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REGENERATIVE BEAM EXTRACTION

Invited Paper

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Abstract

This work is a review of regenerative beam extraction for synchrocyclotrons and isochronous cyclotrons. The linear and nonlinear theories of the system for synchrocyclotrons are briefly reviewed. The problems for the extraction efficiency and external beam quality are discussed. A theory for isochronous cyclotrons is given. Finally, present operating features of the systems and future prospects are summarized.

1. Introduction

A main physical problem in the beam extraction for high-energy cyclic accelerators is getting a sufficient turn separation at the extraction radius, to locate a septum or a septum magnet to take the circulating beam out of the machine. One of the methods used is the regenerative beam extraction method, invented for a synchrocyclotron by Tuck and Teng[1] and developed by LeCouteur.[2,3,4] This method, now common in synchrocyclotrons, has been applied to the Birmingham 40" cyclotron.[5,6] A similar system has been tested for the Berkeley 88" cyclotron.[7] The feasibility of the system has been investigated for the N.R.D.L. cyclotron.[8]

Regenerative beam extraction is a type of resonance extraction method. By introducing a gradient field perturbation near the extraction radius a resonant excitation of radial betatron oscillations is produced, while keeping the vertical motion stable. The type of the resonance is mainly the 2/2 stop band.

The system is generally divided into the linear regenerative system and the nonlinear one. The linear system operates in the linear region of the main field near maximum radius and consists of the two linear gradient field perturbations which are separated by an angle <180°. One gives a radially outward impulse (peeler) and the other gives a radially inward impulse (regenerator). Often the regenerator alone is used, with the fast fall-off of the main field providing the peeler action.

The isochronous cyclotron has higher energy gain per turn than the synchrocyclotron. Unlike the synchrocyclotron, it has modulation of the betatron motion by the sector focusing, and the effects of nonlinear forces due to sectors. All of these differences require careful consideration.

In Sect. 2, theories for the regenerative beam extraction for the synchrocyclotron are reviewed. In Sect. 3 operating features of the system and possible improvements for synchrocyclotrons are discussed. In Sect. 4 the system for isochronous cyclotrons is described. Through all of the discussions, the main emphasis is given to the linear regenerator, for we can get a much better physical insight through the linear theory.

2. Regenerative Extraction for the Synchrocyclotron

A. LeCouteur's Regenerative Theory

In the linear region of the main field, the radial and the vertical motion of the particle can be approximated as simple harmonic oscillations around the equilibrium orbit. The action of the peeler-regenerator can be written as a series of impulses, the strengths of which are proportional to the penetration into the system. The arrangements are shown in Fig. 1. The operation of the system is shown in Fig. 2. Here the peeler introduces an outward impulse causing a phase delay, and the regenerator introduces inward impulse giving a phase advance. After one revolution, the amplitude is increased and the phase is returned to the initial phase.

When the radius of the equilibrium orbit is the same as the radius of the peeler-regenerator, the action of the system may be described by the transfer matrix, as LeCouteur did.[2]

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\nu_r f & \frac{\sin\nu_r f}{\nu_r} \\ -\nu_r \sin\nu_r f & \cos\nu_r f \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -R & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} \cos\nu_r d & \frac{\sin\nu_r d}{\nu_r} \\ -\nu_r \sin\nu_r d & \cos\nu_r d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ P & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$= \overline{M}_r \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \qquad (2.1)$$

where
- x: displacement from the equilibrium orbit
- x': $(dx)/(d\theta)$ angle with the equilibrium orbit
- d: the angle between the peeler and regenerator
- f: $f = 2\pi - d$
- P: $P = -\frac{r}{B}\frac{\partial B}{\partial r}\theta_P$ at the peeler, "peeler strength"
- R: $R = \frac{r}{B}\frac{\partial B}{\partial r}\theta_R$ at the regenerator, "regenerator strength"
- θ_P, θ_R: azimuthal extension of the peeler and regenerator.

