ON THE DESIGN OF THE MAGNETIC FIELD FOR A 450 MeV PROTON MACHINE

W. Müller and W. Wolff
AEG Forschungs-Institute, Frankfurt/Main
(Presented by W. Wolff)

Considerations on the basic design of a 450 MeV machine was reported last year in Los Angeles[1]. It is intended to start with three sectors in the center and to change over to six sectors gradually at a suitable radial interval. The $\nu_R = 1.5$ resonance is to be obtained at the edge and to be used for extraction by the addition of a third harmonic in the field. To get a high meson production the conditions for shifting this resonance towards higher energies were investigated.

The investigations made use of calculated magnetic fields by the method described in detail in the previous paper[1]. This method deals with magnetic fields where the influence of field overlapping is given by a two-dimensional potential theory. Thus, we avoided obtaining magnetic fields which cannot be realized. Besides a spiral of 70° and an induction of 21 kG were never exceeded. The three-sector configuration in the center of the machine has no further complications in comparison with machines in use.

The Transition from Three to Six Sectors

With respect to a simple design, the transition from three to six sectors was located in the Thomas-range which ends at a radius of 125 cm for a small air gap of 20 cm. To avoid the additional difficulties due to the phase shift of the field harmonics it is proposed to perform the transition in such a way that the three hills are narrowed symmetrically to their centerline. The new sectors rise in the middle of the valley as shown in Fig. 1. The height and width of the two kinds of sectors must be chosen such that isochronism and axial focusing are assured and ν_R does not decrease to avoid the neighbourhood of the $\nu_R = 1$ resonance. To consider the influence of the two flutter factors f_3 and f_6 and their derivatives on ν_R and ν_z the following two approximate formulas were used for first orientation.

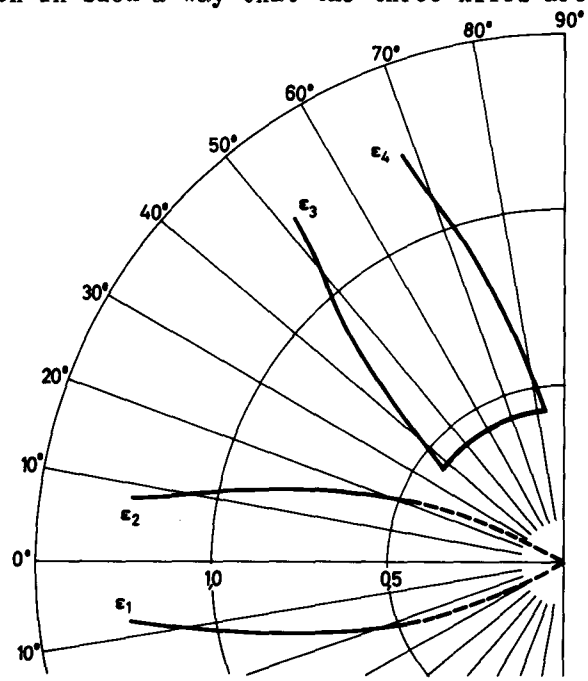

Fig. 1 Transition from three to six sectors.

$$\nu_z^2 = -k + 0.5\,(f_3^2 + f_6^2) \quad , \tag{1}$$

$$\begin{aligned} \nu_R - 1 = \frac{k}{2} &+ \frac{f_3^2}{20}\left\{(1.7 + a_3)^2 - 1 + 1.25a_3\right\} \\ &+ \frac{f_6^2}{128}\left\{(1.5 + a_6)^2 - 1.4 + 1.8a_6\right\} \\ &+ f_3^2 f_6\left\{0.1 + 0.016a_6 + 0.025a_3\right\} \quad , \end{aligned} \tag{2}$$

with $a_n = \frac{R}{f_n}\left(\frac{df_n}{dr}\right)_R$, $n = 3$ and 6.

