LIMITATIONS ON DUTY-FACTOR IMPROVEMENT (VIA PHASE-SHIFTING) USING H^- IONS (*)

M.M. Gordon
Michigan State University, East Lansing

Phase-Slip and Duty-Factor

The approximate equations for the variation of E and φ are: $\frac{dE}{dn} = E_1 \cos\varphi$; $\frac{d\varphi}{dn} = \phi_1(E) = \omega_{rf}T(E) - 2\pi$; where $dn = \frac{d\vartheta}{2\pi}$, E_1 is the peak energy gain/turn, and ϕ_1 is the phase slip/turn. The integral of these equations yields the usual φ versus E relation: $E_1 \sin\varphi(E) = F(E) + \text{constant}$; where $F(E) = \int \phi_1(E)dE$. Consider a beam of particles at an energy E for which $\bar{\varphi} \pm \frac{\Delta\varphi}{2}$ are the maximum/minimum values of $\varphi(E)$. Inserting these values into the $\sin\varphi$ equation, it then follows that: $(\cos\bar{\varphi})\sin\frac{\Delta\varphi}{2} = \text{constant}$, independent of E, E_1, and $\phi_1(E)$. Consequently, the minimum spread in phase, $\Delta\varphi_o$, obtains at E_a when $\bar{\varphi}(E_a) = 0$; also, the maximum phase spread $\Delta\varphi_m$ obtains at E_b when $\bar{\varphi}(E_b) \pm \frac{1}{2}\Delta\varphi_m = \pm 90^\circ$. Hence, the relation between $\Delta\varphi_m$ and $\Delta\varphi_o$ is: $[\sin(\frac{1}{2}\Delta\varphi_m)]^2 = \sin(\frac{1}{2}\Delta\varphi_o)$, which is independent of the path from E_a to E_b (see Fig. 1.). Therefore, use of phase-shifting can increase the duty factor at most by a factor $(\Delta\varphi_m)/(\Delta\varphi_o)$; this improvement is greatest for small $\Delta\varphi_o$, for which $(\Delta\varphi_m)/(\Delta\varphi_o) \cong \sqrt{2/\Delta\varphi_o}$.

Application to H^- Ion Acceleration

With the acceleration of H^- ions, a proton beam can be extracted via electron stripping; thus, the maximum possible phase spread $\Delta\varphi_m$ can be achieved simply by lowering the RF voltage so that the ions accelerate into, but not through, the non-isochronous edge-field. Furthermore, the total phase spread in the beam at E_b can be increased to $2(\Delta\varphi_m)$ by the use of an electron stripping "wire" which is thin enough to strip only part of the beam and to allow the rest to accelerate past it[1]. These latter ions will eventually decelerate past the wire again and the protion of the ions stripped will have phases: $90^\circ < \varphi(E_b) < 90^\circ + \Delta\varphi_m$. For example, if $\Delta\varphi_o \cong 29^\circ$, then $\Delta\varphi_m = 60^\circ$; those ions stripped during acceleration have $30^\circ < \varphi(E_b) < 90^\circ$, and those stripped during deceleration have $90^\circ < \varphi(E_b) < 150^\circ$. In this case, the phase spread is increased from 29° to 120°, yielding a duty factor = 1/3. The energy spread in the resultant proton beam is $\frac{\delta E}{E} = (4A\nu_r^2)/R$, where A is the maximum radial oscillation amplitude, and R is the orbit radius at E; for our machine, $\frac{\delta E}{E} \cong 0.01$.

Discussion of Richardson's Scheme

J.R. Richardson[1] has proposed the use of a sequence of electron-stripping wires covering a narrow r-interval in which B is made to change charply. If each wire strips

(*) Work supported by the National Science Foundation.

