CENTRAL-REGION FACTORS INFLUENCING THE DUTY CYCLE OF A CYCLOTRON BEAM(*)

M. Reiser
Michigan State University, East Lansing.

In an isochronous cyclotron, if negative ions are accelerated and beam deflection is accomplished by single-foil electron stripping, the duty factor of the transmitted beam is essentially determined by the phase-bunching effect, the radial stability limit Δr_{st} of the magnetic field, and the tolerable maximum spread of kinetic energy $\Delta E_k/E_k$ at deflection radius r_d, or the corresponding spread of orbit centers $\Delta r_e = (r_d/2)\ \Delta E_k/E_k$. The two parameters Δr_{st} and/or Δr_e, whichever is smaller, define the tolerable spread of initial orbit centers Δr_{tol} and, since the initial center points of the trajectories depend on the phase ϑ_o where the ions enter into the RF field at the ion source, the acceptable interval of starting phases $\Delta\vartheta_o$. The phase-bunching effect, which occurs in the electric field during the first few revolutions, causes a contraction of the useful phase interval $\Delta\vartheta_o$ and thus determines the final pulse length $\Delta\vartheta$, or the actual duty factor $\Delta\vartheta/2\pi$, of the acceptable beam.

The amount of phase grouping depends strongly on the fraction of time the particles spend within the electric-field region during the early revolutions. If the orbit radius r is much larger than the half width d of the electric-field gap, i.e., the time of gap crossing is a very small compared with the RF period ($r>>d$), phase-bunching is negligible and the final pulse length $\Delta\vartheta$ of the useful beam is equal to the starting-phase interval $\Delta\vartheta_o$. On the other hand, if the first turn is completely within the electric field ($r \leq d$), all particles leaving the source in the accelerating half period of the RF voltage are strongly bunched in phase to a short pulse of about 10^o length at the end of the first revolution, as was shown theoretically by Cohen[1]. Cohen also pointed out that the use of "feelers" or extractor electrodes opposite the source is equivalent to injecting the ions with a high initial velocity and may reduce the amount of phase bunching to some extent.

A typical example of an ideal phase-bunching situation is shown in Fig. 1. The trajectories plotted in this figure have been calculated numerically[2] assuming zero initial velocity for the ions and using the known median-plane potential distribution of an idealized dee geometry. To illustrate the strong grouping in phase, the positions of the particles at times $\vartheta = \vartheta_o + \omega_e t = \pi$ and $\vartheta = 2\pi$ are marked (the starting phase defines the instantaneous voltage $U = U_o \cos(\vartheta_o + \omega_e t)$ at $t = 0$); associated with phase bunching is a very accentuated effect of radial grouping and degrouping.

(*) Research supported in part by National Science Foundation.

The influence of different operating conditions and non-zero initial velocities (i.e., use of an extractor electrode or d.c. injection) on phase grouping and spread of center points in the case $r \leq d$ can be studied analytically with the uniform-field theory[2]. With the coordinate system defined in Fig. 1 and with starting conditions $x_o = y_o = \dot{y}_o = 0$, $\frac{m}{2}\dot{x}_o^2 = eU_i$, the phase ϑ at the end of each full turn, i.e., at times $\vartheta_o + \omega_e t = 2\pi N$ (n = 1, 2, 3, etc.) is in good approximation given by the formula

$$\tan\vartheta = -\frac{x}{y} = -\frac{0.159}{n}\sin^2\vartheta_o + \frac{0.450}{n}\sqrt{\chi\, U_i/U_o}\;\sin\vartheta_o\ , \qquad (1)$$

where $\chi = \frac{d^2B^2e/m}{U_o}$. This formula shows that in the case of zero injection voltage ($U_i = 0$) all particles starting in the interval $-\frac{\pi}{2} \leq \vartheta_o \leq \frac{\pi}{2}$ are bunched in phase to a narrow pulse of 9^o total width at the end of the first turn (n = 1); if the trajectories remain within the electric field during subsequent turns, the total pulse width will be further reduced to 4.5^o after the second (n = 2), 3^o after the third turn (n = 3), etc. The use of an extractor electrode or d.c. injection ($U_i \neq 0$) reduces the amount of phase bunching, and the controlling factor is here the product of the parameter χ and the ratio of injection voltage to dee voltage U_i/U_o.