The equation for ν_R was derived by solution of Hill's equation under the assumption $k \ll 1$ and neglecting higher harmonics than f_6, and terms of higher order. When the transition is finished f_3 has decreased to zero and f_6 has increased continuously from its initial value defined by the pole-face geometry to a final value. This implies that a_6 is always positive but a_3 must change from a positive to negative values. To fulfil the third condition the term with f_3^2 must not decrease more than the terms with k and f_6^2 increase. Accordingly, at the initial radius k must give a considerable contribution to ν_R and the valley must be wide enough that the intermediate sector can be introduced. In our case an initial radius of 43 cm and a final radius of 125 cm proved to be advantageous. Besides, ν_z can always be kept above the value 0.2 provided the initial value of f_6 is large enough; this can be achieved by making the sectors sufficiently narrow.

By means of a suitable computer program a magnetic field shape was found leading to the values of f_3 and f_6 calculated as above, (see Fig. 2). The edges of this field are shown in Fig. 1. Fig. 3 shows the behaviour of ν_R in the range of transition. Both figures demonstrate that the transition is realizable under the above mentioned conditions.

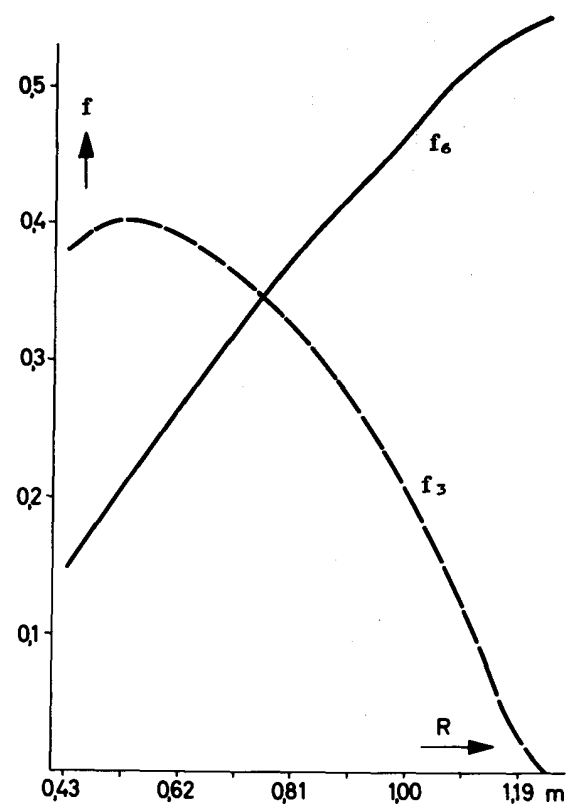

Fig. 2 Radial dependence of f_3 and f_6 harmonic of field flutter.

Magnetic Field of the Outer Edge

In the remaining radial range no special difficulties arise excepting the shaping of the field at the outer edge.

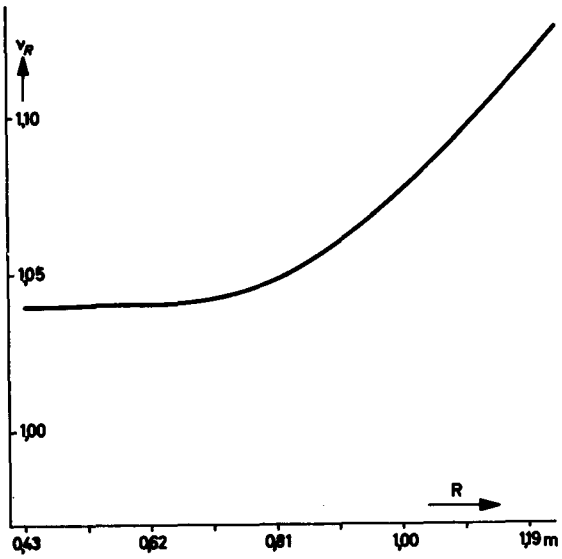

Fig. 3 ν_R in the range of transition from three to six sectors.

No requirements are imposed on ν_R in the intermediate range and ν_z can be realized by a suitable choice of spiral angles. At the outer edge, however, it is desirable to postpone the $\nu_R = 1.5$ resonance to obtain higher energies. Here the field parameters are fixed largely by requirements of construction and economy.