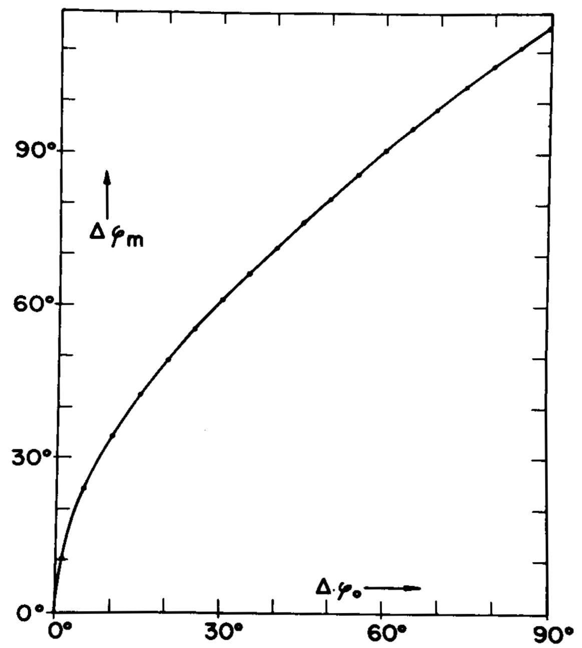

Fig. 1 Maximum possible phase-spread ($\Delta\varphi_m$) versus minimum phase-spread ($\Delta\varphi_0$) for a cyclotron beam, independent of RF voltage, frequency, and magnetic field variation.

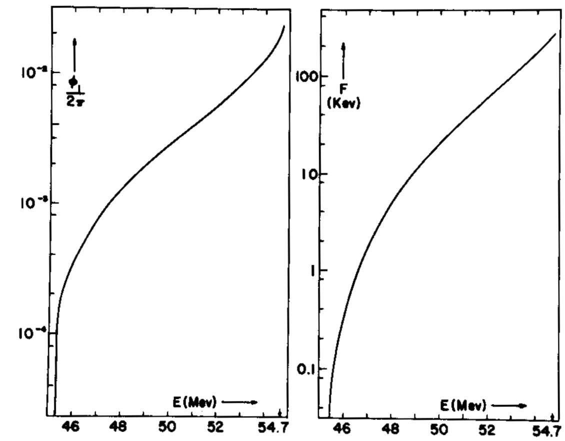

Fig. 2 Fractional phase slip/turn = $\phi_1/2\pi$ versus energy, and "threshold voltage" $F(E) = \int\phi_1(E)\,dE$ versus energy in the edge region of magnetic field "83.3". (See also, Fig. 1 of preceding paper).

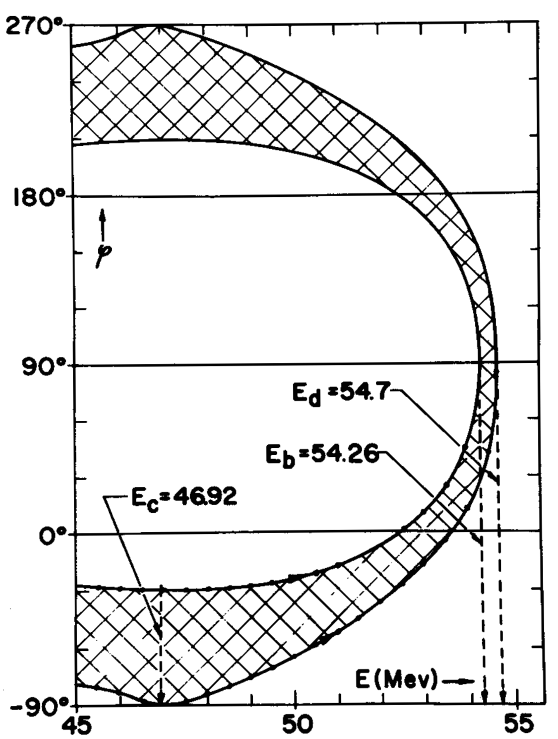

Fig. 3 Phase-spread versus energy for H^- ions which start at $E_a \simeq 0$ with $\bar{\varphi} = 0$, $\Delta\varphi_0 = 29°$, and accelerate to highest possible energy for this field (E_d = 54.7 MeV), in such a way that $d\varphi/dE$ is maximized at E_b; E_1 = 122.4 KeV/turn, $\Delta\omega_{rf}/\omega_{rf} = -3.18 \times 10^{-4}$.