For the coordinates of the center points at times $\vartheta_o + \omega_e t = 2\pi n$ one obtains $x_c = 0$ and

$$\frac{y_c}{d} = \frac{\sin\vartheta_o}{\chi} - \sqrt{\frac{2}{\chi}\frac{U_i}{U_o}}\ . \qquad (2)$$

From this formula one can draw the following conclusions: (a) the instantaneous centers of curvature of each particle trajectory move in such a way that they are always at the same point after each turn, i.e., the distribution of center points remains constant throughout the acceleration process, (b) d.c. injection causes a shift but does not change the total spread of center points; the width of the acceptable starting-phase interval is, hence, also unchanged.

Phase measurements, which show that the pulse length $\Delta\vartheta$ of the beam is typically about 10^o to 30^o, indicate that the normal cyclotron situation, where the electric field penetrates deep into the dees, is ideal for phase grouping and can be well described by the previous uniform-field formulas. If the phase-bunching effect is to be reduced, one must limit the electric field to the gap between dees and/or operate at conditions such that $r > d$ on the first turn already. The general tendency in this case is clear; the shorter the time of gap crossing compared with the RF period, the smaller is the phase-bunching effect. Also, one expects that simultaneously the spread of center points becomes smaller than in the large-gap case. In the idealized case of infinitesimal gap width, for example, the initial center point coordinate y_c is simply

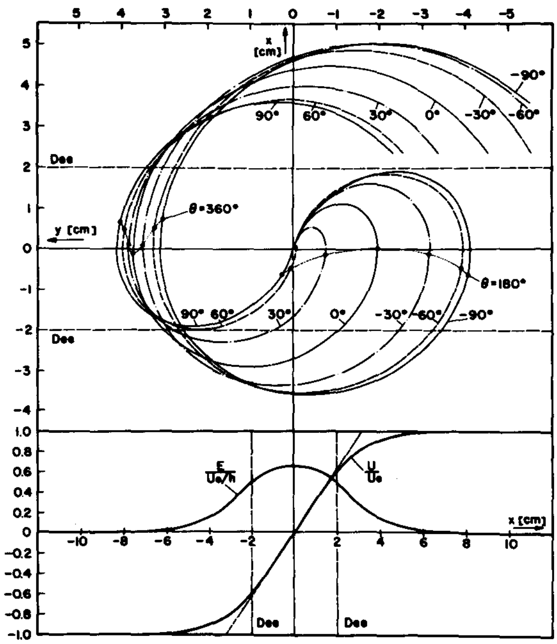

Fig. 1 Electric-field distribution and initial trajectories of H^- ions with different starting phases ϑ_0 in an open-dee geometry. Dee spacing $2k = 4$ cm, dee height $2h = 4$ cm; dee-to-ground voltage $U = U_0 \cos(\vartheta_0 + \omega_e t)$, $U_0 = 70$ kV, $\omega_e/2\pi = 21$ Mc/s; magnetic field $B = 13.774$ kG.

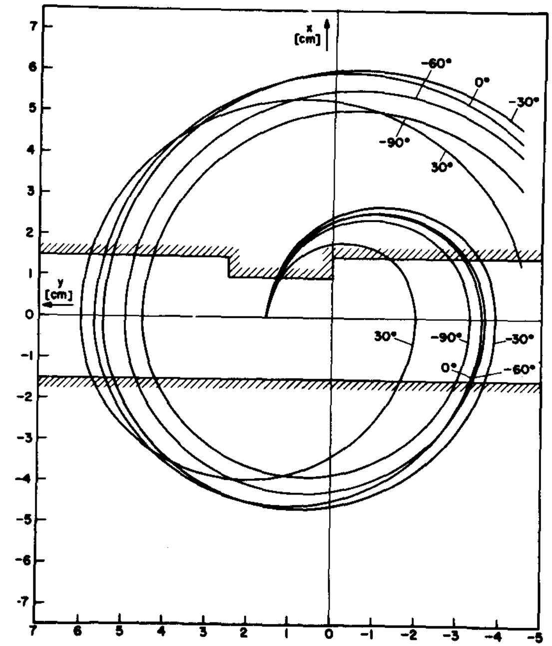

Fig. 2 Initial H^- trajectories in a narrow-gap geometry; dee voltage $U_0 = 70$ kV, injection voltage $U_i = 0$.

