.

The axial motion in the regenerative system has been studied. It is found that a wide variety of different regenerator and peeler strengths give a good axial stability (see also ref. 10,11).

Comparing both types of extraction systems, we must conclude that they will give about the same results. The only important differences lie in the larger oscillation amplitudes and the lower attainable energy in the regenerative system.

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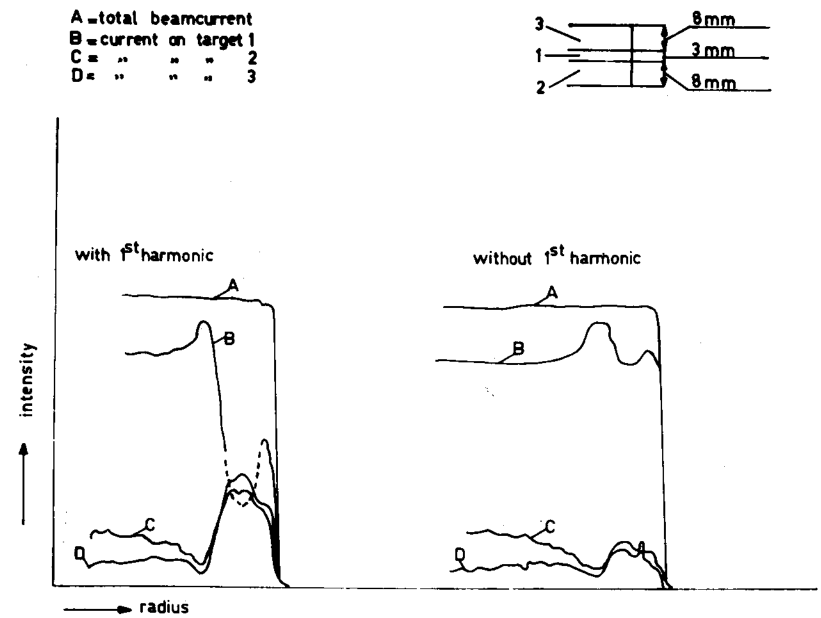

Fig. 1 The intensity distribution of the beam as a function of radius, measured with a threefinger target.

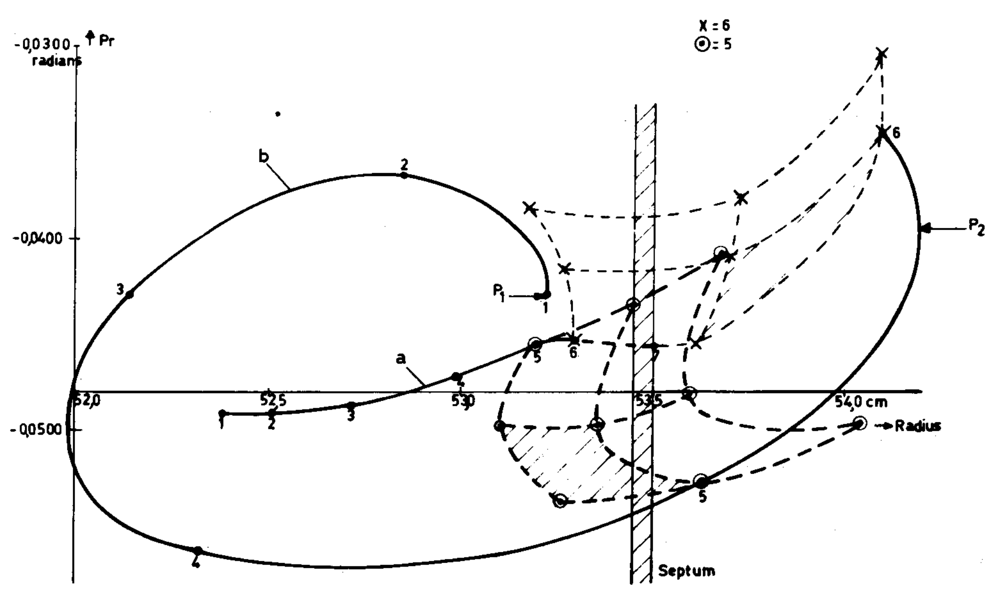

Fig. 2 The motion of a phase space figure during acceleration. Line a represents a particle moving on an "equilibrium orbit"; line b represents a particle with a rather large radial oscillation amplitude