The matrix element of $\overline{M}_r = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$ is given by:

$$\alpha = \cos 2\pi\nu_r + \frac{P}{\nu_r}\sin 2\pi\nu_r - \frac{R}{\nu_r}\sin\nu_r f$$

$$\times \cos\nu_r d - \frac{PR}{\nu_r^2}\sin\nu_r f\sin\nu_r d$$

$$\beta = \frac{\sin 2\pi\nu_r}{\nu_r} - \frac{R}{\nu_r^2}\sin\nu_r f\sin\nu_r d$$

$$\gamma = -\nu_r\sin 2\pi\nu_r + P\cos 2\pi\nu_r - R\cos\nu_r f$$

$$\times\cos\nu_r d - \frac{PR}{\nu_r}\cos\nu_r f\sin\nu_r d$$

$$\delta = \cos 2\pi\nu_r - \frac{R}{\nu_r}\cos\nu_r f\sin\nu_r d$$

For the vertical motion, the transfer matrix $\overline{M}_V$ can be obtained by the following changes:

$$\nu_r \rightarrow \nu_z$$

$$P \rightarrow -P$$

$$R \rightarrow -R$$

The stability condition is given by the trace of the matrix $\overline{M}$

$$\alpha + \delta > 2:\quad \text{unstable}$$

$$\alpha + \delta < 2:\quad \text{stable}$$

For proper operation of the system, it is necessary to keep the vertical motion stable and to excite the radial motion. Then after n turns the radial motion can be written as:

$$\begin{pmatrix}x\\x'\end{pmatrix}_n = \overline{M}_r^n\begin{pmatrix}x_0\\x_0'\end{pmatrix} = K_1 e^{n\lambda}\begin{pmatrix}u\\v\end{pmatrix}$$

$$+ K_2 e^{-n\lambda}\begin{pmatrix}u'\\v'\end{pmatrix} \qquad (2.2)$$

where e^{λ} and $e^{-\lambda}$ are the eigenvalues of the matrix $\overline{M}_r$ and can be found from the trace:

$$\frac{e^{\lambda}+e^{-\lambda}}{2} = \frac{\alpha+\delta}{2} = \cos 2\pi\nu_r + \frac{P-R}{2\nu_r}\sin 2\pi\nu_r$$

$$-\frac{PR}{2\nu_r}\sin\nu_r d\cos\nu_r d \qquad (2.3)$$

and $\begin{pmatrix}u\\v\end{pmatrix}$ and $\begin{pmatrix}u'\\v'\end{pmatrix}$ are the corresponding eigenvectors
K_1, K_2 are determined by $\begin{pmatrix}x_0\\x_0'\end{pmatrix}$

For a sufficiently large n, the last term of Eq. (2.2) will be damped out, and the phase of the oscillation is given by an eigenvector $\begin{pmatrix}u\\v\end{pmatrix}$. The node of the oscillation of the motion between the peeler and the regenerator is given by

$$\cot\chi = \frac{\frac{R}{2\nu_r}\sin\nu_r(f-d) + \frac{d}{2\nu_r}\sin 2\pi\nu_r}{\sin 2\pi\nu_r - \frac{R}{\nu_r}\sin\nu_r d\cos\nu_r f} \qquad (2.4)$$

$$-\frac{\frac{PR}{2\nu_r^2}\sin\nu_r d\sin\nu_r f \pm\sqrt{(\alpha+\delta)^2-1}}{\sin 2\pi\nu_r - \frac{R}{\nu_r}\sin\nu_r d\cos\nu_r f}$$

where the position of the node χ/ν_r is measured from the position of the peeler.

One selects the parameters of the system to minimize the vertical motion. The magnetic channel is put at 60°-80° from the node of the oscillation for large x and x' at extraction.

B. The Linear Regenerative Theory for a General Case

The amplitude of the radial oscillation of the internal beam is typically a few inches. A particle with some radial amplitude can reach the peeler-regenerator at the same radius as a centered beam of higher energy. This is shown in Fig. 3. For this lower energy beam LeCouteur's transfer matrix should be extended to account for this factor.[16] When we take the difference between the radius of the equilibrium orbit and the radius of the peeler-regenerator as Δa,

then

$$\begin{pmatrix}x\\x'\end{pmatrix}_i = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} - \Delta a\begin{pmatrix}y_0\\y_0'\end{pmatrix} \qquad (2.5)$$

where $y_0 = P\beta - R(\sin\nu_r f)/(\nu_r)$

$y_0' = P\delta - R\cos\nu_r f$

After n turns

$$\begin{pmatrix}x\\x'\end{pmatrix}_n = \overline{M}_r^n\begin{pmatrix}x_0\\x_0'\end{pmatrix} - \Delta a\left(\overline{M}_r^{n-1} + \ldots + \overline{M}_r + 1\right)\begin{pmatrix}y_0\\y_0'\end{pmatrix}$$