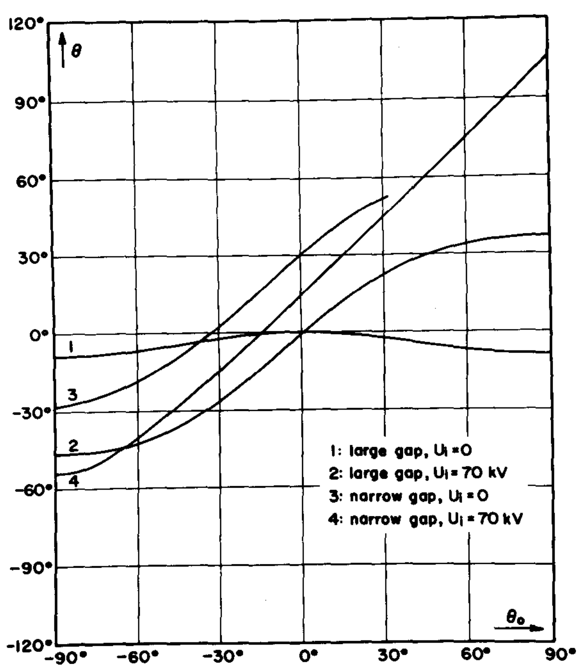

Fig. 3 Phase ϑ of the ions at the end of the first turn as a function of starting phase ϑ_0 in four different cases of central geometry.

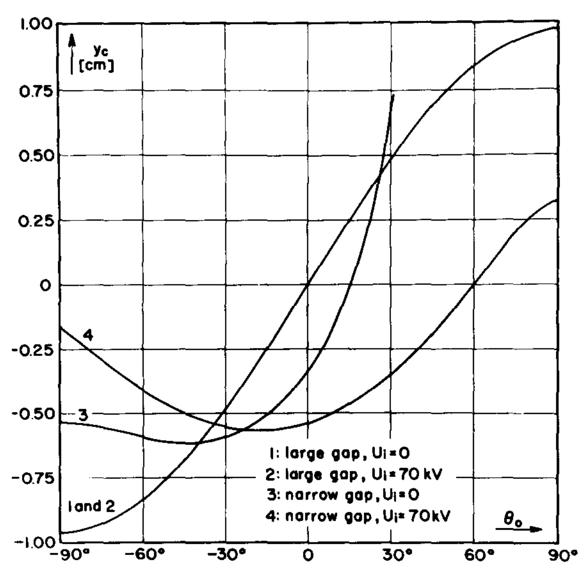

Fig. 4 Center-point coordinates y_c at the end of the first turn as a function of starting phase ϑ_0.

defined by the radius of curvature r_1 after the first gap crossing, i.e.,

$$y_c = -r_1 = -r_o \sqrt{U_i/U_o + \cos\vartheta_o} \quad . \tag{3}$$

This formula indicates that, contrary to the large-gap situation, the spread of orbit centers shrinks and hence the acceptable phase interval $\Delta\vartheta_o$ increases with the injection voltage if d.c. injection is employed.