There remains only to investigate the influence of the radial derivatives of ν_R. General information may be obtained again from an approximate formula for ν_R.

$$\nu_R^2 = \tau_0 + \frac{\tau_1^2 + \sigma_1^2}{2(N^2 - 4\tau_0)} \tag{3}$$

with

$$\tau_0 \cong 1 + k + \frac{f_6^2}{N^2-(1+k)} \cdot \left\{ G_0 \left(1 - \frac{2+k+a_6}{1+k}\right) + a_6(2+k) + \tfrac{1}{2}b_6 - \tfrac{1}{2}N^2 \tan^2\zeta \right\},$$

$$\tau_1 \cong f_6(2+k+a_6) \left\{ 1 - \frac{f_6^2}{(1+k)(N^2-(1+k))} \left(G_0 + a_6(2+k) + \tfrac{1}{2}b_6 - \tfrac{1}{2}N^2 \tan^2\zeta\right) \right\},$$

$$\sigma_1 \cong f_6 N \tan\zeta ,$$

$$G_0 = 1 + 2K + \tfrac{1}{2}\ell ,$$

$$\ell = \frac{R^2}{\bar{B}(R)} \cdot \left(\frac{d^2\bar{B}}{dr^2}\right)_R ,$$

$$k = \bar{k} - \frac{1}{2(1+\bar{k})} \cdot \frac{f_6^2}{N^2 - (1+\bar{k})} (\ell + b_6) ,$$

$$\bar{k} = \frac{R}{m} \cdot \left(\frac{dm}{dr}\right)_R .$$

$\bar{k}$ is that part of field index which is due to the mass increment only. If a_6 is approximately chosen as $-(2 + k)$ then the always positive term τ_1^2 vanishes, and τ_0 decreases. A further decrease of τ_0 can be achieved by a negative second derivative b_6 of the flutter. So, the conditions for small ν_R at the outer edge are

$$a_6 = -(2 + k), \; b_6 < 0. \qquad (4)$$

Negative derivatives can be obtained if the angular width of the sectors is increased. To fulfil the conditions of (4) the field B_{max} in the hill has to decrease at the radius of final energy. We got a realizable field fall-off by measurements on a model with a Rose shim. The beginning of the field fall-off was chosen such that the conditions (4) are met at 450 MeV. Fig. 4 shows the behavior of ν_R and the flutter f_6. With this radial dependence of flutter we obtained the value $\nu_R = 1.48$ an energy of 460 MeV.

In comparison with this case Fig. 5 shows the behavior of ν_R if the angular width of the hill is kept constant. Due to the positive derivative of f_6 the value $\nu_R = 1.48$ appears at 403 MeV.

Besides the smaller ν_R in the first case there is a further advantage because the effective radius of the machine is increased considerably by extending the range of acceleration into the range of field fall-off. However, it is necessary to enlarge the width of the hill by about 1°.

The radial dependence of all field quantities and ν_z are shown in Fig. 6.

Method of Calculation

The next problem is to find the sector profile which gives the above calculated field in the midplane within the allowed tolerances. This calculated field can be used