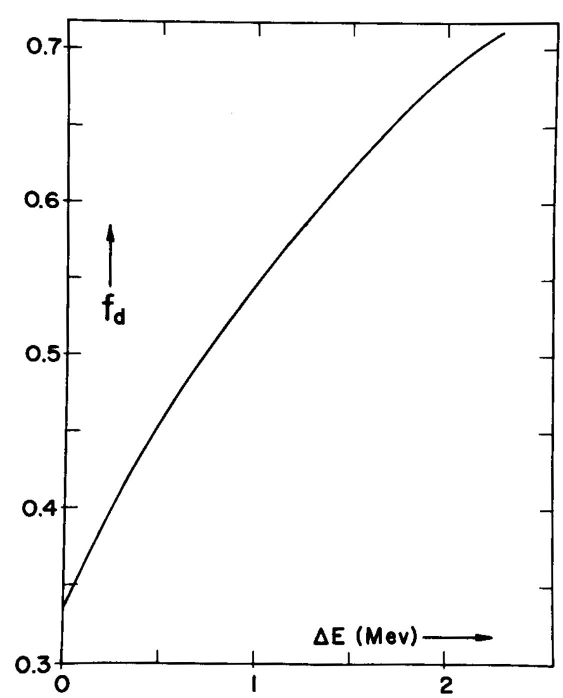

Fig. 4 Duty-factor f_d versus energy-spread ΔE, as obtained from data of Fig. 3, assuming ion stripping occurs (during acceleration and deceleration) in energy range from $E_b - \Delta E$ to E_b = 54.26 MeV.

only a small fraction of the H^- ions during acceleration and deceleration, then the total phase spread of the emergent protons could be very large. Consider then the edge region of the field "83.3" discussed in the preceding paper, and assume for simplicity that the field is perfectly isochronous ($T(E) = 2\pi/\omega_{rf}$) for $E \lesssim 45$ MeV (see Fig. 2). Instead of modifying the field, let the RF voltage and frequency be lowered such that E_b is close to the peak energy obtainable in this field (54.7 MeV), and also such that $\frac{d\varphi}{dE}$ is maximized at $E = E_b$; in this way, the greatest benefit is derived from the sharply falling edge field. The optimum parameters are obtained by specifying the limiting case in which the ion starting at $E_a (\cong 0)$ with $\varphi(E_a) = -\frac{1}{2}\Delta\varphi_o$ reaches a minimum phase $\varphi(E_c) = -90^o$ at E_c, then attains $\varphi(E_b) = +90^o - \Delta\varphi_m$ at E_b, and finally achieves $\varphi(E_d) = +90^o$ at $E_d = 54.7$ MeV. Since the minimum of $\sin\varphi$ occurs at E_c, then: $\phi_1(E_c) = 2\pi(\omega_{rf}/\omega^*_{rf} - 1)$, where $\omega^*_{rf}/2\pi$ is the reduced RF frequency. In addition, E_b, E_c, and E_d must satisfy: $E_c\phi_1(E_c) - F(E_c) = F(E_b) - E_b\phi_1(E_c) = K[F(E_d) - E_d\phi_1(E_c)]$, where $K = K(\Delta\varphi_o) < 1$. Assuming (as before) $\Delta\varphi_o \cong 29^o$ ($\Delta\varphi_m = 60^o$), then $K = 0.6$; specifying $E_d = 54.7$ MeV and using the given values of $\phi_1(E)$, $F(E)$ for this field, then $E_c = 46.92$ MeV and $E_b = 54.26$ MeV; furthermore, it follows that $\phi_1(E_c)/2\pi = 3.18 \times 10^{-4}$, and $E_1 = 122.4$ keV/turn. The curves of Fig. 3 show $\varphi(E)$ versus E for the limiting cases $\varphi(0) = \pm 14.5^o$, as derived from the above data. Fig. 4 shows the resultant duty-factor f_d versus ΔE obtained by assuming ion stripping over the energy-range $E_b - \Delta E$ to $E_b (= 54.26$ MeV). (This ΔE does not include δE described in the preceding section. Clearly, a large ΔE is not practical; moreover, because of the φ-E correlation, any energy selection performed on the resultant proton beam will diminish f_d accordingly. Assuming a reasonable value of $\Delta E = 0.75$ MeV, then $f_d = 0.51$, which is a 52% improvement over f_d for $\Delta E = 0$; the f_d improvement for given ΔE increases rapidly as $\Delta\varphi_o$ decreases. Note that the foregoing calculations are designed specifically to maximize f_d without considering the consequent difficulties in accelerating through the $\nu_r = 1$ and $\nu_r = 2\nu_z$ resonances. Note also that for a given ΔE, f_d can approach unity only by a drastic reduction in E_1 (and hence, E_b) such that effective operation of the machine would be practically impossible.

References

1. J.R. Richardson, UCLA-52 (September, 1962).

DISCUSSION

RICHARDSON : I would like to point out that the suggestion was in a report on the use of negative ions in meson factories primarily designed for the production of secondary beams where one percent energy variation is of no importance. I quite agree with you that for low energy machines the advantages are not quite as great.