where

$$\overline{M}_r = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

By explicit uses of the eigenvectors and eigenvalues of the matrix $\overline{M}_r$

$$\begin{pmatrix}x\\x'\end{pmatrix}_n = e^{n\lambda}\left[K_1 - \Delta a\frac{\ell_1}{e^{\lambda}-1}\right]\begin{pmatrix}u\\v\end{pmatrix}$$

$$+ e^{-n\lambda}\left[K_2 + \frac{\Delta a}{1-e^{\lambda}}\ell_2\right]\begin{pmatrix}u'\\v'\end{pmatrix}$$

$$+\frac{\Delta a}{e^{\lambda}-1}\ell_1\begin{pmatrix}u\\v\end{pmatrix} - \frac{\Delta a}{1-e^{\lambda}}\ell_2\begin{pmatrix}u'\\v'\end{pmatrix} \qquad (2.6)$$

For a sufficiently big value of n,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_n = e^{n\lambda}\left[K_1 - \Delta a \frac{\ell_1}{e^{\lambda}-1}\right]\binom{u}{v} + \Delta a \binom{c}{c'} \qquad (2.7)$$

where

$$c = \frac{(1-\delta)\, y_0 + \beta\, y_0'}{(e^{\lambda}+e^{-\lambda}) - 2}$$

$$c' = \frac{\gamma\, y_0 + (1-\delta)\, y_0'}{(e^{\lambda}+e^{-\lambda}) - 2}$$

We may discuss Eq. (2.7) for the following three typical cases,

a) $K_1 - \Delta a \frac{\ell_1}{e^{\lambda}-1} = 0$; for this case, then $\binom{x}{x'}_n = \Delta a \binom{c}{c'}$. The $\binom{x}{x'}$ is independent of the number of turns, and therefore we may define $\Delta a\binom{c}{c'}$ as the new equilibrium orbit with peeler-regenerator. The vector $\Delta a\binom{c}{c'}$ has the maximum positive displacement just after the regenerator for $\nu_r < 1$, and at just before the peeler for $\nu_r > 1$.

b) $K_1 - \Delta a \frac{\ell_1}{e^{\lambda}-1} < 0$; for this case, the amplitude is increasing, but with negative displacement. This means the particle will be out of the system and may precess away. This factor provides an improvement on the energy spread of the external beam by keeping away the lower energy beam.

c) $K_1 - \Delta a \frac{\ell_1}{e^{\lambda}-1} > 0$; for this case, the particle is regenerated. But the entrance of the channel

$$\left(\frac{dx}{d\theta}\right)_{\Delta a>0} > \left(\frac{dx}{d\theta}\right)_{\Delta a=0} \quad \text{when } \nu_r < 1, \text{ through}$$

the term $a\binom{c}{c'}$. Therefore the different energy of the particle provides the different $(dx)/d\theta$ at the same channel entrance and may cause a difficulty for the beam transport.

Finally, for the vertical motion, there is A.G. focusing when particles enter both peeler and regenerator, and the motion can be described by the transfer matrix as mentiones in Sec. 2.A. But the lower energy particle which has a big amplitude of radial oscillation may stay in the peeler or regenerator for many turns and may not have A.G. focusing in this period. This may be one of the main beam losses in this system.

C. The Nonlinear Regenerator

Since the linear regenerator is operating in the linear region of the main field, the extraction occurs several inches inside the fast fall-off region, and the energy is lower than the maximum possible. LeCouteur[3,4] investigated the possible operation of the regenerator at the end of the linear region, getting a peeler action from the fall-off of the main field. To keep the vertical motion stable and the phase of the oscillation constant, one should increase the strength of the regenerator nonlinearly to match the nonlinear fall-off of the main field. The maximum radius of the regenerator should be less than the radius of the $\nu_r = 2\nu_z$ resonance. For a semi-quantitative discussion, the linear theory may be used. An approximate evaluation of the peeler action from the fall-off of the main field can be made. For a design of a system, all the existing analytical methods may not be accurate enough, and numerical integration is required.

3. Operating Features and Improvements for the Synchrocyclotron

The linear as well as the nonlinear regenerative extraction system was first successfully operated at the Liverpool synchrocyclotron[9] and it is now a common method in synchrocyclotrons (for example, Refs. 10, 11, 12). The extraction efficiencies are about 1-10% and the energy spread of the external beam is about 0.1-1%. Another extraction method is used at the Tokyo synchrocyclotron. There the n = 1 resonance gives an extraction efficiency of about 50%.[13] The machine has relatively good internal beam quality to pass dangerous resonances, such as $\nu_r = 2\nu_z$. The regenerative beam extraction method is a relatively good one compared to previous systems, such as extraction by Coulomb scattering. Though the main purpose of the system was to produce good turn-separation, the method also gave good beam quality. This might be a general characteristic of the resonance extraction method.