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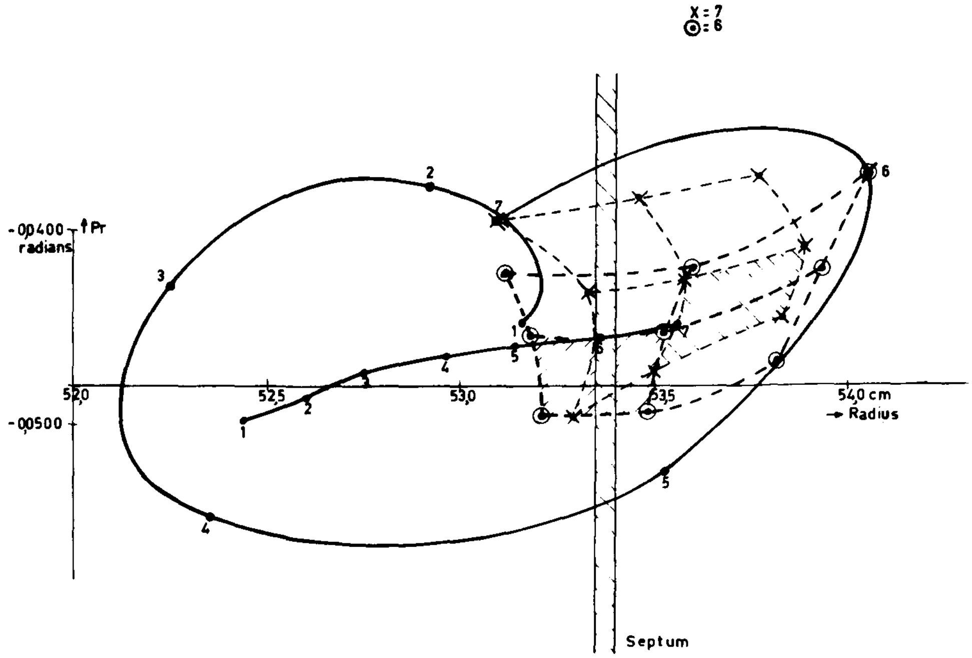

Fig. 3 The motion of a phase space figure with a small radial oscillation amplitude during acceleration

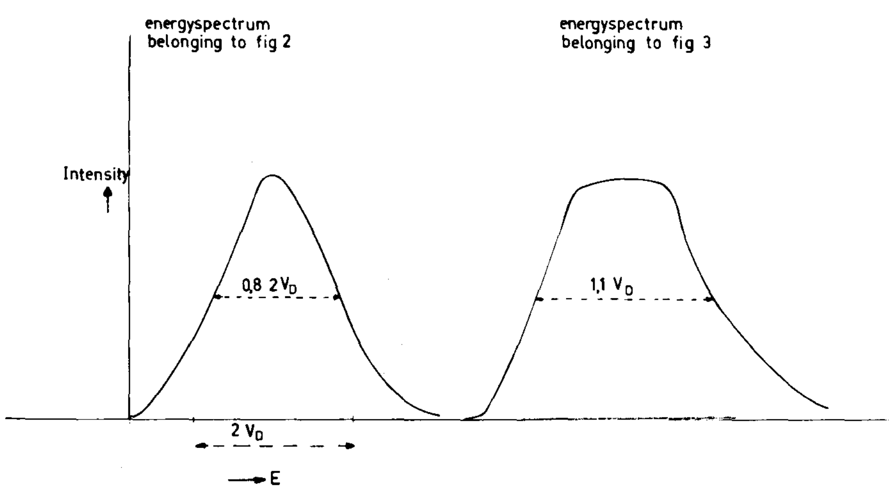

Fig. 4 The energy spectrum of the external beam

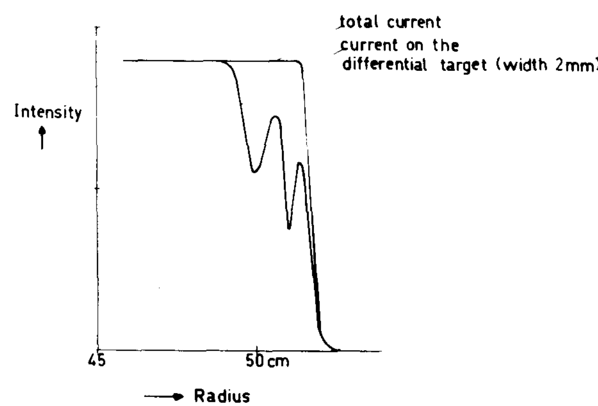

Fig. 5 The intensity of the internal beam as a function of radius measured with a differential target

