BEAM OPTICS STUDIES FOR THE Mc^2 CYCLOTRON

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From recent beam-extraction studies on the Mc^2 electron analogue, it is estimated that the energy spread in the Mc^2 proton beam will be about 1% and that the phase-space volume will be about 80 mm mrad in both the horizontal (radial) and vertical planes. The divergence of the beam is estimated at about ± 8.8 mrad in both directions. For purposes of these in ial studies the phase-space distribution was taken as shown in Fig. 1. This distribution is more conservative for designing an analyzing magnet than the more probable elliptical distribution would be.

To transport and shape the beam, simple systems of quadrupole-doublet lenses were considered. For a maximum pole-tip field of 10,000 gauss, it was found that the minimum focal length attainable with quadrupole elements, 38 cm long, 12.7 cm aperture, and arranged in doublets, was about 1.5 m. The maximum focal length which could accept the phase-space volume of Fig. 1 was found to be about 4.2 m.

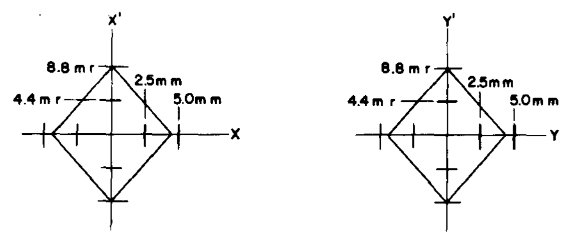

Fig. 1 Assumed phase space of extracted beam.

With a simple two-lens system the horizontal and vertical magnification factors shown in Fig. 2 are possible. These results are for a 3.66 m focal length for lens Q_1. Note that when the first element of both lenses is defocusing in the horizontal plane, the horizontal width of the object can be demagnified by better than a factor of five if lens Q_2 has the minimum focal length of 1.5 m. This arrangement could be used to reshape the beam phase-space distribution (Fig. 1) to : 0.15 cm horizontal extent, ± 25 mrad horizontal divergence, 1.3 cm vertical extent, and 3.5 mrad vertical divergence. The resulting small horizontal width is favorable for energy analysis of the beam. It is assumed that these beam conditions

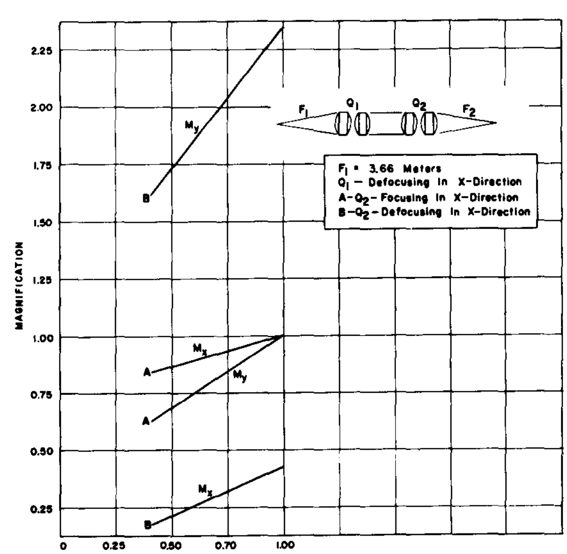

Fig. 2 Magnification for a two-lens system.

(*) Operated for the USAEC by Union Carbide Corporation.

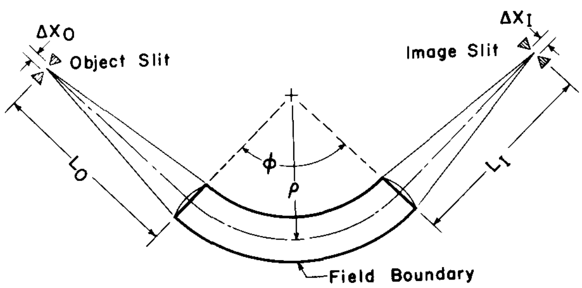

Fig. 3 Simple energy analysis system.

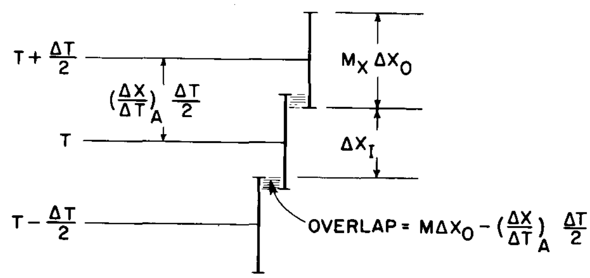

Fig. 4 Illustration of energy dispersion.

can be achieved at the object position of an analyzing magnet.