The low extraction efficiency is mainly due to poor internal beam quality, and naturally the efficiency will be improved by an improvement of the internal beam. The following are possible reasons for the low extraction efficiency and the possible improvements in the extraction system itself.

a) In the synchrocyclotron, individual particles have good turn separations by the action of the regenerator, but there is no real turn separation of the whole beam. Therefore, we inevitably lose some fraction of the beam by striking the beginning of the septum. Therefore the efficiency will be improved by an increase of the ratio between the turn separation of the individual particle and the thickness of the septum. But most of the operating systems are optimized for this and we cannot expect a major improvement by this change.

b) When we take a group of particles whose equilibrium radius is the same as the regenerator radius, all the particles have approximately the same nodes given by the eigenvector $\binom{u}{v}$ [Eq. (2.4)]. The particles are diverging out of a node and converge into the other side of the node. When we put the entrance of the magnetic channel at a point where $(dx)/(d\theta)$ is positive, the beam is diverging at this position, and may be partially lost inside the channel. For an improvement of this factor, one may introduce a radial focusing element just before the magnetic channel, or by weakening the strength of the regenerator at the final part of the regeneration, or by introducing the so-called "compressor" behind the regenerator. A regenerator with compressor gave 15% extraction efficiency for the Göttinger synchrocyclotron.[14]

c) As mentioned in Sect. 2.B, a particle of different energy has a different x' at the entrance of the channel and this causes difficulty of beam transport and loss of beam. To improve this situation the following system might be used. We introduce another weak peeler, which is a time varying perturbation at the regenerator azimuth. It is at a smaller radius than the regenerator (Fig. 4). The particle can be accelerated up to full energy without entering the regenerator because of the action of the peeler-type field. When the particle gets the full energy, the acceleration is turned off and the peeler type perturbation is slowly decreased. Then the particle will be slowly pulled to the regenerator side and will be captured into the regenerator. Since the energies of all the particles differ only by an amount due to the phase oscillation, and the perturbation is well localized in azimuth and radius, all the particles have approximately the same $(dx)/d\theta)$ at the same point of the entrance of the magnetic channel. For stable vertical motion, the peeler-type field should be not so strong and the selection of radius should be careful for a stable off centered beam. If this idea works, there are three advantages, namely, a uniform energy, a good duty cycle, and a higher extraction efficiency.

d) The main loss of beam might be a vertical loss as described in Sect. 2.B. To improve this, we should keep the lower energy particle out of the system. When the particle entered the system the peeler should act at the same turn with the regenerator for A.G. focusing in the vertical motion. The modification which is mentioned in c) may work for this improvement.

For a conclusion of this section, one should investigate the reasons for beam losses in a particular machine. If one loses the beam during regeneration, it might be vertical loss. If one loses the beam at the entrance of the channel it might be radial loss. The above mentioned ideas may greatly improve present systems.

4. The Regenerator for Isochronous Cyclotrons

The main differences between the isochronous cyclotron and the synchrocyclotron are, as described in the introduction, higher energy gain per turn, modulation of the betatron oscillation, and the nonlinear force due to sectors. None of these effects is negligible, and it requires numerical integration for a design of a system. For a design guidance and for a qualitative discussion, an analytical study may still be helpful.

A. Azimuthal Position of the Peeler-Regenerator

It has been shown that the main harmonic component for regenerative action is the second order gradient harmonics[15,16] and the type of resonance is the 2/2 stop band. When we analyze the peeler-regenerator field by a Fourier series, there are considerable amounts of other harmonics such as the first and the third.