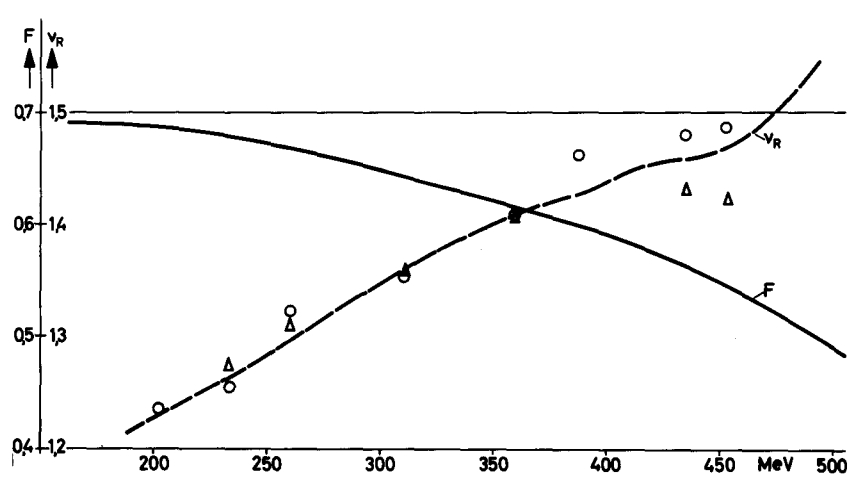

Fig. 4 Field flutter F and ν_R as functions of energy (Case 2-constant hill field B_{max}).
o Calculated by Runge-Kutta integration, with 0.1 cm radial amplitude.
Δ Calculated by Runge-Kutta integration, with 1 cm radial amplitude.
--- Calculated by semi-numerical method.

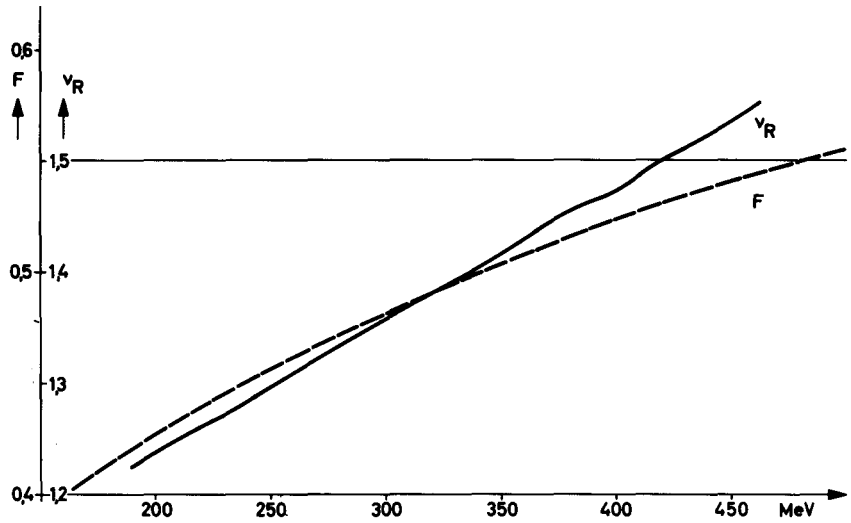

Fig. 5 Field fluttor F and ν_R as functions of energy (Case 1-constant angular width of hills).

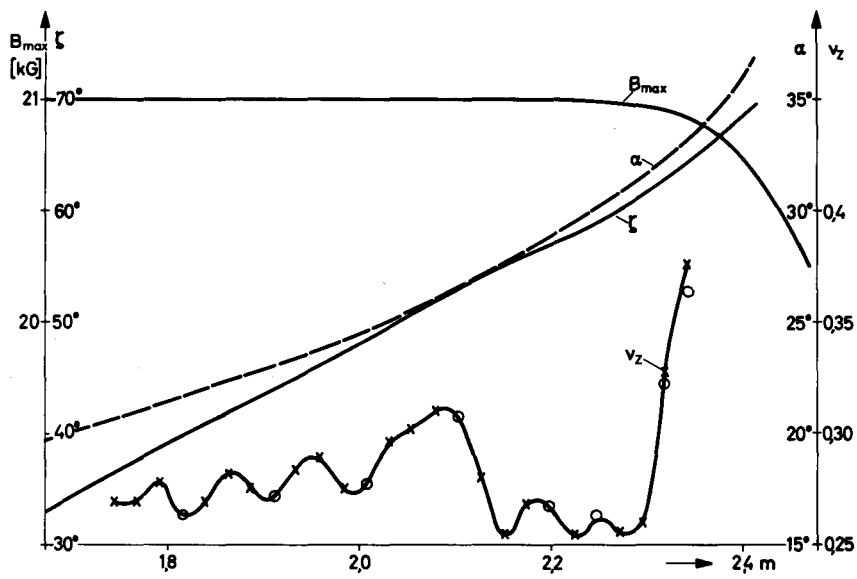

Fig. 6 Radial dependence of B_{max}, spiral ζ, sector width α and ν_z.
o Runge-Kutta points.
x Calculated by semi-numerical method.

only as first approximation because of the assumption of infinite permeability and of two dimensions in the potential theory. The final calculations must be made for measured fields obtained from a magnetic model.