To obtain a quantitative estimate as to the extent of duty-cycle improvement to be achieved by reducing the width of the electric-field region, the particular case of an isochronous cyclotron, accelerating H^- ions, was considered, assuming U_o = 70 kV and B = 13.774 kG. Calculations were made for (a) a large electric-field gap of total width 2d = 8.0 cm and (b) a small gap with 2d = 3.0 cm and a source-to-puller spacing of 1.0 cm. Both normal injection (U_i = 0) and d.c. injection with U_i = 70 kV were studied in each case. In the large-gap case formulas (1) and (2) were used (with d = 4.0 cm and χ = 4.15) to determine phases and orbit centers at the end of the first turn, and the small-gap calculations were made with a computer code[2]. Fig. 2 shows the geometry and initial trajectories in the narrow-gap case with U_i = 0. In contrast to the open-dee situation in Fig. 1 the pattern of motion is very uniform, the particles starting in the interval $-90^\circ \leq \vartheta_o \leq 0^\circ$ being strongly bunched radially. The results of the calculations for the four cases are plotted in the next two figures. These plots demonstrate how the phase-bunching effect (Fig. 3) in the large gap (curve 1) can be substantially reduced by d.c. injection (curve 2) or by decreasing the gap width (curve 3) and, finally, be completely eliminated by d.c. injection in the small-gap case (curve 4), while simultaneously the spread of orbit centers (Fig. 4) is gradually shrinking in a large interval of starting phases. Since in the actual cyclotron one would employ a phase shift between first and second gap crossing to improve electric focusing (the influence of electric defocusing on the duty cycle has been discussed in Ref. 2), two more computer runs in the narrow-gap case were made, one with U_i = 0 and a puller-to-dee angle α of 25° and one with U_i = 70 kV and $\alpha = 15^\circ$. The results, which were quite similar to cases 3 and 4 respectively, are not shown on the figures but are considered in the final comparison of Table I. This table shows the position and the width of the acceptable starting-phase interval $\Delta\vartheta_o$ and the pulse length $\Delta\vartheta$ at the end of the first turn in all six cases for an assumed tolerance limit Δr_{tol} of 0.5 cm. These figures demonstrate the significant improvements of duty cycle which can be achieved by limiting the electric field, i.e., from less than 1% in the ideal phase-bunching situation (case 1) to 24% in case 5 and finally to the very high value of 38% in the case of d.c. injection. This improvement is associated with a remarkable increase of the acceptable starting-phase interval $\Delta\vartheta_o$ and hence of the amount of useful beam.

Table I

Acceptable starting-phase interval and duty cycle for different situations of central geometry with Δr_{tol} = 0.5 cm.

Case	ϑ_o	$\Delta\vartheta_o$	ϑ	$\Delta\vartheta$	Duty Cycle (%)
1	-15° to 15°	30°	-0.6° to 0°	0.6°	0.17
2	-15° to 15°	30°	-13° to 13°	26°	7.2
3	-90° to 17°	107°	-28° to 44°	72°	20.0
4	-90° to 55°	145°	-55° to 70°	125°	34.7
5	-90° to 24°	114°	-8° to 77°	85°	23.6
6	-104° to 58°	162°	-46° to 91°	137°	38.1

Case 1 large gap (width 8 cm), normal injection
Case 2 large gap (width 8 cm), d.c. injection
Case 3 small gap (width 3 cm), puller angle 0, normal injection
Case 4 small gap (width 3 cm), puller angle 0, d.c. injection
Case 5 small gap (width 3 cm), puller angle 25°, normal injection
Case 6 small gap (width 3 cm), puller angle 15°, d.c. injection

Summarizing the results of this analysis, one can say that it should be possible to substantially reduce the phase-bunching effect, which is the main cause for the low duty factor of conventional cyclotrons, and produce a central beam with a pulse length approaching 180° by (a) operating at low magnetic fields and high voltages to increase the radius of curvature, (b) limiting the electric field to a narrow gap by attaching diaphragms to the dees in the center, and (c) if necessary employing a d.c. injection scheme. The duty factor is then limited only by the transmission capability of the magnetic field and the tolerable maximum energy spread of the extracted beam.

References

1. B.L. Cohen, Rev. Sci. Instr. 24, 589 (1953).
2. M. Reiser, MSU-Cyclotron Report 17, March, 1963.

DISCUSSION

VERSTER : You pointed out that the r/d ratio determines phase grouping and also the orbit centre scatter. Some general theory might connect the phase spread and the orbit centre spread using Liouville's theorem. Has anyone formulated such a theory, in a general way?

REISER : We find that if you have phase grouping, in other words, if you have a small

$\Delta\vartheta$ or a small azimuthal extension of the ion pulse or "sausage" of ions, you have a large spread of orbit centres associated with this strong effect of radial bunching and debunching that you saw in the first figure. As you increase $\Delta\vartheta$ the "sausage" of ions extends over a larger azimuth, of course, while its radial dimensions are small. I think this is in agreement with the theorem.

BLOSSER : We would be very pleased to see the people at Philips work out the theory which Verster mentions.