The betatron motion in the isochronous cyclotron has a modulation due to sectors and these harmonics of the modulation may couple to the harmonics of the peeler-regenerator bump and create a second harmonic.[17] The second harmonic of the peeler-regenerator and the second harmonic from the coupling may be added together vectorially. When these two harmonics are in phase, the regeneration is stronger, and it is weaker when they are out of phase. But, the regenerative action can also be treated by a transfer matrix as in the synchrocyclotron and it is obtained by A. A. Garren.[18] The main results are the following:

We take the main magnetic field as

$$B = B_0 \{1 + h_N \cos N [\theta - X(r)]\} \qquad (4.1)$$

The azimuthal positions of the peeler and regenerator are θ_1 and θ_2. The trace of the transfer matrix is given by

$$\begin{aligned} e^{\lambda} + e^{-\lambda} = \alpha + \delta = \cos 2\pi\nu_r \\ + \frac{\sin 2\pi\nu_r}{\nu_r}\left[P(1-d_1) - R(1-2d_2)\right] \\ - \frac{PR}{\nu_r^2}\Big[(1-2d_2-2d_1)\sin\nu_r f \sin\nu_r d \\ + (a_1-a_2)\sin\nu_r(f-d)\Big] \qquad (4.2) \end{aligned}$$

(For the vertical motion $\nu_r \to \nu_z$, $P \to -P$, $R \to -R$.)
Where a and d are:

Case 1) When the spiral focusing is dominant

$$\begin{pmatrix} a_i \\ d_i \end{pmatrix} = h_N \tan\gamma \left[\begin{array}{l} \begin{array}{cc} \text{radial} & \text{vertical} \end{array} \\ \left(-\frac{\nu_r}{N^2-4} \quad \frac{\nu_z}{N^2}\right) 2\cos N(\theta_i - X) \\ \left(\frac{-N}{N^2-4} \quad \frac{1}{N}\right) \sin N(\theta_i - X) \end{array}\right]$$

Case 2) When the flutter focusing is dominant

$$\begin{pmatrix} a_i \\ d_i \end{pmatrix} = h_N \left[\begin{array}{l} \left(\frac{3N}{(N^2-1)(N^2-4)} \quad -\frac{1}{N(N^2-1)}\right) \sin N(\theta_i - X) \\ \left(\frac{2}{N^2-4} \quad \frac{1}{2N(N^2-1)}\right) \cos N(\theta_i - X) \end{array}\right] \qquad (4.4)$$

Case 3) When both are about the same strength Eqs. (4.3) and (4.4) should be added, and

$$\tan\gamma = r\,\frac{\partial x}{\partial r}$$

From the Eqs. (4.2, 4.3, and 4.4) we can see the best position of the peeler-regenerator is

1) spiral focusing: $\theta_i - X = \pm\frac{\pi}{N}$ when $\tan\gamma \gtrless 0$

2) flutter focusing: the center of a valley.

The above discussion was based only on the linear theory. When we take into account the nonlinear forces due to sectors, azimuthal position of the

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bump may be quite critical to the amplification of the radial motion.

B. Radial Position of the Peeler-Regenerator

At the isochronous region, the value of ν_r is bigger than unity, and decreases in the fringing field. One may operate the system at the isochronous part of the field, but it requires a stronger system. The vertical motion should be investigated carefully for the stronger system. When one can shape the field near $\nu_r = 1$ for a few inches, a relatively weaker peeler-regenerator may be used. Finally as in the nonlinear regenerator for the synchrocyclotron, one may use only the regenerator at near $\nu_r = 1$, using the fall-off of the main field as a peeler. The vertical motion should be investigated carefully for this case, since the higher energy gain may introduce the $\nu_r = 2\nu_z$ resonance during regeneration.

C. Acceleration Effects

The accelerator effects can be neglected during regeneration for the synchrocyclotron and the system is peeling off the particles with the bigger amplitude of the radial oscillations. In the isochronous cyclotron, the higher energy gain introduces a number of effects in the regenerative system. First, the higher acceleration allows coherent oscillation of the radial motion and the total phase space area for one energy beam can be captured into the system in one turn and this improves the energy spread of the external beam in addition to the effect which is described in synchrocyclotrons. Second, the node of the oscillation is changing during regeneration by the changes of the energy and it is hard to predict where the node is at the extraction moment. Third, the different amounts of energy gain by the difference of the rf phase during regeneration introduce energy spread and reduction of the extraction efficiency. All of these effects require numerical integration.

D. Duty Cycle

When we accelerate the particle by higher dee voltages, the final energies of the particles and the rf phase may have a certain relation, and the duty cycle may be small when the energy of the extracted beam is analyzed. On the other hand, when we accelerate the particles by a low dee voltage, the final energy of the particle and the rf phase may not have a simple relation, and therefore, the duty cycle will be increased for an analyzed beam. For this low dee voltage the regenerative extraction might be very useful.