Because ν_R approaches the resonance at the outer edge it has to be calculated very exactly. To avoid the influence of the statistical measurement errors on the calculated values of ν_R and ν_z a semi-numerical method has been developed by us. We expect that the results are less influenced than, for example, the results of Runge-Kutta-calculations because all quantities containing field values are formed by integration. The essentials are described in the following.

For the relative deviation x_e of the equilibrium orbit from the reference circle

r = R we get from the radial equation of motion by neglecting terms higher than quadratic,

$$x_e'' + x_e = 1 - (1 + \frac{3}{2} x_e'^2 + 2x_e + x_e^2) . B(\vartheta) + 2x_e'^2 + 2x_e \qquad (5)$$

$$= h(x_e, x_e') ,$$

with

$$B(\vartheta) = \frac{1}{\bar{B}(R)} . B_z(r_e(\vartheta), \vartheta) , \quad r_e(\vartheta) = R(1 + x_e(\vartheta)).$$

The solution results from iteration by calculating the (n+1)th approximation of Eq. (5), i.e. the differential equation,

$$x_{e,n+1}'' + x_{e,n+1} = h(x_{e,n} , x_{e,n}')$$

For this purpose it is necessary to transform the expression on the right side of Eq. (5) to the form of a trigonometric series. This is done in the following way: $B_z(r,\vartheta)$ is stored in the computer for equidistant values of the two arguments r and ϑ. At each iteration step $B_z(r_e(\vartheta), \vartheta)$ is computed from quadratic interpolation formulas. Now a harmonic analysis of $B(\vartheta)$ can be made. For the intended transformation of $h(x_{e,n}, x_{e,n}')$, only differentations, additions, and multiplications of trigonometric series are necessary. Each of these operations leads to a simple law by use of which the coefficients of the resulting series can be calculated from the coefficients of the given series. This enabled us to have the transformation performed by the computer. The iteration is stopped as soon as the desired accuracy is obtained.

The differential equations for relative axial and radial deviations x and y from the equilibrium orbit are

$$x'' + x \{2(1 + x_e + \frac{3}{2}x_e'^2) . B(\vartheta) + (1 + 2x_e + x_e^2 + \frac{3}{2}x_e'^2) . K(\vartheta) - 1\} - 4x'x_e' = 0 \qquad (6)$$

$$y'' + y \{x_e' . L(\vartheta) - (1 + 2x_e + x_e^2 + \tfrac{1}{2}x_e'^2) . K(\vartheta)\} + y'\{x_e' B(\vartheta) - 2x_e'\} = 0 \qquad (7)$$

with

$$K(\vartheta) = \frac{R}{\bar{B}(R)} . \frac{dB_z}{dr} (r_e(\vartheta), \vartheta)$$

$$L(\vartheta) = \frac{1}{\bar{B}(R)} . \frac{dB_z}{d\vartheta} (r_e(\vartheta), \vartheta) .$$

Eq. (6) and (7) are transformed to the standard form of Hill's equation

$$u'' + G(\vartheta)\, u = 0. \qquad (8)$$

The periodic function $G(\vartheta)$ must be written for both cases as a trigonometric series whose coefficients can be calculated again as described above. The rigorous solution of Eq. (8) is then obtained by use of the method given by McLachlan[2]. The accuracy of this method was controlled by comparison with Runge-Kutta calculations. There was good agreement as shown in Fig. 4 and 6.

To investigate the influence of statistical deviations of field values, artificially generated measurement errors were superposed on the analytically calculated field. It turned out that an average error of $1^{o}/oo$ corresponding to a maximum error of 0.5% leads to an error of about $3^{o}/oo$ in ν_R and about 5% in ν_z.

References

1. W. Müller and W. Wolff, Nucl. Instr. and Meth. 18-19, 447 (1962).
2. N.W. McLachlan, Theory and Application of Mathieu Functions, Oxford, p. 127 (1947).