For a proposed experimental investigation of nuclear structure with the primary proton beam of the Mc^2 Cyclotron, an energy dispersion of 0.4 MeV/cm at the image slit would be desirable. To achieve this dispersion criterion, the simple analysis system shown in Fig. 3 is considered. It consists of an object slit, an analyzing magnet with a field-shape index $n = 1/2$, and an image slit. The width at the image slit of a slice of beam ΔT broad in energy is related to the width of the beam at the object position, ΔX_0, and the dispersion of the analyzing magnet, $(\Delta X/\Delta T)_A$, by :

$$\Delta X_I = \Delta T(\Delta X/\Delta T)_A - M_x \Delta X_0 , \qquad (1)$$

where M_x is the horizontal magnification of the analysis system, and aberrations are neglected. Fig. 4 illustrates the conditions at the image slit that lead to Eq. (1). As stated previously, it is desired that ΔX_I be 2.5 cm for an energy spread of 1 MeV.

The energy dispersion produced by an $n = 1/2$ magnet of radius ρ is given by

$$(\Delta X/\Delta T)_A = \frac{\gamma}{\gamma + 1} \, \frac{2\rho(1 + M_x)}{T} , \qquad (2)$$

where $T = m_0(\gamma - 1)c^2$, and m_0 is the rest mass of a beam particle. The magnification is given by

$$M_x = M_y = (L_I/\sqrt{2}\,\rho)\sin(\phi/\sqrt{2}) - \cos(\phi/\sqrt{2}), \qquad (3)$$

where L_I is the image distance and ϕ is tha angle of bend, as shown in Fig. 3. The relations (2) and (3) were used in plotting the radius of curvature and the object-slit width required to give 0.4 MeV/cm energy dispersion for 810 MeV protons, as shown in Fig. 5. Note that for a given object width a required energy dispersion can be attained with a smaller radius of curvature if an increase in magnification can be tolerated. For a mean field of 10,000 gauss, the radius of curvature would be fixed at 492 cm. If the horizontal extend of the beam at the object position can be limited to 0.15 cm by beam optical means, then the desired energy dispersion can be attained when the analysis system is operated to produce a magnification of about 2.7.

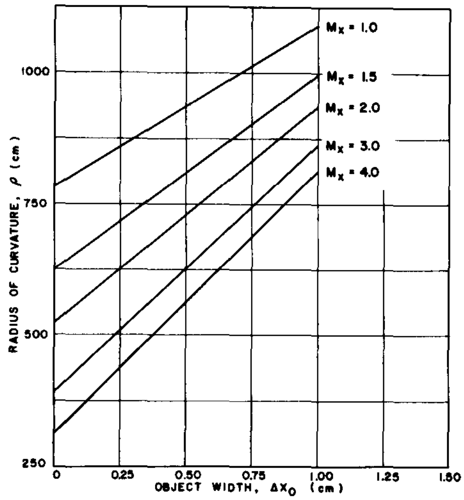

Fig. 5 Conditions to achieve 0.4 MeV/cm energy dispersion with an n = 1/2 magnet.

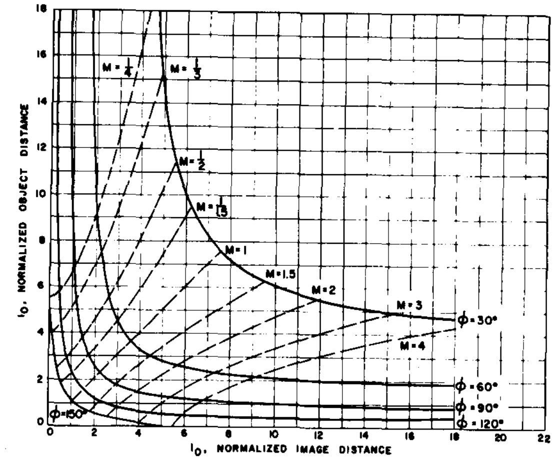

Fig. 6 Double-focusing condition for an n = 1/2 magnet.

To limit the vertical extent of the beam so that it can be transmitted through a magnet with a 12.7 cm gap, the entrance edge of the magnet should be no more than about 124 cm from the object slit. The remaining features of the analysis system are determined by the condition for double focusing :

$$\sqrt{2}\tan(\phi/\sqrt{2}) = 2(\ell_0 + \ell_I)/(\ell_0\ell_I - 2), \qquad (4)$$

where ℓ_0 and ℓ_I are object and image distances respectively in units of radius of curvature.

A plot of Eq. (4) in terms of normalized image and object distances is shown in Fig. 6; the magnification contours result from Eq. (3). It will be noticed that for a value of ℓ_0 = 124 cm/492 cm = 0.25 and a magnification of 2.7 the double-focusing condition is satisfied for a bending angle of 2.5 rad and an image distance L_I = 3.8 x 492 cm = 1870 cm. In this design, the maximum horizontal beam width would be about 71 cm and would occur within the magnet. Because of operation at a magnification of 2.7, the beam would still be about 50 cm wide in the horizontal plane at the exit end of the magnet. At the image position, the vertical extent of the beam is about 3.5 cm and the horizontal extent is determined by energy resolution requirements.

Thus, if aberrations can be made unimportant by magnet design and by final shimming, the above system would provide the desired energy dispersion within a reasonable spot size and with high transmission.