E. Present Design Features of the Regenerative System for Isochronous Cyclotrons

The regenerative systems for the Birmingham 40" cyclotron and the Berkeley 88" cyclotron are electrostatic devices. The Birmingham system has the peeler and the regenerator. It may operate at a radius $\nu_r > 1$, or at a radius $\nu_r < 1$. This system gave the maximum extraction efficiency of 70% and 0.5% energy spread at the normal operating condition. In the loss of 30%, 14% was due to vertical loss and 16% was collected at the first 4" septum.[19]

For the Berkeley 88" cyclotron, the system has only the regenerator and operates at a radius where $\nu_r < 1$. The system gave about 50% extraction efficiency and about 0.3% energy spread at the best condition.[7] The reason of the loss is unknown yet.

For the N.R.D.L. cyclotron which will accelerate protons up to 100 MeV, the system is designed for the magnetic peeler-regenerator and the strength of the system can be changed for the different energies. The magnetic field at the peeler-regenerator is carefully shaped to give about $\nu_r = 1$. The powerful system may give the beam out by itself, though the magnetic channel is located for an efficient beam transport.[8]

5. Conclusion

For the synchrocyclotron, the regenerative extraction method is the most successful method at the moment, and the system may be further improved if the quality of the internal beam were improved, or the vertical focusing were improved at the beginning of the regeneration.

For the isochronous cyclotron, the system may be very useful when the turn separation by the energy gain is small. For high dee voltages, it would be hard to obtain further gain of efficiency compared to conventional methods.

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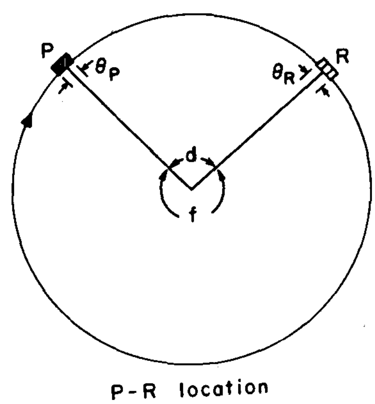

Fig. 1. Position of the peeler-regenerator.

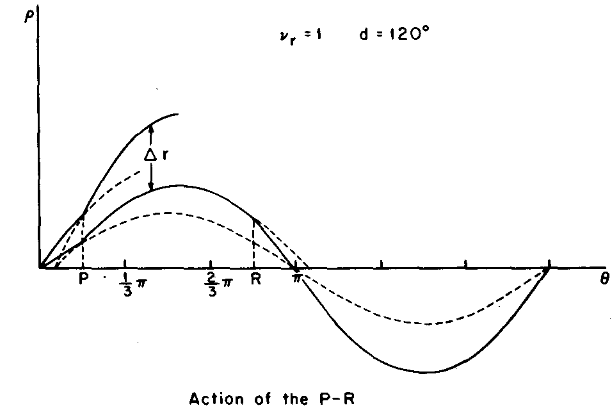

Fig. 2. Action of the peeler-regenerator.

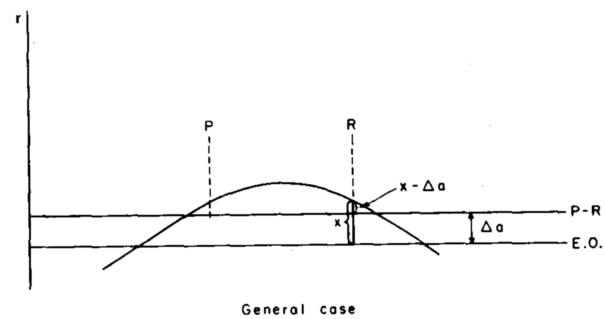

Fig. 3. A general case for the equilibrium orbit less than the radius of the regenerator.

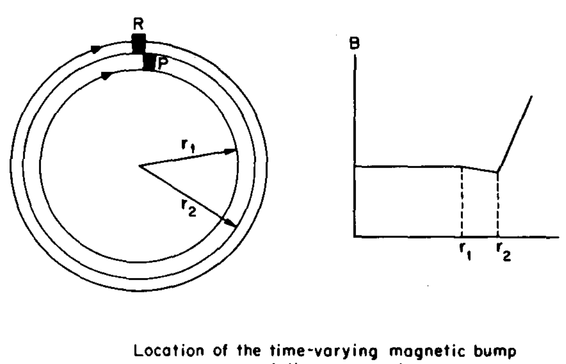

Fig. 4. An additional perturbation to improve acceptance and the vertical motion.