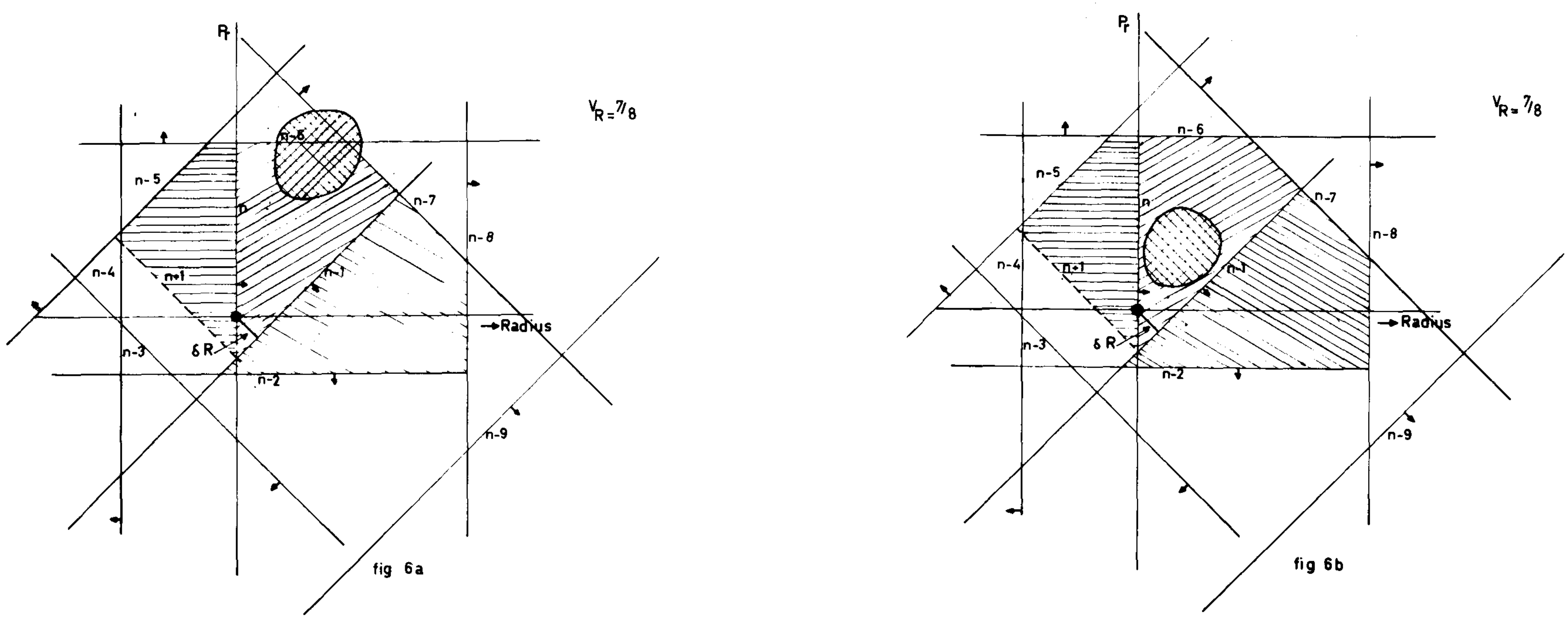

Fig. 6. Discrete energy regions in phase space. The "roundshaped" area in Fig. 6a represents a beam which can have separated peaks in the energy spectrum. In Fig. 6b a beam is shown which has only one peak in the energy spectrum. From these figures it is clear that a well-centered beam will have a bad energy resolution as many discrete energy regions are covered by the phase space area occupied by the beam. The radial frequency in these figures is $\nu_R = 7/8$; R is the radial separation between two seccessive equilibrium orbits.

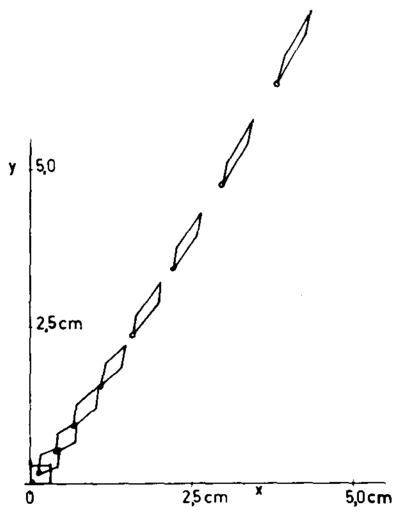

Fig. 7 The motion of a phase space figure in the regenerator system, shown in fig. 8.

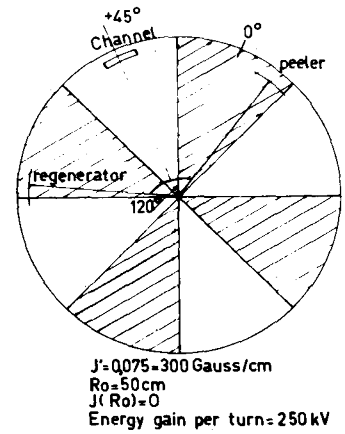

Fig. 8 An example of a regenerative system for which some calculations have been made.

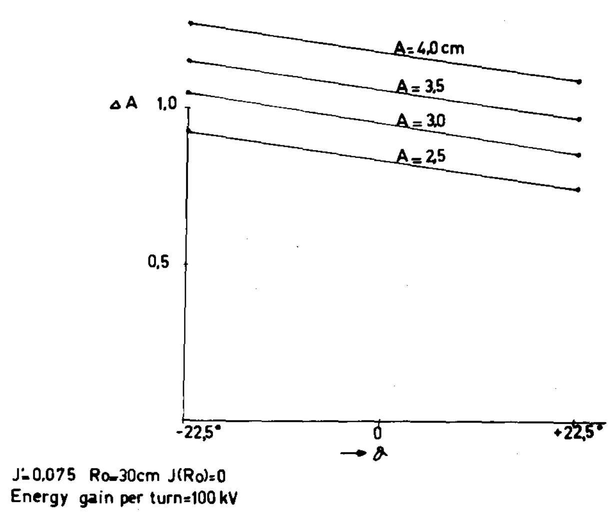

Fig. 9 The increase of the radial amplitude as a function of azimuthal positions of the regenerative system for different amplitudes (the azimuthal positions are given in fig. 8).